



Intrinsic features of flow-induced stability of a square cylinder

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Vortex-induced vibrations and galloping of an elastically mounted square cylinder are investigated for cylinder mass ratio $m^* = 2\text{--}50$, damping ratio $\zeta = 0\text{--}1.0$, mass-damping ratio $m^*\zeta = 0\text{--}50$ and flow reduced velocity $U_r = 1\text{--}80$. We home in on the effects of m^* , ζ , $m^*\zeta$, $(m^* + m_{a0}^*)\zeta$ and $(m^* + m_{ae}^*)\zeta$ on the critical reduced velocity U_{rc} marking the onset of galloping, where m_{a0}^* is the quiescent-fluid added mass ratio and m_{ae}^* is the effective added mass ratio. Vibration responses, forces, vibration frequencies and added mass ratios are studied and discussed. The different branches of vortex-induced vibrations have different dependencies of m_{ae}^* on U_r . The m_{ae}^* in the initial branch is positive and drops rapidly with U_r , but that in the lower branch is negative and declines gently. In the galloping regime, m_{ae}^* jumps from negative to positive at the onset of galloping, declining slightly with increasing U_r . Our results and prediction equations show that when $\zeta = 0$, U_{rc} is independent of m^* for $m^* \geq 5$, albeit slightly higher for $m^* = 3$. The latter is ascribed to mode competition. When $\zeta > 0$, U_{rc} linearly increases with increasing ζ . Detailed analysis substantiates that $m^*\zeta$ or $(m^* + m_{a0}^*)\zeta$ does not serve as the unique criterion to predict the galloping occurrence. Here, we propose a new combined mass-damping parameter $(m^* + m_{ae}^*)\zeta$ in the relationship between galloping onsets and structural properties, which successfully scales all data of U_{rc} at different m^* and ζ values.

Key words: flow–structure interactions, vortex shedding

1. Introduction

Flow-induced vibration (FIV) of slender structures is a result of fluid–structure interactions, which can be observed in many engineering fields, such as offshore drilling platforms and suspension bridges. When fluid flow passes these slender structures, they may undergo vortex-induced vibration (VIV) and/or galloping, both of which are commonly known as FIV. Blevins (1990) defined galloping as a large-amplitude and

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low-frequency self-excited vibration of non-circular structures at a reduced velocity higher than a threshold. The galloping vibration is initiated when the overall damping of the system is negative because of negative flow-induced damping, given that the structural damping is positive (Blevins 1990; Païdoussis, Price & Delangre 2010; Qin, Alam & Zhou 2017, 2019).

Structural mass ratio m^* , damping ratio ζ , natural frequency f_n , Reynolds number Re , reduced velocity U_r , structure shape and turbulent intensity all play important roles in galloping. Barrero-Gil, Sanz-Andres & Roura (2009) concluded that a square cylinder cannot gallop at $Re < 159$ with $m^*\zeta = 0.25, 1.25$ and 2.5 , while Joly, Etienne & Pelletier (2012) observed galloping for $Re > 140$ with $m^* = 20, \zeta = 0$. The latter threshold Re value is much lower than the former. When Re was systematically increased with $m^* = 10, \zeta = 0$, Sen & Mittal (2011) found that the oscillation frequency abruptly decreases at $Re > 170$ when a square cylinder vibrates in both transverse and streamwise directions, which points to the occurrence of galloping instability. The galloping vibration regime ($Re > 170$) corresponded to 2S (two single vortices in one oscillation period) and 2P (two paired vortices) vortex shedding modes. The 2S and 2P modes were associated with high-amplitude ($Re < 215$) and very-high-amplitude ($Re \geq 215$) vibrations, respectively. At $Re \geq 220$, the transverse vibration amplitude when the cylinder was only allowed to vibrate in the transverse direction was much smaller than that when it was allowed to vibrate in both transverse and streamwise directions. For the transverse vibration, the 2S mode prevailed. Sen & Mittal (2015) investigated the influence of m^* on galloping instability for $m^* = 1, 5, 10$ and $20, \zeta = 0$ and $Re = 50\text{--}250$. Galloping was only found for high $m^* = 5, 10$ and 20 with the onset of galloping at $Re = 186, 174$ and 169 , respectively. With increasing m^* , the critical Re (or critical reduced velocity U_{rc}) marking the onset of lock-in and galloping increases and decreases, respectively. They varied U_r by increasing the free-stream flow velocity (i.e. Re), where the Strouhal number and forces of a fixed cylinder are very sensitive to Re . Given that they increased U_r by varying Re , their m^* effect is contaminated by the Re effect, as is shown later. They identified three vortex-shedding modes: 2S, C(2S) and 2P + 2S. Modes 2S and C(2S) are involved in VIV, whereas the galloping vibration features 2S mode for $A^* (= A/D) < 0.7$ and 2P + 2S mode for $A^* > 0.7$, where A is the vibration amplitude and D is the cylinder width.

For a square cylinder with $m^* = 10$, Bhatt & Alam (2018) reported the occurrence of galloping at $Re = 200$ but not at $Re = 100$. In the case of galloping vibrations, they observed N(2S) vortex shedding mode, where the wake is akin to the wake of a 2S mode. Zhao (2015) examined the FIVs of a square cylinder and a rectangular cylinder at $m^* = 10, Re = 200$, and found that U_{rc} for galloping increases with increasing cylinder cross-sectional aspect ratio α , with $U_{rc} = 13$ for the square cylinder. Zhao *et al.* (2019) experimentally investigated the effect of m^* on the vibration response of a square cylinder at $\zeta = (1.31\text{--}2.58) \times 10^{-3}$. When m^* was increased, the combined VIV–galloping response weakened and ceased to exist for $m^* \geq 11.31$. Sourav & Sen (2019) for $Re \leq 250$ identified the threshold $m^* = 3.4$ above which VIV and galloping were separated. Han & Langre (2022) at low Reynolds numbers pointed out that galloping may occur even for a very low mass ratio, and there is no critical mass ratio for galloping of a square cylinder. The galloping onset is delayed at a low mass ratio.

The effects of m^* , ζ and $m^*\zeta$ on the response of a freely vibrating circular cylinder were numerically investigated by Bahmani & Akbari (2010) for $Re = 80\text{--}160$. When m^* or ζ was increased, both A^* and U_r ranges of VIV shrank. They further noted that the oscillator system behaves nonlinearly with m^* and ζ . Rabiee & Farahani (2020) numerically investigated FIV of a heated square cylinder with $\zeta = 0, 0.01, 0.05, 0.1, 0.25$

and 0.5 at $Re = 80, 90, 220$ and 250 . They also observed an inverse relationship between A^* and ζ . At $Re = 220$ involving galloping vibrations, as ζ increases from 0 to 0.1 and 0.25, A^* reduces by 36 % and 91 %, respectively.

As Re , U_r , m^* and ζ all play key roles in vibration generation, they all must simultaneously exceed their threshold values for the onset of galloping. In general, if any of these parameters falls short of the corresponding threshold value with the remaining parameters being sufficiently high, the cylinder will execute VIV alone, and the transition to galloping will not take place.

Sen & Mittal (2016) found that the wake mode type is a direct function of the synchronization type; it is a 2S or C(2S) mode for 1:1 synchronization between the oscillation and shedding frequencies, a 2P + 2S mode when the synchronization is very close or equal to 1:3 and a 3(2S) or unstable 2S mode when the synchronization is a little far from 1:3. Zhao *et al.* (2014) experimentally observed an increased A^* for 1:3 synchronization, deviating from the trend of the typical galloping response. Yao & Jaiman (2017) found a low-frequency galloping instability and a kink in the amplitude response around 1:3 synchronization for a triangular cylinder. Daniel, Todd & Yahya (2021) observed amplitude deviations from a typical galloping response at higher synchronizations.

Beating phenomena in instantaneous vibration responses were observed in some studies on VIV of a circular cylinder. Khalak & Williamson (1999) indicated that this beating is associated with the transition between the lower and upper branches, and hence between the 2S and 2P modes. Voorhees *et al.* (2008) pointed out that the difference in frequencies between the cylinder vibration and the vortex shedding gives rise to the beating. Shen, Chan & Wei (2018) reported similar observations. The modulation frequency equals the difference between the two frequencies. In the galloping branch, the vibration of a square cylinder is generally quasi-periodic, but it is periodic only when the vortex-shedding to oscillation frequency ratio $f_s/f_o = 3$ (Zhao *et al.* 2014; Zhao 2015; Sen & Mittal 2016). We focus on not only the effect of m^* but also ζ and a combined mass-damping parameter on the vibration responses of a square cylinder, and revisit the mechanisms and roles of the 1:3 and 1:5 synchronizations further.

1.1. Objective

The transverse dynamic response of a cylinder can be simplified as a spring–damper–mass system expressed in non-dimensional form as

$$\ddot{Y} + 4\pi \left(\frac{1}{U_r} \right) \zeta \dot{Y} + \left(\frac{2\pi}{U_r} \right)^2 Y = \frac{C_L}{2m^*}, \quad (1.1)$$

where Y , \dot{Y} and \ddot{Y} are respectively the instantaneous displacement, velocity and acceleration of the cylinder in the transverse direction. The non-dimensional parameters are defined and listed in table 1, where c and k are the damping and spring constants of the cylinder system, respectively. Parameter m is the mass of the cylinder per unit length and ρ is the density of the fluid. Terms f_o and f_s are the cylinder vibration frequency and vortex shedding frequency, respectively. Force F_L is the lift force acting on the unit span of the cylinder.

In (1.1), U_r , ζ and m^* are the parameters involved in FIVs while C_L is the flow-induced lift force that depends on U_r , ζ , m^* and Y . One can estimate C_L from (1.1) when U_r , ζ , m^* and Y are all known. Undoubtedly, ζ and m^* are the structural parameters, the latter

Amplitude ratio: $A^* = A/D$	Lift force coefficient: $C_L = F_L / (0.5\rho DU_\infty^2)$
Reduced velocity: $U_r = U_\infty / (f_n D)$	Normalized time: $t^* = tU_\infty / D$
Mass ratio: $m^* = m / (\rho D^2)$	Normalized pressure: $p^* = p / (\rho U_\infty^2)$
Damping ratio: $\zeta = c / (2\sqrt{km})$	Frequency ratio: $f_r = f_o / f_n$
Natural frequency: $f_n = 1 / 2\pi\sqrt{k/m}$	Reynolds number: $Re = \rho U_\infty D / \mu$
Strouhal number: $St = f_s D / U_\infty$	Critical damping: $c_c = 4\pi m f_n$

Table 1. Definitions of non-dimensional parameters.

indicating the heaviness of the structure. On the other hand, $U_r (= U_\infty / f_n D)$, as defined in table 1), consists of a structural parameter (i.e. f_n) and a flow parameter (i.e. U_∞ / D), the former representing the structural rigidity or strength (how many oscillations per second) while the latter indicating the flow strength (how many D the flow can travel per second) or forcing frequency in other words. Thus, U_r represents a competition between the structural rigidity and flow strength. That is, the smaller the U_r , the higher the structural rigidity and/or the smaller the flow strength, and *vice versa*. If U_r with a constant $f_n D$ is varied by changing U_∞ (i.e. Re), it acts as a flow parameter. Conversely, if U_r is varied via f_n , keeping U_∞ (i.e. Re) constant, it acts as a structural parameter. When the effect of U_r on A^* (i.e. A^* versus U_r graph) is investigated with increasing U_r via U_∞ , the effect involves the combined influence of U_r and Re . On the other hand, when the same is done via f_n , it solely involves the influence of U_r . When Re is kept constant, a better presentation of the U_r effect is provided.

We aim to investigate the effects of m^* ($= 2-50$), ζ ($= 0-1$), $m^*\zeta$ ($= 0-50$) and U_r ($= 1-80$) on vibration and frequency responses, flow structures, added mass and forces of an elastically mounted square cylinder at $Re = 170$. This investigation further explores whether $m^*\zeta$ is an appropriate parameter to characterize FIV. If not, what is the appropriate parameter? To guarantee the occurrence of galloping and avoid the three-dimensional effect, we select $Re = 170$ based on investigations by Barrero-Gil *et al.* (2009), Joly *et al.* (2012) and Sen & Mittal (2011).

2. Methodology

2.1. Governing equations and numerical set-up

The flow is assumed to be incompressible, viscid and two-dimensional while the physical properties of the fluid are constant. The governing equations to simulate the flow field around an elastically mounted rigid square cylinder are the continuity and Navier–Stokes equations, which can be written in a non-dimensional form as

$$\text{continuity: } \nabla \mathbf{u}^* = 0 \tag{2.1}$$

and

$$\text{momentum: } \frac{\partial \mathbf{u}^*}{\partial t^*} + \mathbf{u}^* \nabla \mathbf{u}^* = -\nabla p^* + \frac{1}{Re} \nabla^2 \mathbf{u}^*, \tag{2.2}$$

where p^* ($= p / \rho U_\infty^2$), \mathbf{u}^* and t^* ($= tU_\infty / D$) are the normalized static pressure, normalized velocity and normalized time, respectively. Term \mathbf{u}^* is composed of two velocity components $\mathbf{u}^* = (u^*, v^*) = (u/U_\infty, v/U_\infty)$ in the streamwise and transverse directions, respectively. Parameter D is the width of the square cylinder. Reynolds number is defined as $Re = \rho U_\infty D / \mu$, where μ is the viscosity of the fluid, ρ is the fluid density and U_∞ is the

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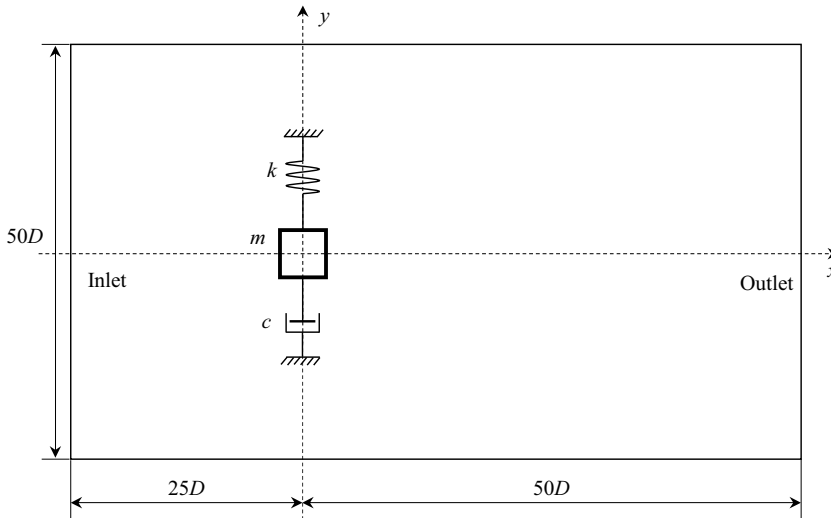


Figure 1. A schematic of the flow configuration and computational domain.

free-stream flow velocity. The reduced velocity $U_r = U_\infty / (f_n D)$, where $f_n (= 1/2\pi\sqrt{k/m})$ is the natural frequency of the cylinder.

Ansys-Fluent 17.2 based on the finite-volume method is utilized as the solver. The second-order upwind scheme and the central differencing scheme are used to discretize the convective and diffusion terms, respectively. A first-order implicit formulation is adopted for time discretization because of its unconditional stability (Manson, Pender & Wallis 1996) and compatibility with the dynamic mesh (Shaaban & Mohany 2018). The pressure-correction-based iterative algorithm SIMPLE (semi-implicit method for pressure-linked equations) proposed by Patankar (1980) is employed for coupling the velocity and pressure fields.

The dynamic response of the cylinder system is given by (1.1). The fourth-order Runge–Kutta method is employed to solve this second-order differential equation at each time step, where the fluid forces acting on the cylinder are composed of pressure and shear stress forces, obtained directly from the ANSYS-Fluent 17.2 solver. The lift force is provided on the right-hand side of (1.1), which is integrated to advance the cylinder motion. In the next time step, the cylinder displacement, velocity and acceleration are updated, and the equation of fluid motion is integrated to complete the fluid–solid coupling. At each time step, the deformation of the computational domain is managed by the dynamic meshing tool in ANSYS-Fluent 17.2, with the mesh updated using the Laplace smoothing and layering methods. In the subsequent time step, the field variables (u , v and p) within the entire computational domain are updated based on each node's information, which involves solving the governing equations: the continuity (2.1) and the Navier–Stokes (2.2) equations.

2.2. Domain size, grid resolution and time-step dependence test

A schematic of the cylinder configuration is shown in figure 1. The computational domain is rectangular, scaled with D as a length of $L_u + L_d = 75D$ in the flow direction and a height of $H = 50D$ in the transverse direction. It gives a blockage ratio $D/H = 2.0\%$, which is less than the 3.3% used in Zheng & Alam (2017) and Bhatt & Alam (2018). The boundary

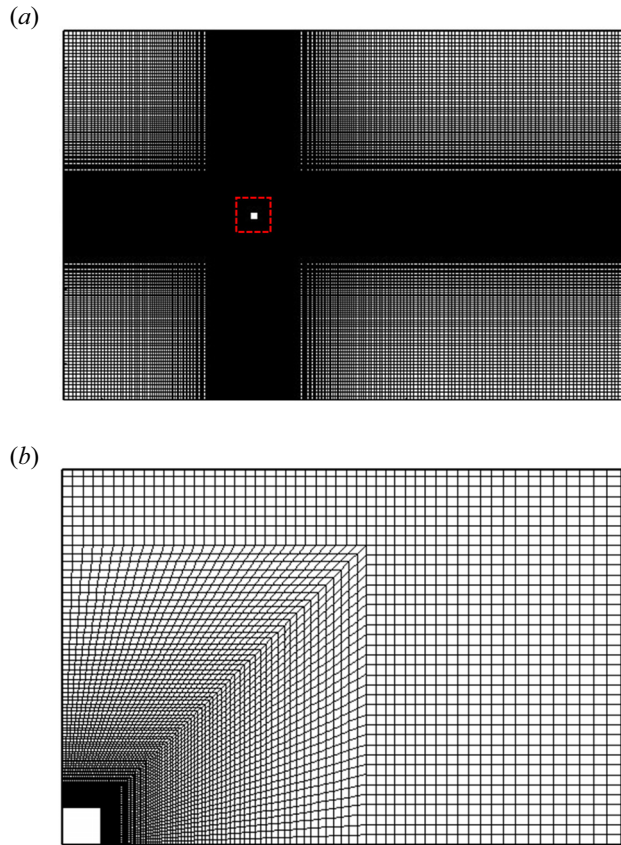


Figure 2. (a) Global view and (b) zoomed-in view of the meshes around the square cylinder.

conditions at the inlet are $u^* = 1.0$, $v^* = 0$. Symmetry conditions ($v^* = 0$, $\partial u^*/\partial y^* = 0$ and $\partial p^*/\partial y^* = 0$) are applied to both upper and lower boundaries. The inlet boundary is set as the velocity inlet ($u^* = 1$, $v^* = 0$), while the outlet boundary is given as $\partial u^*/\partial x^* = 0$, $\partial v^*/\partial x^* = 0$ and $\partial p^*/\partial x^* = 0$. No-slip boundary conditions are deployed on the cylinder surface. The initial flow field in the computation domain is given as $u^* = 1$, $v^* = 0$ and $p^* = 0$.

Figure 2(a) shows a global view of grid distributions in the entire computational domain, and figure 2(b) shows a zoom-in view of grid distributions around a quadrant of the square cylinder. The entire flow field is given a structured quadrangular grid system. The oscillating cylinder is surrounded by a central box of size $8D \times 8D$ (figure 2a). The central box of high grid density moves with the vibrating cylinder, while the remaining grids in the domain are stationary. A dynamic mesh scheme is utilized to move the cylinder including the central box and to adjust the mesh accordingly. The motion of the cylinder is defined by the user-defined function. At each time step, smoothing and layering are applied at each new position of the cylinder.

Grid and time-step independence tests were performed for a vibrating cylinder with $m^* = 10$ and $U_r = 7$ (table 2). Four grids M1 = 22 292, M2 = 44 799, M3 = 76 622 and M4 = 145 232 were tested, each with time steps $\Delta t = 0.001$, 0.002, and 0.005. The values of A^* , $f_r = f_o/f_n$ and C_L' are presented in table 2, where f_o was estimated from the fast Fourier transform (FFT) of the Y signal, A^* was obtained from the root-mean-square value

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Δt (s)	Number of nodes	A^*	f_r	C'_L
0.005	22 292 (M1)	0.0829 (21.11 %)	1.0838 (3.87 %)	0.1603 (27.14 %)
	44 799 (M2)	0.0654 (9.17 %)	1.1258 (2.65 %)	0.2038 (3.39 %)
	76 622 (M3)	0.0594 (4.71 %)	1.1556 (0.32 %)	0.2107 (0.95 %)
	145 232 (M4)	0.0566	1.1594	0.2127
0.002	22 292 (M1)	0.0833 (21.85 %)	1.0996 (4.49 %)	0.1905 (23.52 %)
	44 799 (M2)	0.0651 (8.60 %)	1.149 (1.61 %)	0.2353 (3.02 %)
	76 622 (M3)	0.0595 (3.87 %)	1.1675 (0.27 %)	0.2424 (0.04 %)
	145 232 (M4)	0.0572	1.1643	0.2423
0.001	22 292 (M1)	0.0853 (22.74 %)	1.0823 (6.67 %)	0.1939 (26.87 %)
	44 799 (M2)	0.0659 (9.41 %)	1.1545 (0.95 %)	0.246 (2.97 %)
	76 622 (M3)	0.0597 (3.52 %)	1.1654 (0.25 %)	0.2533 (0.04 %)
	145 232 (M4)	0.0576	1.1625	0.2534

Table 2. Grid and time-step independence tests for vibrating cylinder at $Re = 170$, $m^* = 10$ and $U_r = 7$.

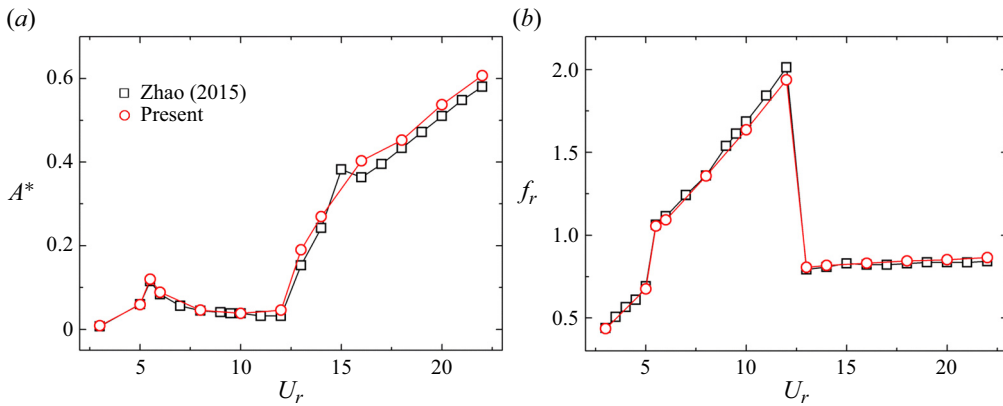


Figure 3. Comparisons of (a) response amplitude A^* and (b) frequency ratio f_r ($=f_o/f_n$) of a square cylinder at mass ratio $m^* = 10$, damping ratio $\zeta = 0$ for $Re = 200$.

of Y as $A^* = Y_{rms} \times \sqrt{2}/D$ and C'_L was calculated from the lift signal. The percentage deviation is provided in parentheses with increasing node numbers. When the grid system increases from M1 to M4, the deviations get smaller. The deviations in the results between M3 and M4 for $\Delta t = 0.001$, 0.002 and 0.005 are less than 3.52 %, 3.87 % and 4.71 %, respectively. Grid M3 is considered, given that the node number for M4 is approximately twice that for M3 and the deviations in the results between M4 and M3 are small. The deviations in the results for M3 are less than 4.3 % between $\Delta t = 0.002$ and 0.001 , while it is 13.08 % between $\Delta t = 0.005$ and 0.002 . Grid M3 and $\Delta t = 0.002$ are thus chosen for further successive simulations. There were 240 nodes on each side of the cylinder surface for M3. The first layer of elements had a radial distance of $0.005D$ from the cylinder surface.

2.3. Validation

In addition to the grid and time-step validation, the model for FIVs was validated for a square cylinder vibration at $Re = 200$ for $U_r = 3-22$. This $Re = 200$ is chosen to compare the results from Zhao (2015). Figure 3 compares the vibration and frequency responses

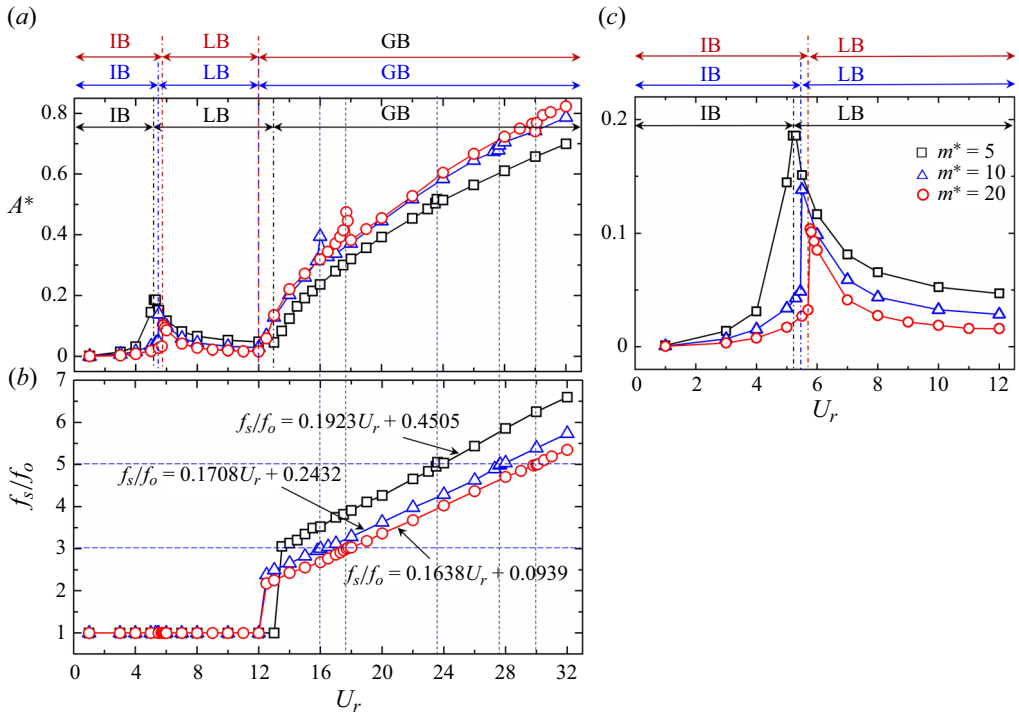


Figure 4. Dependence on reduced velocity U_r and mass ratio m^* of (a) response amplitude A^* and (b) f_s/f_o . (c) Zoom-in view of A^*-U_r plot in (a) for $U_r = 1-12$. Here, $\zeta = 0$ and $Re = 170$.

between the present and Zhao’s works for a cylinder with $m^* = 10$, $\zeta = 0$, and $Re = 200$. The present results of A^* and f_r concur well with those of Zhao (2015).

3. Results and discussion

3.1. Effect of m^* on vibration response

We present here the effect of m^* on vibration and frequency responses at $\zeta = 0$ (figure 4). The cylinder responses can be divided into three branches, namely the initial branch (IB), lower branch (LB) and galloping branch (GB). For $m^* = 20$, the vortex excitation regime (including IB and LB) appears when $U_r \leq 12$, whereas GB occurs when $U_r > 12$. Following Bhatt & Alam (2018), the identifications of IB and LB are made based on the relationship between St and U_r shown in figure 5(a). The Strouhal number St – the dimensionless vortex shedding frequency of the vibrating cylinder – was estimated from the power spectral density functions of the lift signals. Here, St_0 is used to denote the dimensionless shedding frequency of a fixed cylinder (figure 5a). The IB corresponds to $St < St_0$ while the LB to $St > St_0$. The boundary between IB and LB is characterized by a jump in St , e.g. between $U_r = 5.7$ and 5.77 for $m^* = 20$, between $U_r = 5.45$ and 5.5 for $m^* = 10$ and between $U_r = 5.2$ and 5.3 for $m^* = 5$ (figure 5a). The value of A^* increases with increasing U_r in IB, which is accompanied by a declining St and a constant $f_s/f_o = 1.0$ (figures 4a,b and 5a). The $f_s/f_o = 1.0$ indicates the lock-in (figure 4b). On the other hand, f_r grows with increasing U_r (figure 5b).

A marked rise in A^* also characterizes the boundary between IB and LB (figure 4c). In the LB regime, A^* after reaching a peak declines with increasing U_r . The LB is further

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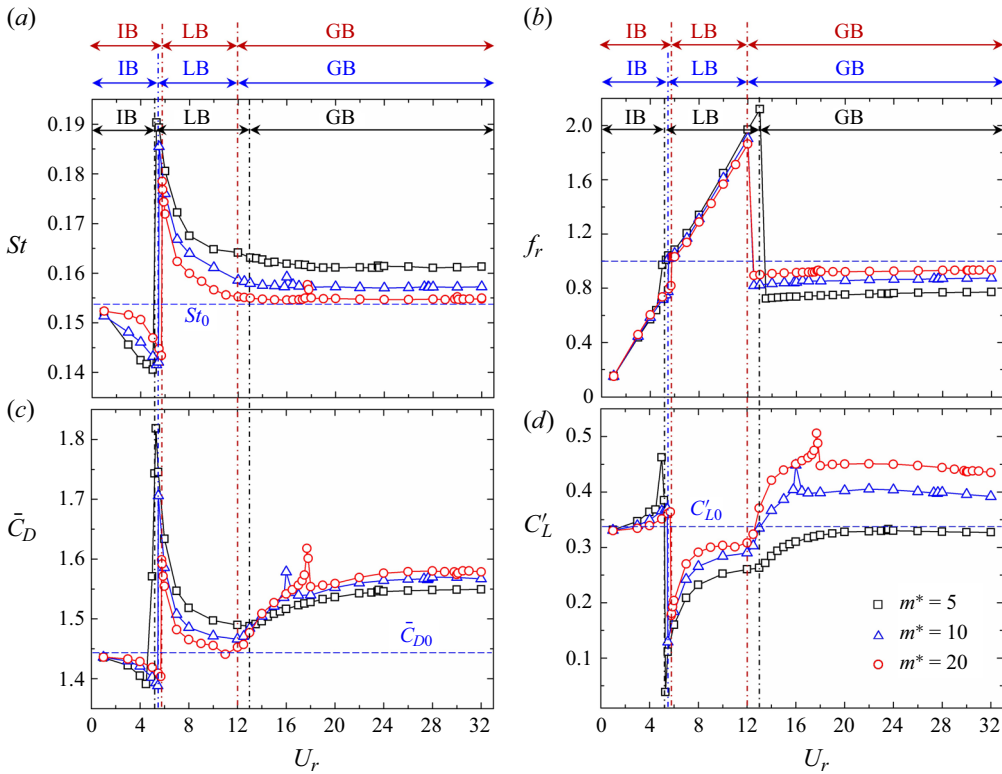


Figure 5. Variations of (a) Strouhal number St , (b) oscillation frequency ratio f_r , (c) time-mean drag coefficient \bar{C}_D and (d) fluctuating lift C'_L with reduced velocity U_r for different mass ratio m^* values. Here, damping ratio $\zeta = 0$ and $Re = 170$.

characterized by $f_s/f_o = 1.0$ (lock-in, figure 4b), declining St and increasing f_r . These three attributes are similar to those in IB. How do IB and LB differ? Why are there jumps in St and f_r between IB and LB? The distinct phase lag ϕ between C_L and Y^* ($= Y/D$) answers these two questions: $\phi = 0^\circ$ in IB and $\phi = 180^\circ$ in LB (not shown), given $\zeta = 0$. The jump in ϕ from 0° to 180° occurs at the boundary between IB and LB.

The GB can be identified from the recovery in the increase of A^* (figure 4a) and jump in f_s/f_o (figure 4b) or from the plunge in f_r (figure 5b). Therefore, the critical reduced velocity U_{rc} for the onset of galloping is 13, 12 and 12 for $m^* = 5, 10$ and 20, respectively. In GB, A^* increases rapidly with U_r . Deviating from this trend, A^* surges at 1:3 (i.e. $f_s/f_o = 3$) and 1:5 (i.e. $f_s/f_o = 5$) synchronizations (figure 4a,b). The surge is stronger at 1:3 synchronization than at 1:5 synchronization. The onset of galloping for $m^* = 5$ coincides with 1:3 synchronization. Sen & Mittal (2016), Yao & Jaiman (2017) and Daniel *et al.* (2021) all observed 1:3 synchronization. Here, we find that 1:5 synchronization also plays a role in the vibration responses. The vibration response is generally quasi-periodic (amplitude varying with time) but is periodic (constant amplitude) for 1:3 and 1:5 synchronizations. Sen & Mittal (2016) reported that the vibration response for $m^* = 5$ was quasi-periodic for the entire GB because they ignored the 1:5 synchronization.

Term f_s/f_o in GB generally increases linearly with increasing U_r except for the occurrence of synchronizations around $f_s/f_o = 3$ or 5 (figure 4b). The linear relationship

between f_s/f_o and U_r can be expressed as

$$f_s/f_o = 0.1638U_r + 0.0939 \quad \text{for } m^* = 20, \quad (3.1a)$$

$$f_s/f_o = 0.1708U_r + 0.2432 \quad \text{for } m^* = 10, \quad (3.1b)$$

$$f_s/f_o = 0.1923U_r + 0.4505 \quad \text{for } m^* = 5. \quad (3.1c)$$

The value of St is more or less independent of U_r in the galloping regime while f_r being smaller than 1.0 slightly increases with U_r (figure 5a,b).

Parameter m^* plays a crucial role in A^* . It is a well-accepted argument that an increase in m^* reduces A^* (e.g. Bahmani & Akbari 2010). In the vortex excitation regime (including IB and LB), a larger m^* leads to a smaller A^* and a postponement of the A^* peak. On the other hand, in the GB regime, the scenario is the opposite: the larger the value of m^* , the larger is A^* and the smaller is U_r for galloping onset (figure 4a). That is, the vortex excitation regime shrinks, and the onset of galloping advances when m^* is increased. The surge in A^* at 1:3 synchronization grows with increasing m^* . In addition, a larger m^* in the GB regime leads to (i) reductions in f_s/f_o and gradient of f_s/f_o with U_r (figure 4b; (3.1)), (ii) a decrease of St and (iii) an augmentation of f_r .

The maximum A^* of a square cylinder in the VIV regime is much smaller than that of a circular cylinder case (Sen & Mittal 2011, 2015; Li *et al.* 2019). Li *et al.* (2019) for a square cylinder at $Re = 150$ demonstrated that the maximum A^* is 0.17, 0.14 and 0.11 at $m^* = 5, 10$ and 20, respectively. Zhao (2015) observed the maximum A^* of 0.11 at $m^* = 10$ and $Re = 200$. The maximum A^* in our study ($Re = 170$) is 0.18, 0.14 and 0.1 at $m^* = 5, 10$ and 20, respectively. Thus, the observed influence of m^* on the vibration response within the VIV regime in our study appears to be consistent with the previous findings. Experimental results, typically obtained at higher Re values, feature larger A^* values in the galloping regime (Zhao *et al.* 2014) than the numerical results at lower Re values.

3.2. Effect of m^* on fluid forces

Figure 5(c,d) illustrates dependencies of \bar{C}_D and C'_L on U_r for different m^* values. The corresponding \bar{C}_{D0} and C'_{L0} for the fixed cylinder are represented by blue dashed lines. In IB, with increasing U_r , \bar{C}_D ($< \bar{C}_{D0}$) declines and C'_L ($> C'_{L0}$) grows for all m^* values. A larger m^* corresponds to a larger \bar{C}_D but a smaller C'_L . The scenario is the opposite in LB, i.e. $\bar{C}_D > \bar{C}_{D0}$ and $C'_L < C'_{L0}$, the former declining and the latter growing. On the other hand, \bar{C}_D augments in GB, with surges at 1:3 synchronization. Although rising in the early part of GB (i.e. up to 1:3 synchronization for $m^* = 10$ and 20, and 1:5 synchronization for $m^* = 5$), C'_L does not appreciably vary in the late part of GB.

Parameter m^* has distinct influences in different regimes. Increasing m^* renders increased \bar{C}_D and decreased C'_L in IB but the reverse correspondence in LB. In GB, relationships of \bar{C}_D and C'_L with m^* are straightforward, both increasing with m^* . A fixed cylinder, complementing $f_n = \infty$, corresponds to $U_r = 0$. Parameters \bar{C}_D , C'_L and St all approach their fixed cylinder values when U_r decreases toward $U_r = 0$.

3.2.1. Quiescent fluid added-mass force and flow-induced force

According to Lighthill (1986), the lift force (F_L) can be decomposed into a ‘potential force’ component and a ‘vortex force’ component, where the ‘potential force’ is caused by the potential added-mass force and the ‘vortex force’ is due to the dynamics of vorticity in the flow. Given that the fluid is viscous, Alam (2022) proposed a quiescent-fluid added-mass

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force F_{La0} (replacing the potential force) and a flow-induced force F_{Lf} (replacing the vortex force). Naturally, the magnitude of the quiescent-fluid added-mass force differs from that of the potential force, depending on the body shapes and orientations (Chen, Alam & Zhou 2020). Alam (2022) further proved that the quiescent-fluid added-mass force must be considered to correctly estimate the flow-induced force and the phase lag between force and displacement. The total lift force is expressed as

$$F_L = F_{La0} + F_{Lf}. \quad (3.2)$$

Normalizing all forces by $(1/2\rho U_\infty^2 D)$,

$$C_L(t) = C_{La0}(t) + C_{Lf}(t). \quad (3.3)$$

Here, $C_L(t)$ is the lift force coefficient, defined as $C_L(t) = F_L(t)/(0.5\rho DU_\infty^2)$, and $F_L(t)$ is the total force on the cylinder in the y direction. Coefficients $C_{La0}(t)$ and $C_{Lf}(t)$ are the instantaneous quiescent-fluid added-mass force coefficient and flow-induced force coefficient, respectively.

Force $F_{La0}(t)$ is given by

$$F_{La0}(t) = -m_{a0}\ddot{y}(t) \quad (\text{neglecting quiescent - fluid damping force}). \quad (3.4)$$

Here, m_{a0} is the quiescent-fluid added mass and is expressed as $m_{a0} = m_{a0}^* \times m_d$, where m_{a0}^* is the quiescent-fluid added-mass ratio and $m_d = \rho D^2$ is the fluid mass displaced by the cylinder.

The quiescent-fluid added mass and potential added mass are the added masses measured in the quiescent fluid (still and viscid fluid) and potential fluid (inviscid, incompressible fluid), respectively. The former is generally measured numerically while the latter can be measured both numerically and experimentally by plucking the cylinder in quiescent fluid. When the damping is small, the natural frequency of the cylinder system in a vacuum can be considered as

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}. \quad (3.5)$$

When the cylinder is submerged in a fluid (e.g. water), the natural frequency of the cylinder oscillation changes because of the added mass m_a generated by the fluid. The modified natural frequency f_{nf} of the cylinder in a fluid can be expressed as

$$f_{nf} = \frac{1}{2\pi} \sqrt{\frac{k}{m + m_a}}. \quad (3.6)$$

Combining (3.5) and (3.6),

$$m_a = m \left[\left(\frac{f_n}{f_{nf}} \right)^2 - 1 \right]. \quad (3.7)$$

When f_{nf} is measured directly from the decay of the cylinder oscillation, all the parameters on the right-hand side of (3.7) are known, which leads to the estimation of m_a . See Chen *et al.* (2020) and Alam (2022) for details of measuring m_a . The quiescent-fluid added-mass coefficient m_{a0}^* is coincidentally about 1.0 for a circular cylinder but approximately 1.5 for a square cylinder with zero incidence angle (Chen *et al.* 2020).

Following (3.3), $C_L(t)$ is then decomposed into $C_{La0}(t)$ and $C_{Lf}(t)$, and their corresponding fluctuating (root-mean-square) coefficients C'_{La0} and C'_{Lf} , respectively, are

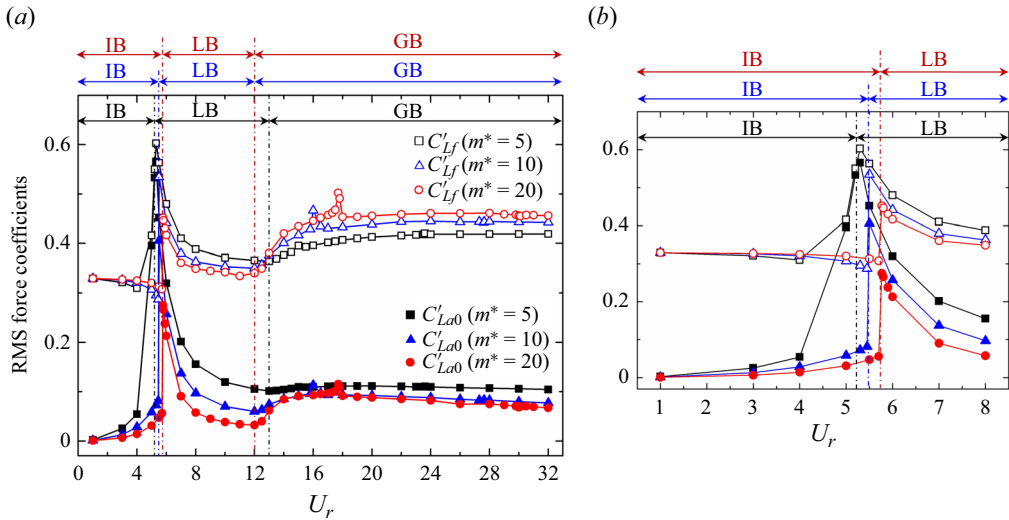


Figure 6. (a) Variations of fluctuating quiescent-fluid added-mass force coefficient C'_{La0} and flow-induced lift coefficient C'_{Lf} with U_r and m^* . (b) Zoomed-in view of (a) for $U_r = 1-8$. Here, $\zeta = 0$ and $Re = 170$.

obtained as presented in figure 6. Coefficient C'_{La0} gets smaller with increasing m^* in all branches. In IB, the growth in C'_{La0} is very high for all m^* values, largely following A^* trends in IB. On the other hand, C'_{Lf} in IB does not change appreciably for the high $m^* = 10$ and 20 but does increase for the low $m^* = 5$. Both C'_{La0} and C'_{Lf} decline in LB for all m^* values. This suggests that the increase of A^* in IB is largely due to the imminent resonance effect (vortex shedding frequency approaching the cylinder natural frequency), while the decrease of A^* in LB results from the decreasing C'_{Lf} and the retreating resonance effect (vortex shedding frequency retreating from the cylinder natural frequency). A larger m^* leads to a smaller C'_{Lf} in LB but the opposite scenario takes place in GB. Interestingly, in GB, although A^* increases with increasing U_r for all m^* values, C'_{Lf} and C'_{La0} gently increase and decrease ($U_r > 16$), respectively. This indicates that flow-induced force (F_{Lf}) and added-mass force (F_{L0}) in GB vary with U_r^{2+} and U_r^{2-} , respectively. There are peaks in C'_{Lf} at $U_r = 16$ and 17.7 for $m^* = 10$ and 20, respectively, where both U_r values correspond to $f_s/f_o = 3$. Similar peaks are also observed for C'_{La0} , albeit weaker than those for C'_{Lf} .

3.3. Role of effective added mass ratio in vibration branches

When a square cylinder oscillates freely in the transverse direction, the effective added mass ratio m_{ae}^* and added damping ratio ζ_a can be presented as

$$m_{ae}^* = \frac{m_{ae}}{\rho D^2} = \frac{F_{L0} \cos \phi}{\rho (2\pi f_o)^2 Y_0 D^2} \tag{3.8}$$

and

$$\zeta_a = \frac{c_a}{c_c} = -\frac{F_{L0} \sin \phi}{8\pi^2 m f_{afn} Y_0}. \tag{3.9}$$

The different branches of VIVs have different relationships of m_{ae}^* with U_r (figure 7e). In IB, the F_L and Y signals are in phase, $\cos \phi > 0$; m_{ae}^* is therefore positive. On the contrary,

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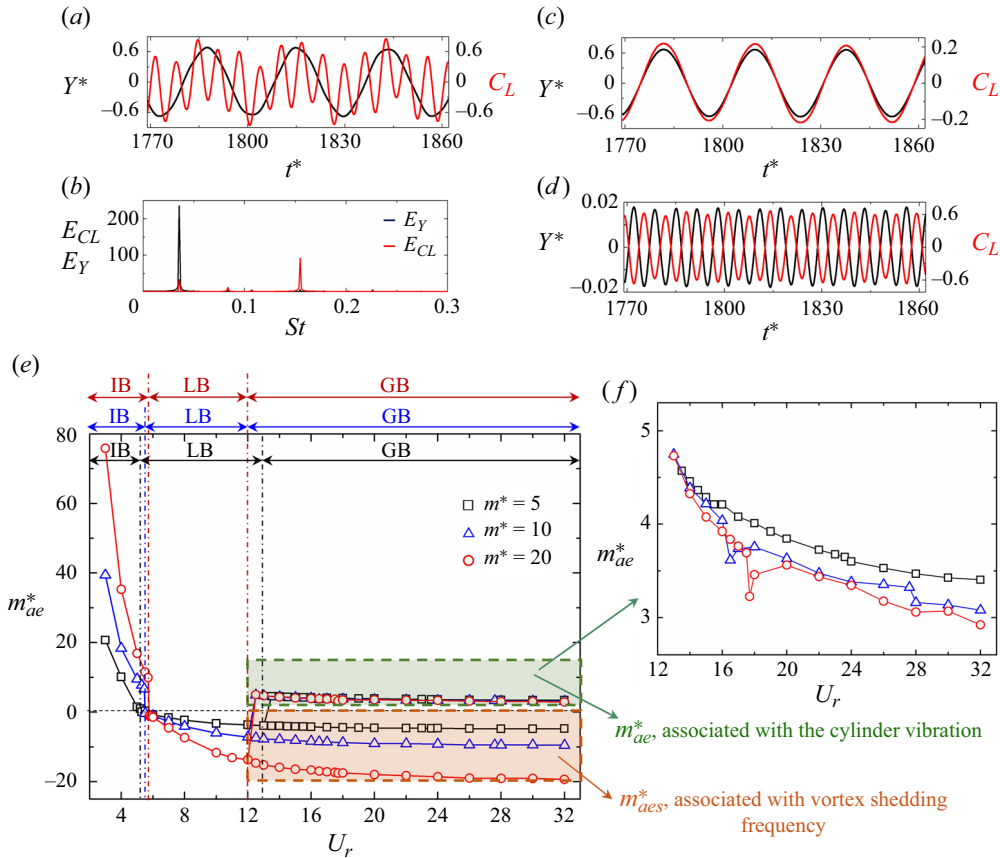


Figure 7. (a) Time histories, (b) power spectral density functions, (c) time histories of low-pass-filtered and (d) time histories of high-pass-filtered Y^* (black lines) and C_L (red lines) at $U_r = 26$ and $m^* = 20$. (e) Variations of effective added mass m_{ae}^* with U_r and (f) zoomed-in view of m_{ae}^* variations in GB. Here, $\zeta = 0$ and $Re = 170$.

the F_L and Y signals are out of phase, $\cos \phi < 0$, in LB, which makes m_{ae}^* negative. The IB and LB are distinguished by m_{ae}^* changing from positive to negative (figure 7e). In the IB and LB, the cylinder motion is sinusoidal. The natural frequency of the vibrating cylinder is then modified because of m_{ae} generated by the surrounding fluid. The modified frequency f_{nf} equals the natural frequency of the cylinder vibrating in the fluid, i.e. $f_{nf} = f_o$:

$$f_o = \frac{1}{2\pi} \sqrt{\frac{k}{m + m_{ae}}}. \quad (3.10)$$

Combining (3.5) and (3.10),

$$m_{ae} = m[(f_n/f_o)^2 - 1]. \quad (3.11)$$

Parameter m_{ae}^* can be presented as

$$m_{ae}^* = \frac{m_{ae}}{\rho D^2} = m^*[(f_n/f_o)^2 - 1] = m^*[(1/f_r)^2 - 1]. \quad (3.12)$$

The equation demonstrates that m_{ae}^* is proportional to $[(1/f_r)^2 - 1]$. Since f_r being < 1.0 linearly grows with U_r in IB (figure 5b), m_{ae}^* being > 0 declines parabolically with U_r . On the other hand, again f_r grows with U_r in LB but now $f_r > 1.0$, hence m_{ae}^* is negative,

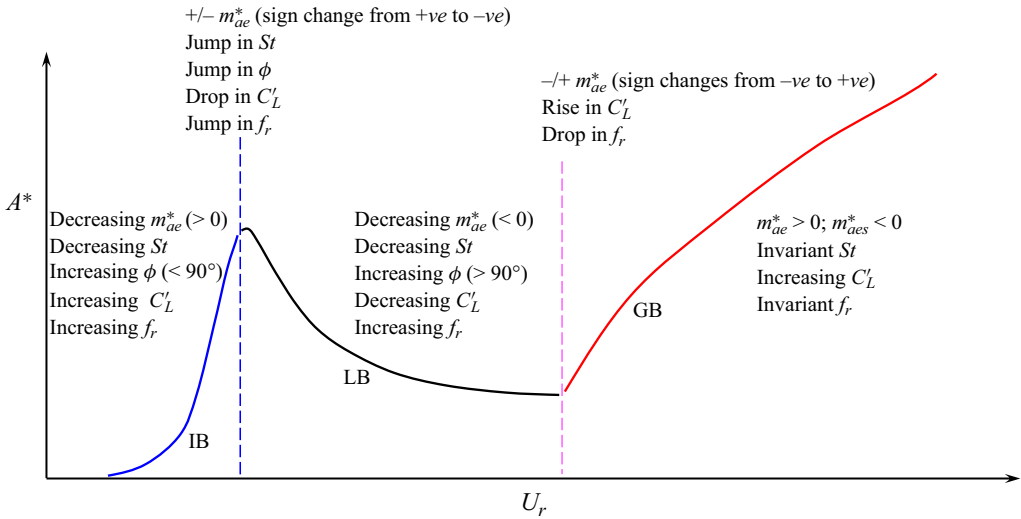


Figure 8. Typical response curve showing major characteristics in IB, LB and GB as well as at their borders.

declining with U_r at a smaller slope than that in IB. A large m^* yields a greater m_{ae}^* magnitude in both IB and LB as m_{ae}^* is directly proportional to m^* (figure 7e; (3.12)).

In GB, both F_L and Y signals are composed of low- and high-frequency components corresponding to the oscillation and vortex shedding frequencies, respectively (figure 7a). These low- and high-frequency components were decomposed using the FFT-filter tool (figure 7c,d). The cutoff frequency for the decomposition was chosen as the average of the high and low frequencies. It can be observed that there are two major peaks in the power spectra (figure 7b) for the vibration in GB. The C_L and Y^* associated with the cylinder oscillation are in phase (figure 7c), while those with the vortex shedding frequency are antiphase (figure 7d). In GB, the effective added mass associated with the cylinder vibration can be referred to as m_{ae}^* while that with the vortex shedding frequency is termed as m_{aes}^* (figure 7e). Here m_{ae}^* jumps from negative to positive at the onset of galloping. The jump in m_{ae}^* is caused by the drop in f_r while m_{ae}^* in GB is positive because $f_r < 1.0$ (figure 5b; (3.12)). The value of m_{ae}^* in GB slightly declines with increasing U_r and m^* (figure 7f), following the increase of f_r with U_r (figures 5b and 7e; (3.12)). On the other hand, m_{aes}^* is negative and declines when U_r is increased, and a larger m^* corresponds to a smaller m_{aes}^* (figure 7e). In IB and LB, the cylinder vortex shedding frequency (f_s) equals the vibration frequency (f_o), i.e. $f_s = f_o$, where only one frequency persists in both F_L and Y signals. Parameter m_{aes}^* in IB and LB thus can be expressed as

$$m_{aes}^* = \frac{m_{ae}}{\rho D^2} = m^*[(f_n/f_s)^2 - 1] = m^*[(1/f_r)^2 - 1]. \quad (3.13)$$

Therefore, $m_{aes}^* = m_{ae}^*$ in IB and LB, where the dependence of m_{aes}^* on U_r is the same as that of m_{ae}^* . The major features of different branches are summarized in figure 8.

3.4. Identifications of IB–LB and LB–GB boundaries

In FIV experiments and simulations, researchers largely get vibration response (A^* versus U_r) and have difficulty in identifying the borders between different branches in A^* versus U_r curves, particularly when the A^* variations with U_r are smooth. Bhatt & Alam (2018)

showed that the borders between IB and LB can be distinguished from the relationship between St and U_r as $St < St_0$ for IB and $St > St_0$ for LB. That is, the IB–LB boundary (i.e. boundary between IB and LB) is accompanied by a jump in St (figure 8). Without measuring St , how can the borders be determined? Here are additional methods to identify the borders. Firstly, the IB–LB boundary can be identified from the change in m_{ae}^* from +ve to –ve, and the LB–GB boundary can be pinpointed from the change in m_{ae}^* from –ve to +ve (figures 7e and 8). Secondly, the IB–LB boundary is characterized by a jump in $Y^* - C_L$ phase lag from $<90^\circ$ to $>90^\circ$ (Williamson & Govardhan 2004; Bhatt & Alam 2018). Thirdly, a dramatic drop and a rise in C'_L mark the IB–LB and LB–GB boundaries, respectively (figures 5d and 8). Fourthly, the IB–LB and LB–GB boundaries undergo a jump and a drop in f_r , respectively. In the literature, the identification of different branches was made based on the relationship between St and U_r . Here, we explore more avenues and generalize them to expand the methodology for identifying these branches. Figure 8 encompasses all possible parameter changes (i.e. m_{ae}^* , St , ϕ , C'_L and f_r), which aids in discerning distinct vibration branches when one of the parameters is measured.

3.5. Beat mechanism different from the classical beat

Next, we focus on the reasons for the uneven features of the vibration amplitude in the GB, taking the case of $m^* = 20$ and $\zeta = 0$ as an example. Except at $U_r = 17.7\text{--}17.8$ and $U_r = 30\text{--}30.1$, the vibrations in GB do not exactly repeat from one period to another (figures 9 and 10), which indicates that there are multiple frequency components, resulting in an obvious beat-like variation in the amplitudes of Y^* histories. As already discussed with figure 4(a,b), $U_r = 17.7\text{--}17.8$ and $U_r = 30\text{--}30.1$ correspond to 1:3 (i.e. $f_s/f_o = 3$) and 1:5 (i.e. $f_s/f_o = 5$) synchronizations, respectively. Interestingly, the beating amplitude in Y^* (i.e. the change in the vibration amplitude due to beating) grows and declines as f_s/f_o respectively approaches and departs from $f_s/f_o = 3$ and/or 5, as does the beating period (figures 9 and 10). The general consensus is that a beat is characterized by amplitude modulation. It is an interference pattern between two different frequencies, perceived as a periodic variation in amplitude. As two frequencies are close to each other (i.e. small difference between them), the beating period becomes longer and it will be infinite (constant amplitude) when the two frequencies are identical.

First, we analyse the beat phenomenon for $f_s/f_o = 3$. The Lissajous diagrams of $C_L\text{--}Y^*$ for $U_r = 15\text{--}20$ presented in figure 11 also demonstrate that the vibration amplitude in GB changes from cycle to cycle except at $U_r = 17.7\text{--}17.8$ (i.e. $f_s/f_o = 3$), where the $C_L\text{--}Y^*$ diagram is an enclosed curve (figure 11e,f), and the frequency in C_L is three times that in Y^* at $U_r = 17.7\text{--}17.8$. It can be deemed ‘1:3 lock-in’ where f_s locks in with $3f_o$. Figure 12 shows power spectra (E_Y and E_{C_L}) and envelopes of Y^* and C_L at $U_r = 17.5$ and 19. For $U_r = 17.5$, $f_s/f_o = 2.951 < 3$, whereas for $U_r = 19$, $f_s/f_o = 3.185 > 3$. In the power spectra of E_Y (figure 12a,e), there is a minor peak (f'_o) on the left-hand side of f_o for $U_r = 17.5$ (figure 12a) and on the right-hand side of f_o for $U_r = 19$ (figure 12e). The beat frequency (i.e. the frequency of the envelope) in Y^* can be represented as $f_{bY} = 1/T_{bY} = |f_o - f'_o|$ (figure 12c,g). The question is, what is the origin of f'_o ? Generally, such a beat frequency is generated from the interference between two frequencies. One can expect this beat frequency is due to the difference between the vortex shedding frequency and cylinder vibration frequency as these two frequencies are predominantly active during the cylinder vibrations. This is, however, not the case here as is shown below.

Similarly, there is a minor peak (f'_s) on the right-hand side of f_s (i.e. $f_s/f_o < 3$) and on the left-hand side of f_s (i.e. $f_s/f_o > 3$) for $U_r = 17.5$ and 19, respectively (figure 12b,f). The corresponding beat frequency is $f_{bL} = 1/T_{bL} = |f_s - f'_s|$ (figure 12d,h).

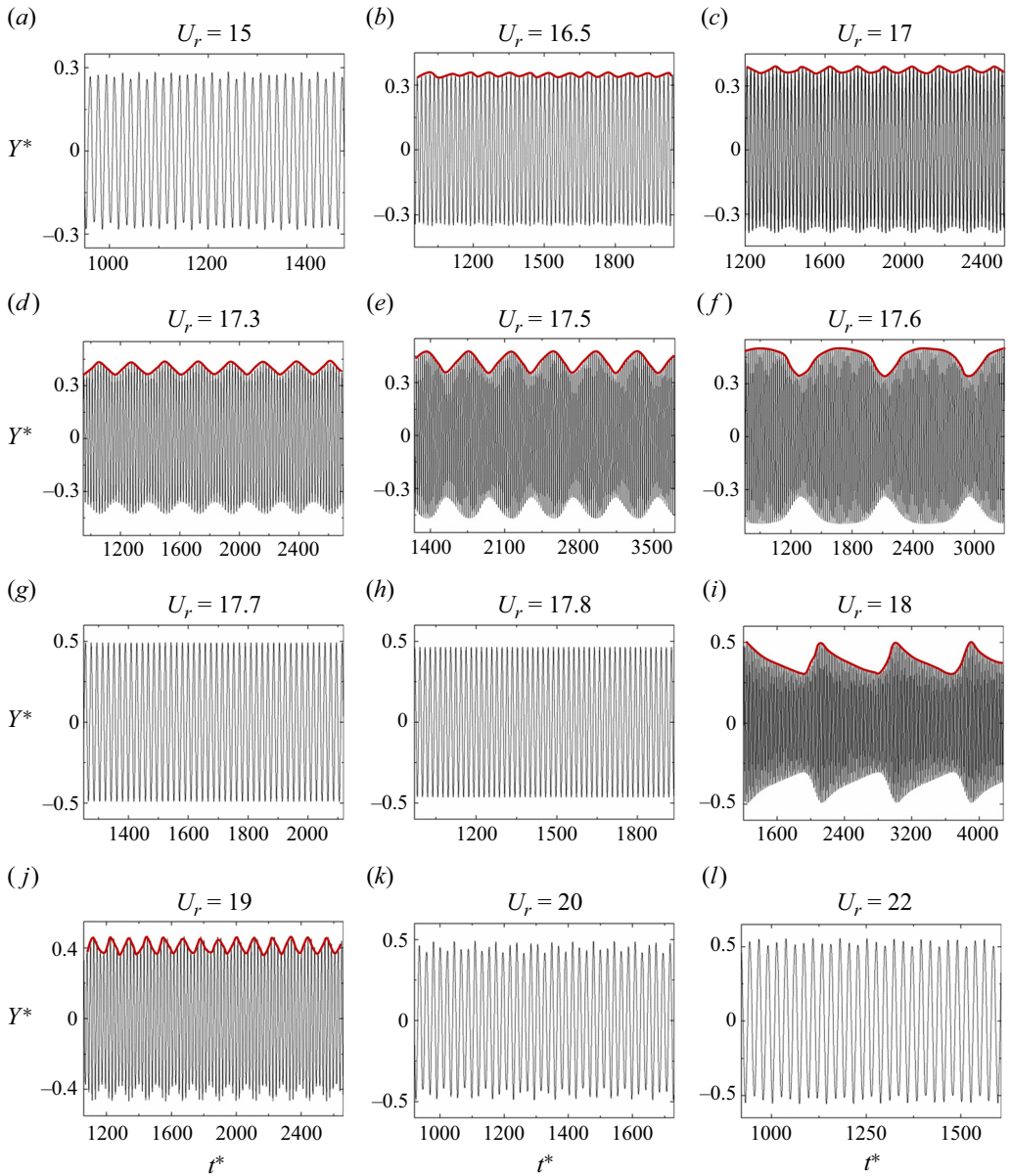


Figure 9. (a–l) Time histories of cylinder displacement Y^* for different U_r . The red curve is the envelope of Y^* . Here, $m^* = 20$, $\zeta = 0$ and $Re = 170$.

For a given U_r ,

$$f_{bY} = f_{bL} \quad (\text{i.e. } T_{by} = T_{bL}) \quad (3.14)$$

and

$$|f_o - f'_o| = |f_s - f'_s|. \quad (3.15)$$

It is found that $f'_s = 3f_o$.

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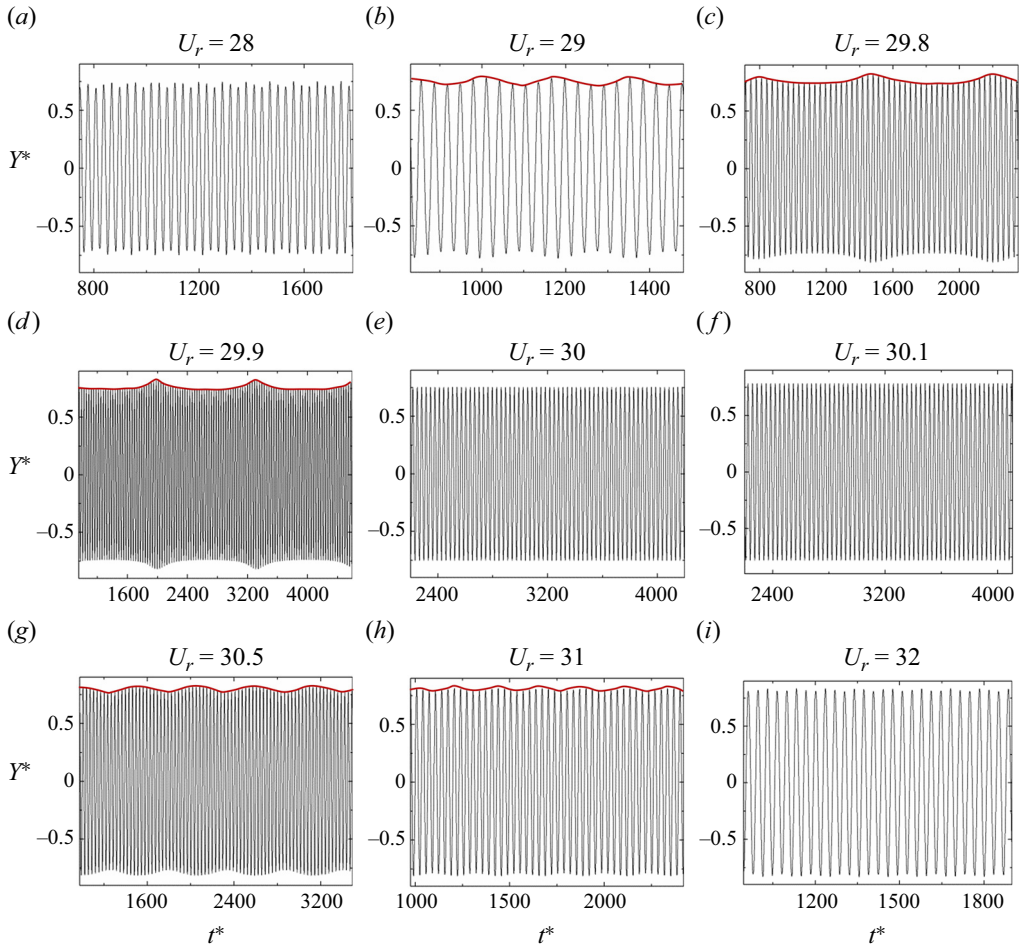


Figure 10. (a–i) Time histories of cylinder displacement response Y^* for different U_r values. Here, $m^* = 20$, $\zeta = 0$ and $Re = 170$.

Using (3.14), it can, therefore, be written as

$$|f_s - f'_s| = |f_s - 3f_o| = |f_o - f'_o|. \quad (3.16)$$

This is to say that the beat frequency in both Y^* and C_L is $f_b = |f_s - 3f_o|$. This explains that the beat frequency will be smaller as f_s/f_o gets closer to 3, and it will be zero (no beating) when $f_s/f_o = 3$. That is, the beating phenomenon is linked to the difference between f_s and $3f_o$, tending to 1:3 synchronization, not directly linked to the difference between f_s and f_o . In GB, since f_s/f_o is proportional to U_r (figure 4b; (3.1)), the beating U_r around $f_s/f_o = 3$ can be predicted from (3.1) for different values of m^* . The other peaks E_{CL} and E_Y emerge at $2f_s - f'_s$ and $2f_o - f'_o$, respectively. A beating phenomenon was also observed in some studies of the FIV of a circular cylinder associated with a mode change or mode competition. Voorhees *et al.* (2008), Zhang *et al.* (2015) and Shen *et al.* (2018) indicated that this beating is due to the difference between the vibration frequency and the vortex shedding frequency (i.e. f_o and f_s), while Pan, Cui & Miao (2007) and Mittal (2017) claimed that the beating is the modulation between the natural frequency f_n of the cylinder and the Strouhal frequency $f_{s, fixed}$ of the fixed cylinder. The beating phenomenon

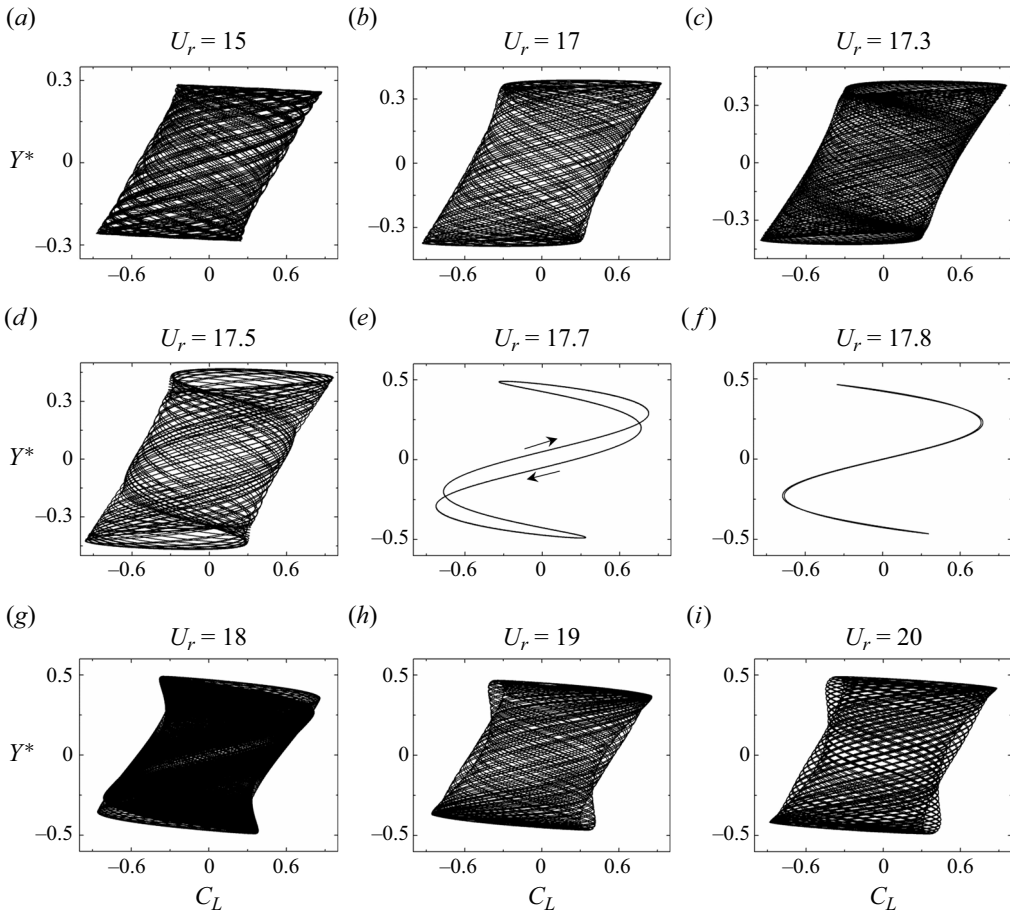


Figure 11. (a–i) Lissajous C_L – Y^* diagrams for different U_r . Here, $m^* = 20$, $\zeta = 0$ and $Re = 170$.

of a vibrating square cylinder in GB, however, is not the same as the cases of the circular cylinder mentioned above. It is the difference between f_s and $3f_o$ that gives rise to the beating phenomenon as f_s/f_o gets close to 3. This observation is made for the first time.

Apart from the 1:3 synchronization mentioned above, we also observed the 1:5 synchronization ($f_s/f_o = 5$) between the vibration frequency and the shedding frequency at $U_r = 23.65$, 27.6 and 30 for $m^* = 5$, 10 and 20, respectively. Here the beat frequency $f_b = |f_s - 5f_o|$ as f_s/f_o gets close to 5. The beat amplitude (figure 10) is, however, much smaller than that in the case involving 1:3 synchronization (figure 9).

3.6. Effects of m^* , ζ , $m^*\zeta$, $(m^* + m_{a0}^*)\zeta$ and $(m^* + m_{ae}^*)\zeta$ on U_{rc} for galloping onset

Next, we analyse the effects of m^* and ζ on U_{rc} for the onset of galloping. To save time and computational resources, we simulated the responses at high U_r values only (figures 13 and 14). Parameter U_{rc} is identified from the abrupt increase in A^* (figure 13) and/or from the sudden decrease in f_r (figure 14). For all ζ values examined, A^* decreases with increasing m^* before the galloping onset (i.e. for $U_r < U_{rc}$), which implies that A^* in LB and/or DB (desynchronization branch) weakens with increasing m^* . However, for a given m^* , A^* for $U_r < U_{rc}$ does not change appreciably with increasing ζ . Figure 14 displays that $f_r > 1.0$

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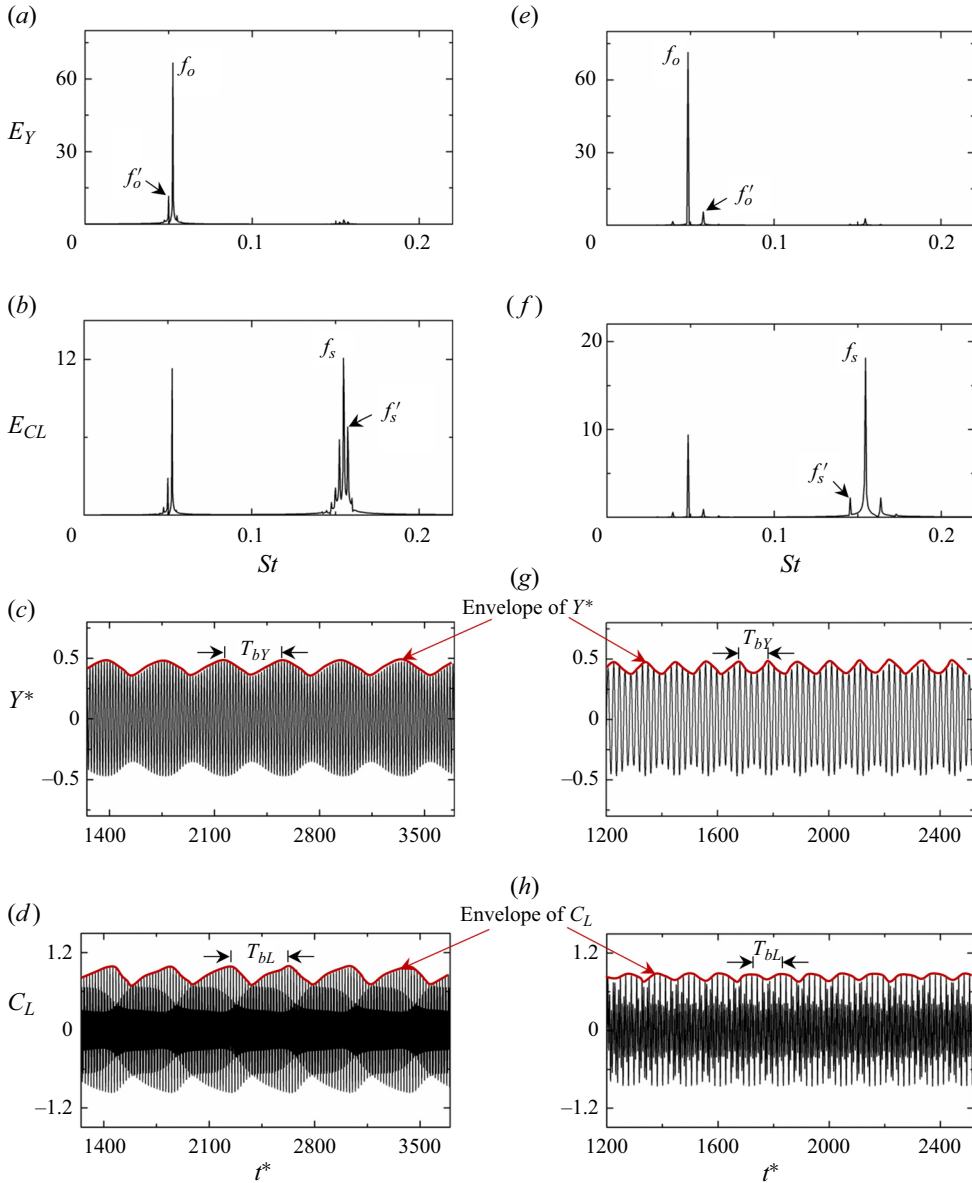


Figure 12. Power spectrum of (a,e) Y^* and (b,f) C_L . Envelopes of (c,g) Y^* and (d,h) C_L . (a–d) $U_r = 17.5$, (e–h) $U_r = 19$. Here, $m^* = 20$, $\zeta = 0$ and $Re = 170$.

for $U_r < U_{rc}$ but $f_r < 1.0$ for $U_r > U_{rc}$. In the latter U_r regime, when m^* is increased, f_r increases to approach 1.0.

To further investigate the influence of m^* and ζ on U_{rc} , more cases of m^* and ζ are also considered. Table 3 summarizes U_{rc} for $m^* = 2, 3, 5, 10, 20, 30, 40$ and 50 and $\zeta = 0, 0.001, 0.01, 0.05, 0.1, 0.2, 0.4, 0.6$ and 1.0 at $U_r \leq 80$. When $\zeta = 0$, U_{rc} is 17, 13, 12, 12, 12 and 12 for $m^* = 3, 5, 10, 20, 30, 40$ and 50 , respectively, i.e. U_{rc} decreases with increasing m^* for $3 \leq m^* < 10$ before being insensitive to m^* for $m^* \geq 10$. Figure 15 shows the dependence of galloping occurrence on $m^* = 2-50$ and $\zeta = 0-1$. No galloping is

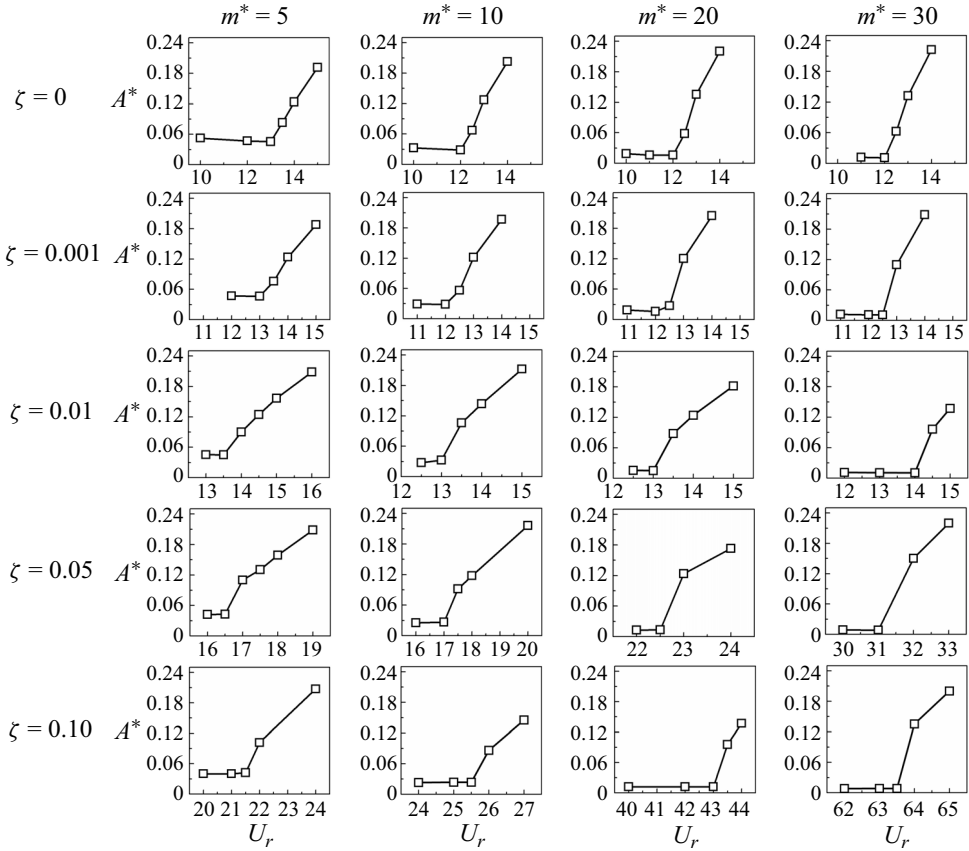


Figure 13. Variations of vibration amplitude A^* with reduced velocity U_r for different m^* and ζ values. Here, $Re = 170$.

observed at $m^* = 2$ for all ζ values examined. Galloping is observed for $m^* = 3-10$ when $\zeta = 0.2$, while the same occurs at $m^* = 5$ only for $\zeta = 0.4$. For $\zeta \geq 0.6$, no galloping is observed, regardless of m^* , until $U_r = 80$ examined. When m^* is increased from 5 (i.e. $m^* \geq 5$), U_{rc} changes insignificantly for $\zeta \leq 0.01$, but significantly for $\zeta \geq 0.05$ (table 3, figure 16a). Parameter U_{rc} , however, significantly increases when m^* is decreased from 5 to 3 for all ζ values (table 3, figure 16b). For all m^* values examined, an increase in ζ makes U_{rc} higher, particularly when $\zeta > 0.01$ (table 3, figure 16b). That is, the effect of ζ on U_{rc} is insignificant for $\zeta \leq 0.01$ but significant for $\zeta > 0.01$, a higher ζ suppressing the cylinder vibration. Joly *et al.* (2012) also found a higher U_{rc} value at $\zeta = 0.1$ than that at $\zeta = 0$.

The three-dimensional view of the dependence of U_{rc} on m^* and ζ shown in figure 17 allows us to observe the variations of U_{rc} with m^* and ζ more intuitively at $\zeta \leq 0.2$. The value of U_{rc} achieves its maximum at $m^* = 30$ and $\zeta = 0.1$. For all m^* values, an increase in ζ leads to a larger U_{rc} . The increase is, nevertheless, larger for a large m^* . Particularly for $\zeta > 0.01$, U_{rc} declines for $m^* \leq 5$ and rapidly increases with increasing m^* . The degree of the decrease and/or increase becomes larger for a higher ζ value.

Some researchers have debated whether $m^*\zeta$ can serve as the FIV criterion of a circular cylinder. Griffin, Skop & Ramberg (1975) and Griffin (1980) used a Skop–Griffin

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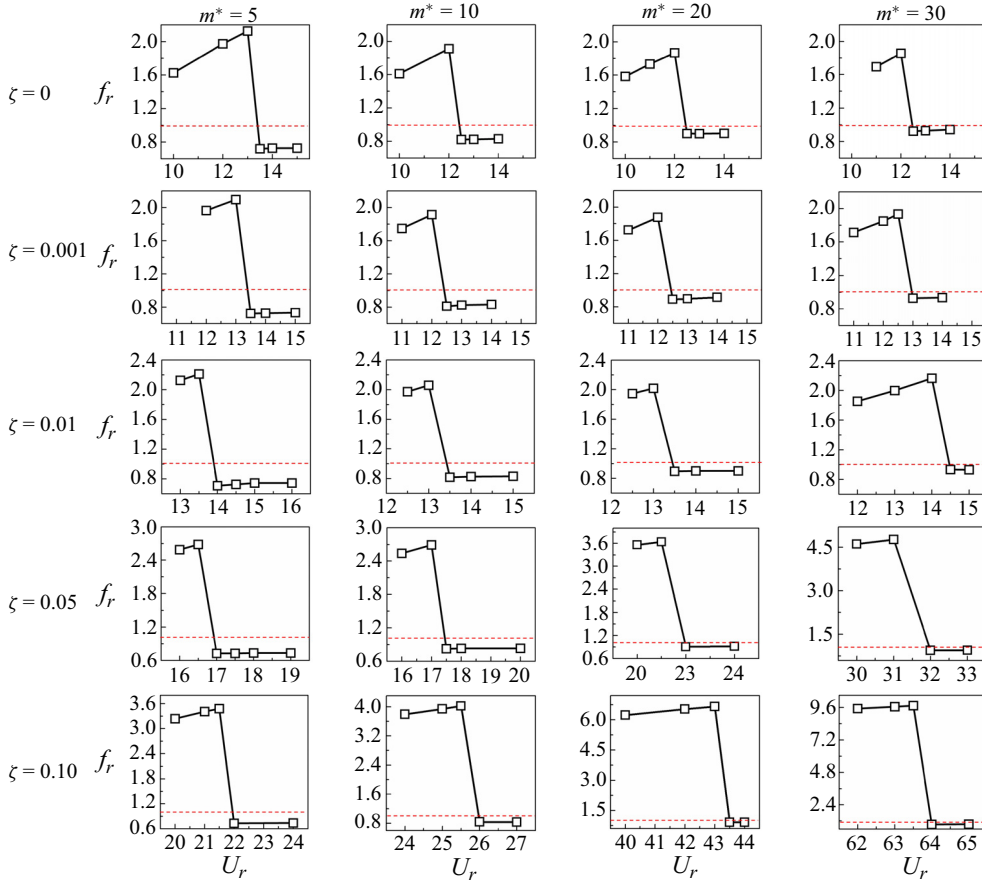


Figure 14. Variations of frequency ratio f_r with reduced velocity U_r for different m^* and ζ . The red dotted line represents $f_r = 1$. Here, $Re = 170$.

ζ	0	0.001	0.01	0.05	0.1	0.2	0.4	0.6	1
$m^* \leq 2$	—	—	—	—	—	—	—	—	—
$m^* = 3$	17	17	18	22	28	44	—	—	—
$m^* = 5$	13	13	13.5	16.5	21.5	34	67.5	—	—
$m^* = 10$	12	12	13	17	25.5	50	—	—	—
$m^* = 20$	12	12	13	22.5	43	—	—	—	—
$m^* = 30$	12	12.5	14	31	63.5	—	—	—	—
$m^* = 40$	12	12.5	14	40	—	—	—	—	—
$m^* = 50$	12	12.5	15	50	—	—	—	—	—

Table 3. Effects of m^* and ζ on U_{rc} . ‘—’ means no galloping observed for $U_r \leq 80$ examined.

parameter $S_G (= 2\pi^3 S^2 m^* \zeta)$, where S is the Strouhal number of the static cylinder) to plot the maximum vibration amplitude A_{max}^* against S_G (i.e. $m^* \zeta$). It is known as the Griffin plot. However, this plot showed a large scatter of A_{max}^* data. Later, Khalak & Williamson (1999) and Govardhan & Williamson (2000) introduced a combined parameter $(m^* + m_{a0}^*) \zeta$ to modify the Griffin plot, where m_{a0}^* (i.e. potential added-mass ratio C_A in

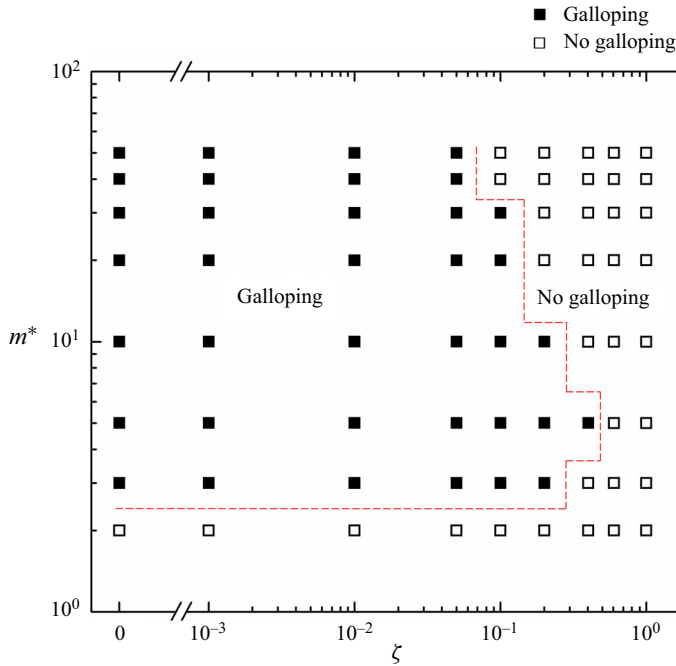


Figure 15. Dependence of galloping occurrence on $m^* = 2-50$ and $\zeta = 0-1$. Here, $Re = 170$.

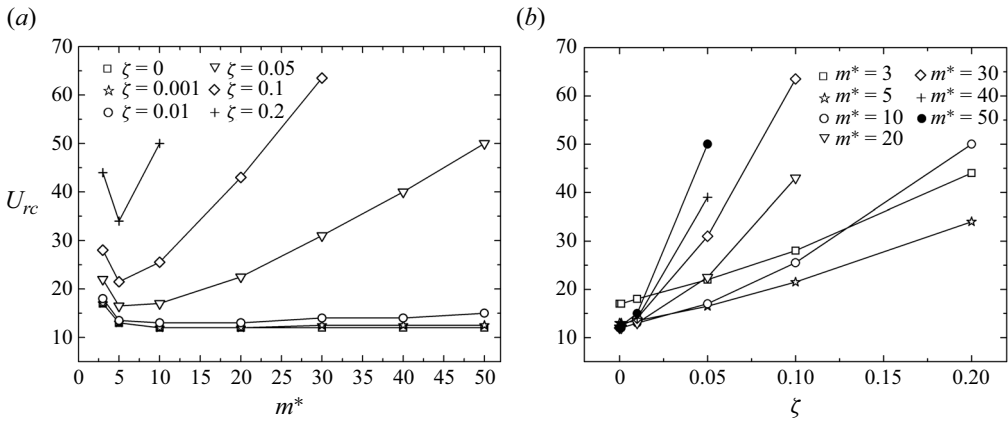


Figure 16. Effect of (a) mass ratio m^* and (b) damping ratio ζ on critical reduced velocity U_{rc} . Here, $Re = 170$.

their investigations) is the quiescent-fluid added-mass ratio and is about 1.0 for a circular cylinder. They demonstrated that $(m^* + m_{a0}^*)\zeta$ collapses the A_{max}^* data well for a wide range of $(m^* + m_{a0}^*)\zeta > 0.06$. Sarpkaya (1978, 1995) stated that the vibration response depends on $m^*\zeta$ for $S_G > 1$ (i.e. $m^*\zeta > 0.4$), while it depends on m^* and ζ separately rather than on $m^*\zeta$ for $m^*\zeta < 0.4$. Blevins & Coughran (2009) for two-degrees-of-freedom vibration found that the ratio of inline to transverse amplitudes is governed by m^* and ζ independently. Bahmani & Akbari (2010) claimed that $m^*\zeta$ can be used to characterize A_{max}^* with an acceptable accuracy. For a cylinder submerged in the wake of another cylinder, Alam (2021) claimed that $m^*\zeta$ (i.e. $m^*\zeta = 0-8$) does not serve as a unique

Features of flow-induced stability of a square cylinder

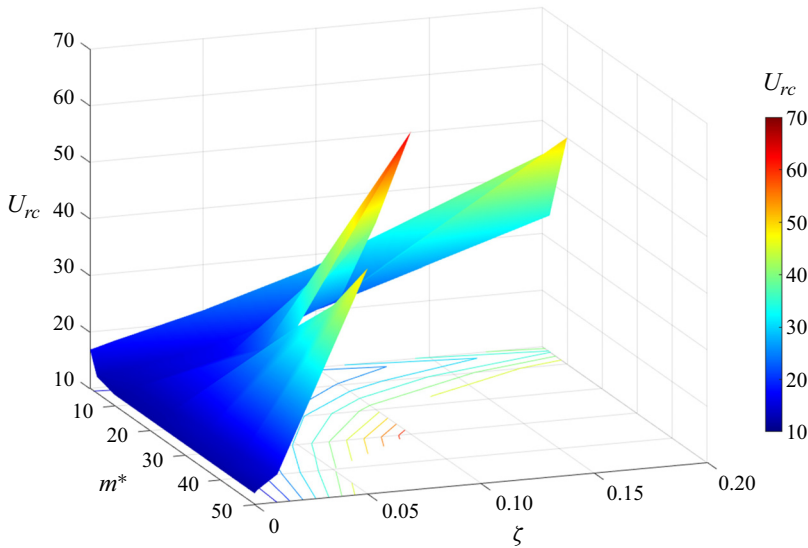


Figure 17. Relationship of U_{rc} with m^* ($= 3-50$) and ζ ($= 0-0.2$). Here, $Re = 170$.

parameter to characterize the vibration amplitude. In addition, Bearman (1984) reported that low values of m^* affect the frequency ratio independently, not $m^*\zeta$. Williamson & Govardhan (2004) and Govardhan & Williamson (2004) demonstrated that the VIV range depends only on m^* at $(m^* + C_A)\zeta < 0.05$. Here, we will see whether $m^*\zeta$ or $(m^* + m_{a0}^*)\zeta$ works well to collapse U_{rc} data. If not, what is the intrinsic parameter that dictates the vibration? Figure 18(a,b) shows the relationships of U_{rc} with $m^*\zeta$ and $(m^* + m_{a0}^*)\zeta$. As shown in figure 18(a), there are more than one U_{rc} data point, significantly different, for a given $m^*\zeta$ (e.g. three U_{rc} values for $m^*\zeta = 1.0$ and four U_{rc} values for $m^*\zeta = 2.0$), which indicates that $m^*\zeta$ cannot serve as the only criterion for galloping onset. In figure 18(b), the U_{rc} data scattering alleviates, which points to a linear increase in U_{rc} with $(m^* + m_{a0}^*)\zeta$ overall at $m^* \geq 5$. But still, there are some scattered data points, and the collapse of U_{rc} data on a line is still not credible. It hints that something is still missing in $(m^* + m_{a0}^*)\zeta$. We find the missing parameter here.

As m^* decreases, f_r in GB deviates considerably from unity, i.e. the vibration frequency differs from the natural frequency (figures 5b and 14). The deviation is attributed to the flow-induced added mass m_{af}^* that should be considered in $(m^* + m_{a0}^*)\zeta$. The effective added mass is the quiescent-fluid added mass plus flow-induced added mass, i.e. $m_{ae}^* = m_{a0}^* + m_{af}^*$ (Alam 2022). While m_{a0}^* is independent of U_r , m_{af}^* is highly dependent on U_r and markedly changes at U_{rc} . Therefore, m_{af}^* is crucial and should be added with m_{a0}^* . That is, in the mass-damping parameter, we have to consider m_{ae}^* , not m_{a0}^* , i.e. $(m^* + m_{a0}^*)\zeta$ should be replaced by $(m^* + m_{ae}^*)\zeta$. As shown in figure 7(f), m_{ae}^* jumps at U_{rc} and declines with increasing m^* and U_r . We consider m_{ae}^* at the galloping onset for $(m^* + m_{ae}^*)\zeta$, and the dependence of U_{rc} on $(m^* + m_{ae}^*)\zeta$ is presented in figure 19. For $(m^* + m_{ae}^*)\zeta > 0.55$, as shown in figure 19(a), the U_{rc} data points collapse well on a line at $m^* \geq 5$, showing that U_{rc} linearly increases with $(m^* + m_{ae}^*)\zeta$. The relationship between U_{rc} and $(m^* + m_{ae}^*)\zeta$ can be represented by a curve-fitting equation:

$$U_{rc} = 17.4(m^* + m_{ae}^*)\zeta + 2.5 \quad (\text{for } (m^* + m_{ae}^*)\zeta > 0.55 \text{ and } m^* \geq 5). \quad (3.17)$$

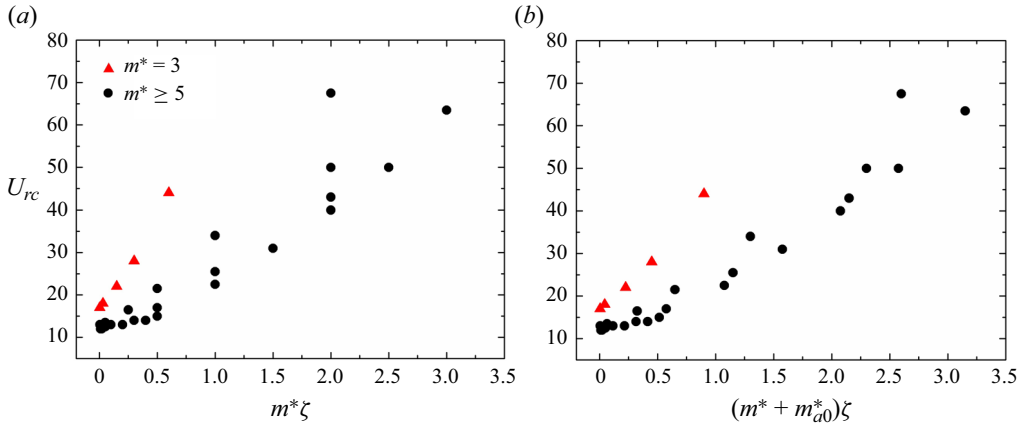


Figure 18. Combined mass-damping parameter (a) $m^*\zeta$ and (b) $(m^* + m_{a0}^*)\zeta$ with critical reduced velocity U_{rc} . Here, $Re = 170$.

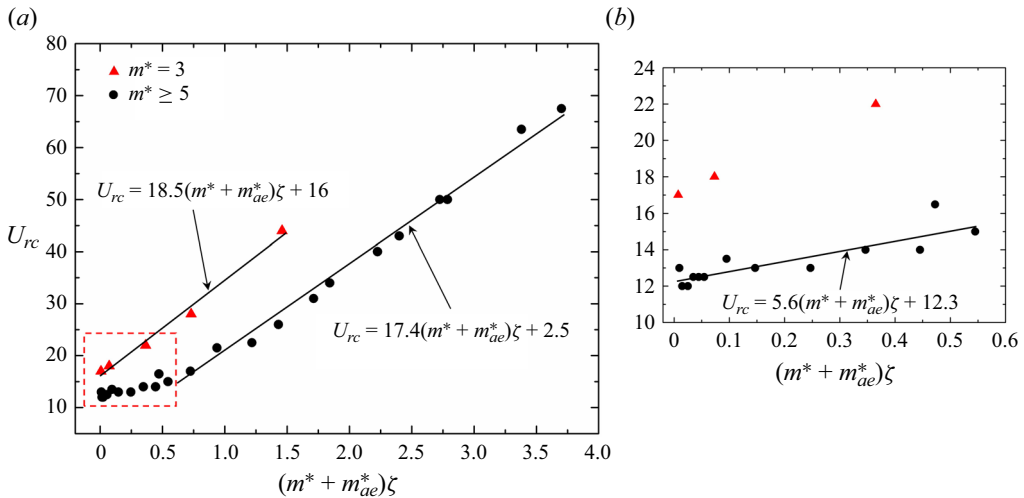


Figure 19. (a) Relationship between critical reduced velocity U_{rc} and combined mass-damping parameter $(m^* + m_{ae}^*)\zeta$. (b) Zoomed-in view of U_{rc} variations at $(m^* + m_{ae}^*)\zeta \leq 0.55$.

For $(m^* + m_{ae}^*)\zeta \leq 0.55$ as shown in figure 19(b), the U_{rc} also increases linearly, albeit with a smaller slope, with increasing $(m^* + m_{ae}^*)\zeta$ for $m^* \geq 5$. The U_{rc} data points for $m^* = 5$ appear more scattered when $(m^* + m_{ae}^*)\zeta > 0.55$. The linear relationship at $(m^* + m_{ae}^*)\zeta \leq 0.55$ and $m^* \geq 5$ can be represented as

$$U_{rc} = 5.6(m^* + m_{ae}^*)\zeta + 12.3 \quad (\text{for } (m^* + m_{ae}^*)\zeta \leq 0.55 \text{ and } m^* \geq 5). \quad (3.18)$$

Equation (3.18) further proves that when $\zeta = 0$ or is very small, U_{rc} becomes independent ($U_{rc} = 12.3$) of m^* , which is consistent with the observation made in figure 16(a).

For $m^* = 3$, U_{rc} again linearly increases with $(m^* + m_{ae}^*)\zeta$, with a larger slope than the case for $m^* \geq 5$ (figure 19a). The relationship between U_{rc} and $(m^* + m_{ae}^*)\zeta$ can be expressed as

$$U_{rc} = 18.5(m^* + m_{ae}^*)\zeta + 16 \quad (\text{for } m^* = 3). \quad (3.19)$$

The slopes of U_{rc} in (3.17) and (3.19) are close to each other although the U_{rc} intercepts differ between the two cases.

The use of $(m^* + m_{ae}^*)\zeta$ in (3.17)–(3.19) can predict U_{rc} . Term $(m^* + m_{ae}^*)\zeta$ can, therefore, be considered a criterion for the prediction of galloping onset. Equation (3.17) also illustrates why galloping is not observed for $m^* = 20$ and $\zeta = 0.2$, where $(m^* + m_{ae}^*)\zeta = 4.76$. The value of U_{rc} predicted by (3.17) is $85.19 > 80$ examined. It can therefore be concluded that the criterion for a fluid–structure system is $(m^* + m_{ae}^*)\zeta$, not $m^*\zeta$ or $(m^* + C_A)\zeta$ or $(m^* + m_{a0}^*)\zeta$. For a given $(m^* + m_{ae}^*)\zeta$, U_{rc} is higher for $m^* < 5$ than for $m^* \geq 5$. There is a threshold of $m^* (= 5)$ after ($m^* \geq 5$) or before ($m^* < 5$) which the relationships between U_{rc} and $(m^* + m_{ae}^*)\zeta$ differ. We further shed light on this threshold in the next section.

3.7. Underlying physics of m^* and ζ effects on galloping onset

3.7.1. Effects of m^* and ζ when $\zeta = 0$

Using the global linear stability analysis method, Meliga & Chomaz (2011), Zhang *et al.* (2015) and Yao & Jaiman (2017) investigated the mechanism of VIVs of a freely vibrating cylinder. They identified two modes responsible for the onset of instability in the fluid–structure system: wake mode (WM) and structure mode (SM). These two leading modes have a strong coupling for low m^* , while they are decoupled for high m^* . Parameter m^* plays an important role in the selection of the leading mode. The essential characteristic of the flutter-induced lock-in is the competition between the two unstable modes (Zhang *et al.* 2015). Li *et al.* (2019) utilized linear stability analysis and direct numerical simulations to investigate the underlying mechanisms of galloping of a square cylinder with zero damping. They concluded that the unstable SM leads to the low-frequency large-amplitude vibration of the cylinder, while the unstable WM results in the high-frequency vortex shedding in the wake. The instability of SM is the primary cause of the galloping phenomenon. The onset of galloping is postponed when SM and WM compete with each other. As such, the galloping phenomenon can be completely suppressed or U_{rc} can be delayed at relatively low- Re and low- m^* (< 5) conditions. In their study, the galloping vibration completely disappeared for low m^* ($m^* < 4$, $Re = 150$) because of the strong competition between the enhanced SM and WM at low m^* (lighter structure). They also introduced a ‘pre-galloping’ region, located between VIV and galloping, where galloping does not occur but the SM is unstable.

Here, we investigate the influence of m^* on galloping by analysing the high-frequency (f_s) and low-frequency (f_o) signals of the vibration. At $\zeta = 0$, U_{rc} remains constant for $m^* \geq 10$ while it increases with decreasing m^* for $3 \leq m^* < 10$. Ultimately, galloping is suppressed for $m^* \leq 2$, i.e. a relatively low m^* inhibits galloping. Given that the maximum U_r examined in this study is 80, galloping may be observed for a smaller m^* (≤ 2) if U_r is further increased (Han & Langre 2022). It has already been shown that the high-pass-filtered C_L and Y^* associated with the vortex shedding are antiphase, not auspicious for the cylinder vibration. Conversely, the low-pass-filtered signals are in phase, auspicious for the cylinder vibration. First, we analyse vibration signals (Y^*) at $m^* = 20$ with $U_r = 12$ and 13, corresponding to pre-galloping and galloping regions, respectively (figure 20a,d). To gain insight into the evolution of the frequency content over time, the continuous wavelet transform (CWT) is performed using the Morlet wavelet (figure 20b,e). The wavelet centre frequency $Fc = 3$ and the wavelet scales equal $2Fc - 2Fc \times 2^{14}$, which allows us to measure frequency in the range of $Fs/2^{14} - Fs/2$ (where Fs is the sampling frequency). Additionally, power spectra of vibration responses in different developing

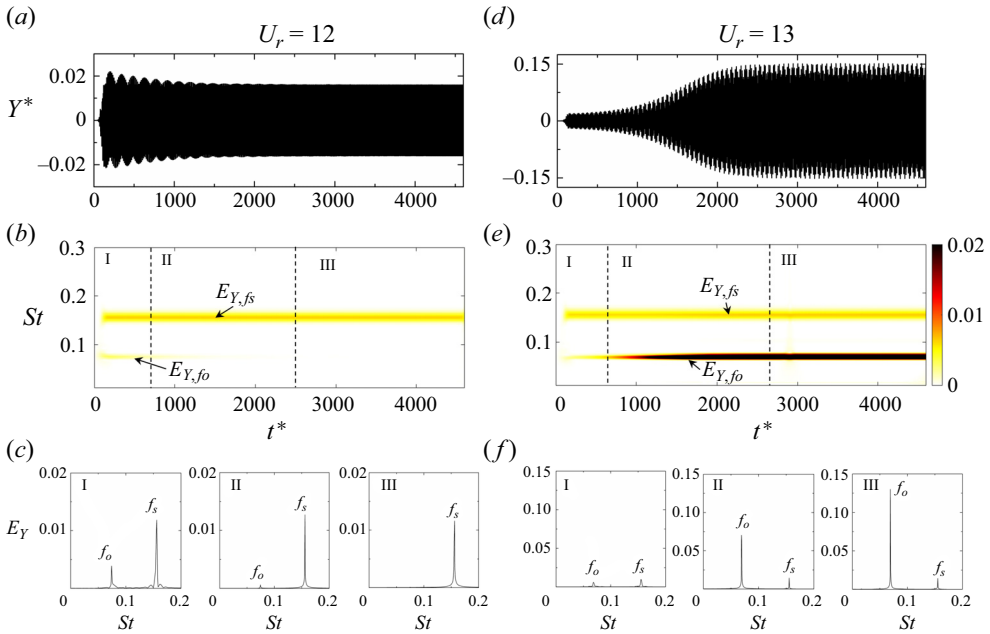


Figure 20. (a,d) Time histories of Y^* . (b,e) Time–frequency spectrum of Y^* based on CWT. (c,f) Power spectral density functions of Y^* at different regimes. (a–c) $m^* = 20$, $U_r = 12$; and (d–f) $m^* = 20$, $U_r = 13$. Here, $\zeta = 0$ and $Re = 170$.

regimes are shown in [figure 20\(c,f\)](#). The cylinder vibration in each signal comprises two distinct frequencies f_o and f_s , associated with SM and WM, respectively. Consider $E_{Y,fo}$ and $E_{Y,fs}$ as the energy intensity at f_o and f_s , respectively.

In the pre-galloping region, the vibration amplitude rapidly develops and reaches its maximum at $t^* \approx 200$ ([figure 20a](#)). At $t^* < 800$, there is an intensified competition between f_o and f_s , where $E_{Y,fo}$ and $E_{Y,fs}$ are considerable ([figure 20c](#)); this transition is marked as regime I. In regime II, $E_{Y,fo}$ rapidly decreases and becomes insignificant while $E_{Y,fs}$ dominates the competition, suppressing the vibration at f_o ([figure 20c](#)). In regime III ($t^* > 2500$), the cylinder vibration solely occurs at f_s , and the amplitude remains constant.

After galloping, $E_{Y,fo}$ increases gradually in regimes I and II, and it remains unchanged in regime III ([figure 20e](#)). In regime I, $E_{Y,fo}$ is lower than $E_{Y,fs}$ ([figure 20f](#)). Parameter $E_{Y,fo}$ increases and $E_{Y,fs}$ remains unchanged from regimes I to II; eventually, $E_{Y,fo}$ becomes larger than $E_{Y,fs}$. This indicates that SM dominates the competition and results in the occurrence of galloping. In regime III, $E_{Y,fo}$ remains unchanged, with $E_{Y,fo}/E_{Y,fs} = 7.92$ ([figure 20f](#)).

The E_Y values can be used to characterize the vibration amplitude. As shown in [figure 21](#), the cylinder vibration is decomposed into low-frequency and high-frequency vibrations after the commencement of galloping. We define the vibration amplitudes of the low- and high-pass-filtered Y^* as A_{low}^* and A_{high}^* corresponding to $E_{Y,fo}$ and $E_{Y,fs}$, respectively. In the pre-galloping region, the vibration amplitude A^* in regime III can also be characterized by $E_{Y,fs}$, which declines gently with increasing U_r ([figures 4a](#) and [21](#)). After the onset of galloping, A_{high}^* remains almost unchanged. The almost unchanged $E_{Y,fs}$ and A_{high}^* nearly before the galloping occurrence and after the galloping occurrence indicate that vortex shedding is not involved in the occurrence of galloping. The value of

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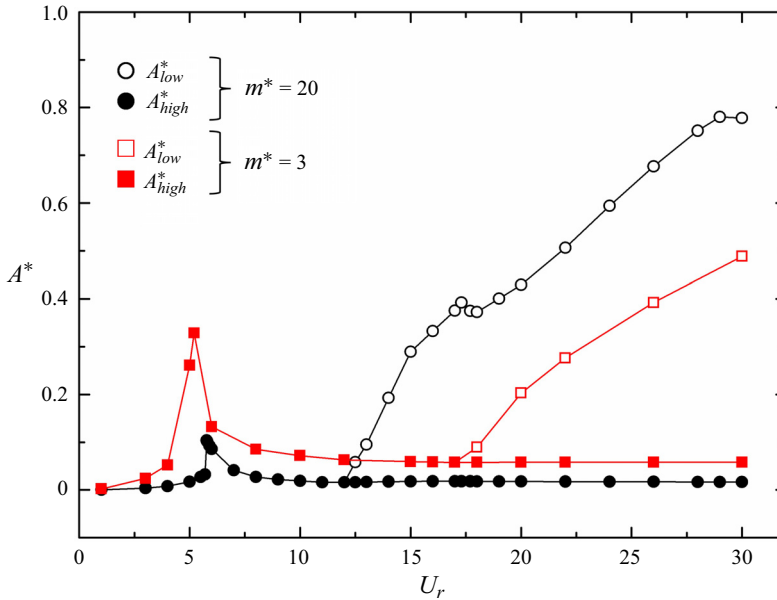


Figure 21. Dependence of A^* on U_r and m^* . Here, $\zeta = 0$ and $Re = 170$.

A_{low}^* , however, increases markedly with the increase of U_r in the galloping regime. It is the low-frequency motion (SM) that is auspicious for the cylinder vibration, and consequently promotes the occurrence of galloping. As shown in figure 20(b,e), $E_{Y,fo}$ at $m^* = 20$ and 13 is significantly greater than that at $U_r = 12$, while $E_{Y,fs}$ is almost the same.

Since U_{rc} is delayed with a decrease in m^* from 5 to 3 and galloping is suppressed for $m^* = 2$ (table 3, figure 16a), we will further investigate the mode competition phenomenon at $m^* = 3$ to evaluate the influence of m^* on the competition. At $m^* = 3$, both A^* in pre-galloping region and A_{high}^* in galloping regime increase compared to that at $m^* = 20$ (figure 21), and the $E_{Y,fs}$ value at $m^* = 3$ is significantly intensified (figures 20b,e and 22b,e). Consequently, the competition between $E_{Y,fo}$ and $E_{Y,fs}$ becomes more intense at $m^* = 3$, with $E_{Y,fo}/E_{Y,fs} = 1.52$ at the galloping onset in regime III (figure 22f). It can be inferred that if m^* is further decreased, $E_{Y,fs}$ could completely overpower $E_{Y,fo}$ and suppress galloping. At the galloping onset, $E_{Y,fo}/E_{Y,fs}$, i.e. A_{low}^*/A_{high}^* , diminishes with decreasing m^* . It can be reasonably inferred that if A_{low}^*/A_{high}^* at the galloping onset drops to 1 with a further decrease in m^* , then galloping will be completely suppressed. Figure 23 presents the influence of A_{low}^*/A_{high}^* on m^* at the galloping onset. It can be seen that A_{low}^*/A_{high}^* decreases linearly with decreasing m^* , and can be represented by a curve-fitting equation:

$$A_{low}^*/A_{high}^* = 0.41m^* + 0.155. \quad (3.20)$$

When $A_{low}^*/A_{high}^* = 1$, the critical m^* predicted by (3.20) is $m_c^* = 2.06 \approx 2$. Equation (3.20) explains why galloping is not observed for $m^* \leq 2$. Additionally, we observed galloping at $m^* = 2.5$ and $U_r = 40$, further verifying the rationality of the prediction of m_c^* . Figure 23 also shows the dependence of A_{low}^*/A_{high}^* on m^* at $Re = 150$ and 160, where A_{low}^*/A_{high}^* declines with decreasing m^* , the declining rate increasing with decreasing Re . When A_{low}^*/A_{high}^* decreases to 1, m_c^* is larger at a smaller Re .

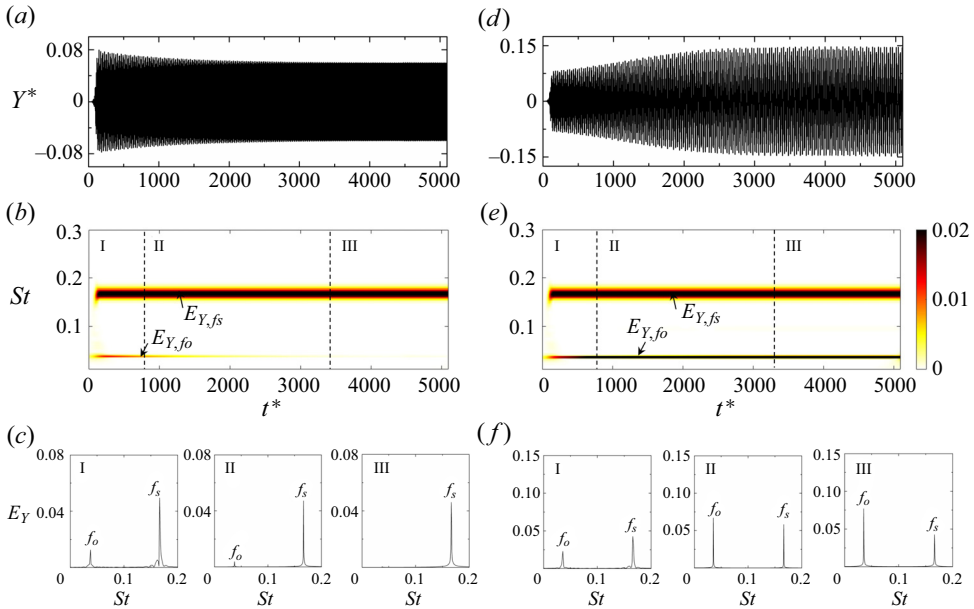


Figure 22. (a,d) Time histories of Y^* . (b,e) Time–frequency spectrum of Y^* based on CWT. (c,f) Power spectral density functions of Y^* at different regimes. (a–c) $m^* = 3$, $U_r = 17$; and (d–f) $m^* = 3$, $U_r = 18$. Here, $\zeta = 0$ and $Re = 170$.

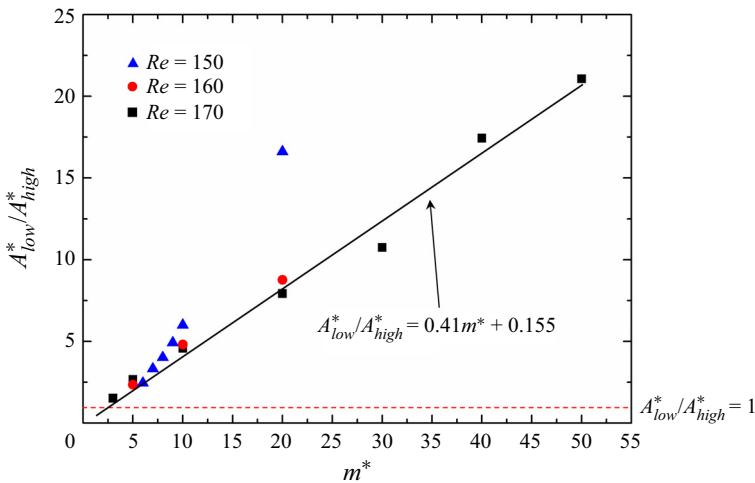


Figure 23. Effect of m^* and Re on A_{low}^*/A_{high}^* at the galloping onset. Here, $\zeta = 0$.

In summary, at $\zeta = 0$, as m^* decreases, the contribution of WM becomes more significant, leading to a delay in the onset of galloping. Eventually, galloping can be completely suppressed at sufficiently low m^* values. At $m^* = 3$, the occurrence of galloping at $\zeta = 0$ depends on the competition between SM and WM while the competition is contingent on m^* . However, at $m^* \geq 5$, the contribution of WM becomes negligible; the cylinder vibration is thus dominated by SM, leading to a constant U_{rc} as m^* increases.

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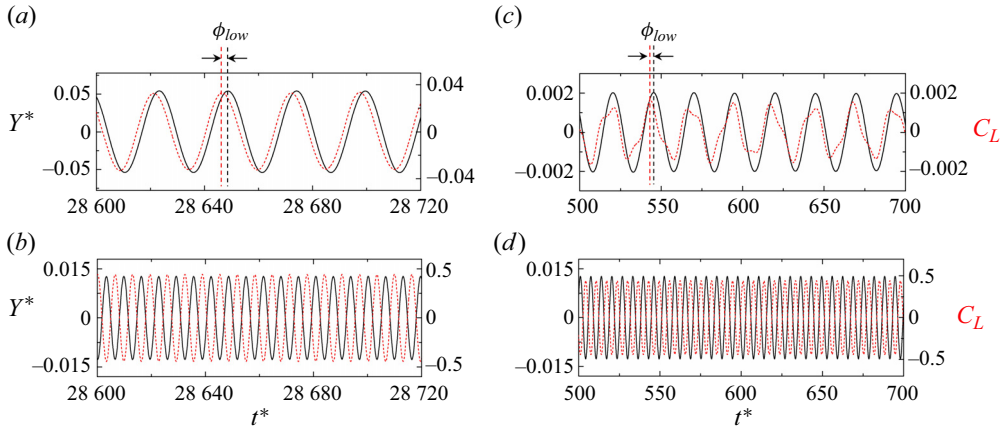


Figure 24. Time histories of (a,c) low-pass-filtered and (b,d) high-pass-filtered Y^* (black lines) and C_L (red lines) in GB. (a,b) $m^* = 20$, $\zeta = 0.05$ and $U_r = 23$ in regime III; (c,d) $m^* = 20$, $\zeta = 0.05$ and $U_r = 22.5$ in regime I. Here, $Re = 170$.

3.7.2. Effects of m^* and ζ when $\zeta > 0$

Moving on to discussing the influence of ζ on galloping, we observe that at $\zeta > 0$, U_{rc} increases with the increase of ζ . Additionally, U_{rc} shows a nonlinear relationship with m^* , initially decreasing and then increasing as m^* increases. When $\zeta = 0$ or is small, U_{rc} is independent of m^* (≥ 5). As previously discussed with figure 7, when the cylinder vibrates in GB without damping ($\zeta = 0$), the phase lag between the low-pass-filtered C_L and Y^* is 0° (figure 7c), while that between the high-pass-filtered C_L and Y^* is 180° (figure 7d). As ζ increases from 0, taking $\zeta = 0.05$ and $m^* = 20$ as an example, where $U_{rc} = 22.5$, the phase lag ϕ_{low} (in degrees) between the low-pass-filtered C_L and Y^* at $U_r = 23$ (beyond the galloping onset) undergoes a shift from 0 to a positive value, i.e. $\phi_{low} \approx 27^\circ$ (figure 24a). As discussed with figure 20(a–c), in regime I of the pre-galloping region, the cylinder vibration consists of both high- and low-frequency components. Therefore, at $\zeta = 0.05$, $\phi_{low} \approx 26.5^\circ$ is achieved in regime I of the pre-galloping region (i.e. $U_r = 22.5$) (figure 24c). The phase lag between the high-pass-filtered C_L and Y^* at $\zeta > 0$, however, remains 180° both before and after U_{rc} (figure 24b,d). Therefore, the presence of damping ($\zeta > 0$) introduces a positive phase lag between the low-pass-filtered C_L and Y^* , both before and after galloping, modifying the coupling between the flow and cylinder vibration.

Figure 25(a) demonstrates the variations of m_{ae}^* , ζ_a and ϕ_{low} with U_r at $m^* = 20$, $\zeta = 0.05$. In GB, $m_{ae}^* > 0$, declining with increasing U_r , in a similar fashion to the case of $\zeta = 0$. The added damping ratio ζ_a compensates the structural damping ratio $-\zeta$ (dashed line), so that $\zeta_a + \zeta = 0$. Notably, ϕ_{low} is positive and rises with increasing U_r . Compared with $\zeta = 0$, it is the positive phase lag that introduces a modification in the coupling between the cylinder vibration and the fluid dynamics.

The energy W_f transferred from fluid to the cylinder for each cycle of oscillation is given by

$$W_f = \int F_L dY = \int_t^{t+T} F_{L0} \sin(2\pi f_o t + \phi) d(Y_0 \sin 2\pi f_o t) = \pi F_{L0} Y_0 \sin \phi. \quad (3.21)$$

When C_L leads Y^* ($\phi > 0$), positive work is done on the cylinder. An increasing ϕ ($< 90^\circ$) provides more energy to the cylinder vibration.

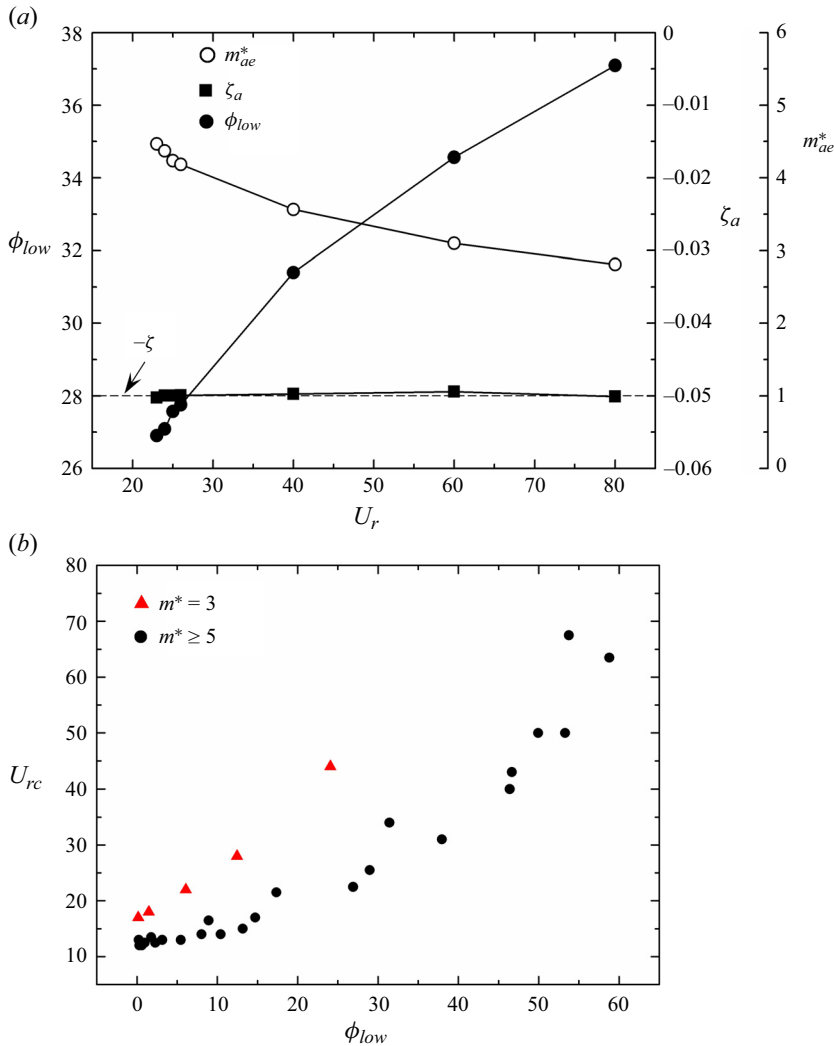


Figure 25. (a) Variations of phase lag ϕ_{low} (deg.), added damping ζ_a and effective added mass m_{ae}^* with U_r at $m^* = 20$ and $\zeta = 0.05$. The dashed line represents $-\zeta$. (b) Dependence of U_{rc} on phase lag ϕ_{low} at the galloping onset. Here, $Re = 170$.

The presence of damping ($\zeta > 0$) introduces a positive phase lag, modifying energy transfer between the flow and cylinder, so the relationship between U_{rc} , ζ and m^* becomes more complex and it is worth understanding the influence of ζ on galloping behaviour. The coupling in the presence of damping is crucial in determining the galloping onset, especially for lower m^* ($m^* < 5$) where the role of vortex shedding becomes significant.

Figure 25(b) shows the dependence of U_{rc} on ϕ_{low} . At $m^* \geq 5$, a larger ϕ_{low} corresponds to a larger U_{rc} overall. The value of U_{rc} at $m^* = 3$, however, is obviously larger than that at $m^* \geq 5$, which is due to the fierce competition between SM and WM at $m^* = 3$. As discussed before, for $\zeta = 0$, U_{rc} at $m^* = 3$ entirely depends on the competition between SM and WM. Therefore, for $\zeta > 0$, because of the presence of non-zero ϕ_{low} , U_{rc} at $m^* = 3$ is influenced not only by the competition but also by the magnitude of ϕ_{low} . On the other hand, when $m^* \geq 5$, the competition is weak, with no influence on the galloping onset.

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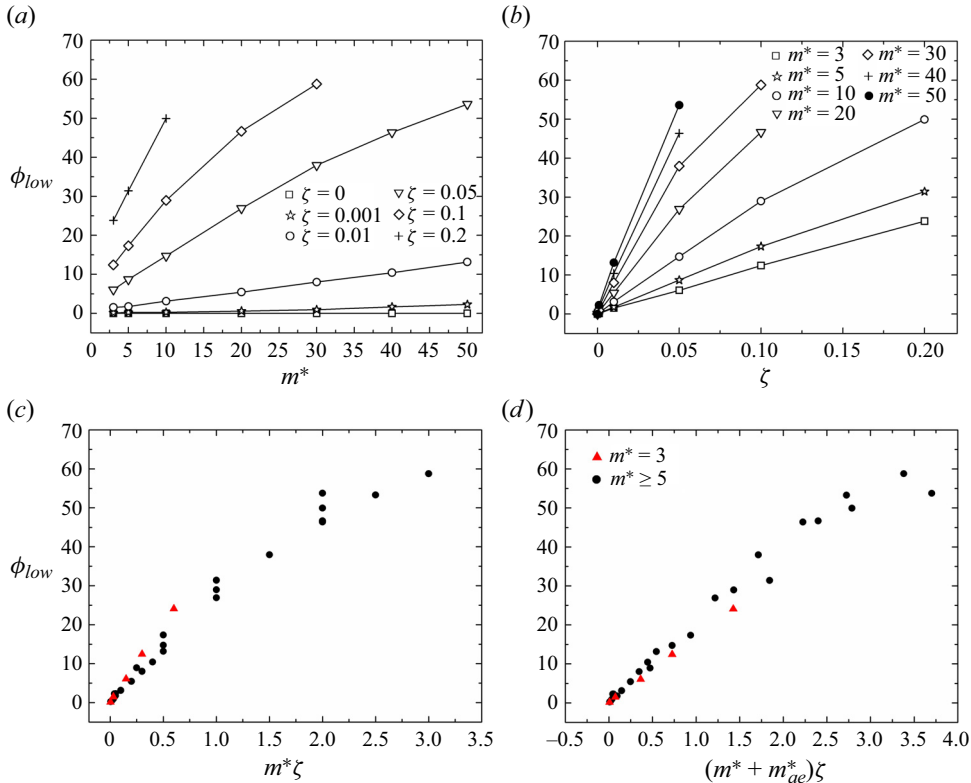


Figure 26. Effect of (a) mass ratio m^* , (b) damping ratio ζ , (c) $m^*\zeta$ and (d) $(m^* + m_{ae}^*)\zeta$ on phase lag ϕ_{low} at the galloping onset. Here, $Re = 170$.

The value of U_{rc} is, therefore, mainly influenced by ϕ_{low} , while a larger m^* and/or ζ yields a larger U_{rc} .

Overall, ϕ_{low} being associated with work done is an important factor in determining the onset of galloping, and its magnitude is influenced by both m^* and ζ . Understanding the interplay between m^* , ζ and ϕ_{low} provides valuable insights into the stability of the fluid–structure system during galloping.

Figure 26(a,b) shows the effect on ϕ_{low} (in degrees) of m^* and ζ at the onset of galloping. The relationship between ϕ_{low} and m^* or ζ is straightforward, where ϕ_{low} increases with m^* (figure 26a) and/or ζ (figure 26b). In the previous sections, the relationship of U_{rc} with $m^*\zeta$, $(m^* + C_A)\zeta$ or $(m^* + m_{a0}^*)\zeta$ has been made clear. Is there any relationship of ϕ_{low} with the same? Figure 26(c,d) shows the dependencies of ϕ_{low} on $m^*\zeta$ and $(m^* + m_{ae}^*)\zeta$. It is evident that there is no clear relationship between ϕ_{low} and $m^*\zeta$ (figure 26c) or $(m^* + m_{ae}^*)\zeta$ (figure 26d), which precludes the use of $m^*\zeta$ or $(m^* + m_{ae}^*)\zeta$ to predict ϕ_{low} effectively. However, when $(m^* + m_{a0}^*)\zeta$ is employed (figure 27), the ϕ_{low} data points collapse well on a line for all m^* values, following a quadratic polynomial relationship between ϕ_{low} and $(m^* + m_{a0}^*)\zeta$. This reiterates that it is $(m^* + m_{a0}^*)\zeta$, not $m^*\zeta$, which intrinsically combines and reflects the mass-damping properties of the system. This relationship between ϕ_{low} and $(m^* + m_{a0}^*)\zeta$ can be obtained as

$$\phi_{low} = -3.1[(m^* + m_{a0}^*)\zeta]^2 + 28.6(m^* + m_{a0}^*)\zeta. \quad (3.22)$$

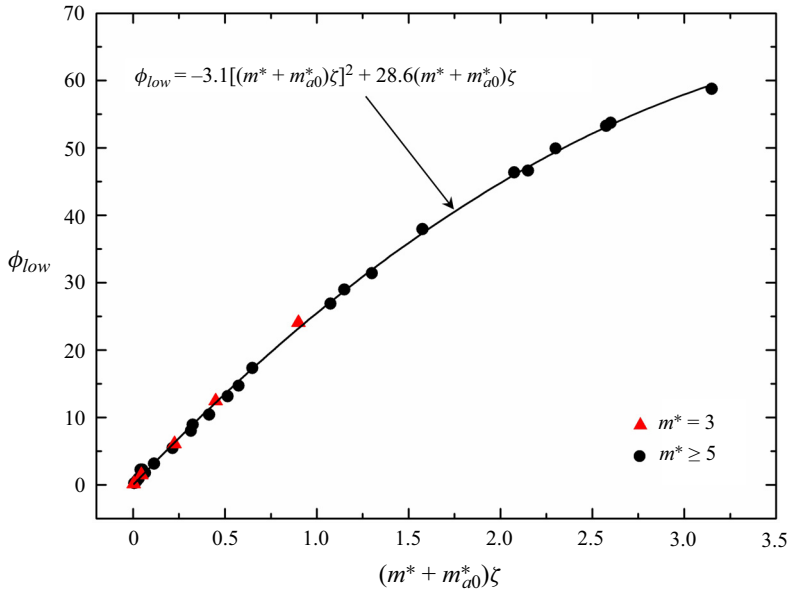


Figure 27. Effect of combined mass-damping parameter $(m^* + m_{a0}^*)\zeta$ on phase lag ϕ_{low} at the galloping onset. Here, $Re = 170$.

This equation allows us to predict ϕ_{low} using the value of $(m^* + m_{a0}^*)\zeta$. Parameter $m_{a0}^* = 1.5$ for a square cylinder with zero incidence angle (Chen *et al.* 2020). As long as m^* and ζ are provided, we can obtain the phase lag ϕ_{low} between the low-pass-filtered C_L and Y^* at the onset of galloping.

As discussed with figure 19, the U_{rc} data points collapse well on $(m^* + m_{ae}^*)\zeta$ at $m^* \geq 5$, where the influence of ϕ_{low} is significantly greater and U_{rc} is mainly determined by ϕ_{low} . At $m^* = 3$, however, U_{rc} is influenced by both the competition and the magnitude of ϕ_{low} . As a result, U_{rc} at $m^* = 3$ deviates from the fitting curve. The competition affects U_{rc} at $m^* = 5$ to some degree, where U_{rc} at $m^* = 5$ is 13, that is, slightly larger than 12 at $m^* \geq 10$. However, the influence of ϕ_{low} on U_{rc} at $m^* = 5$ is greater than that at $m^* = 3$. Particularly, at $(m^* + m_{ae}^*)\zeta > 0.55$, U_{rc} is largely determined by ϕ_{low} , so that the U_{rc} data points fit well on a line at $m^* \geq 5$ and $(m^* + m_{ae}^*)\zeta > 0.55$ (figure 19a). At $(m^* + m_{ae}^*)\zeta \leq 0.55$, however, the U_{rc} data points at $m^* \geq 5$ all lie above the fitted curve for $(m^* + m_{ae}^*)\zeta > 0.55$ (figure 19b), as ϕ_{low} at $(m^* + m_{ae}^*)\zeta \leq 0.55$ is not large enough to curb the mode competition.

4. Conclusions

Flow-induced vibrations of a square cylinder with mass ratio $m^* = 2\text{--}30$, damping ratio $\zeta = 0\text{--}1.0$ and mass-damping ratio $m^*\zeta = 0\text{--}50$ are numerically investigated. The investigation covers five aspects: (i) effect of m^* on the vibration response, forces and frequency response, (ii) roles of effective added mass m_{ae}^* in different vibration branches, (iii) characteristic changes at the borders between different branches, (iv) combined effect of m^* and ζ on critical reduced velocity U_{rc} and phase lag ϕ_{low} for galloping onset and (v) underlying mechanisms of the influences of m^* and ζ on galloping.

The effect of m^* is investigated on vibration responses, frequency responses, forces and effective added-mass ratio. An increase in m^* reduces U_r range of VIVs and advances the galloping onset. Parameter m^* has distinct influences in IB, LB and GB regimes. In IB, a larger m^* leads to a smaller A^* , C'_L and C'_{La0} , but a larger St , \bar{C}_D and C'_{Lf} . In LB,

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a larger m^* corresponds to a smaller A^* , St , \bar{C}_D , C'_{La0} and C'_{Lf} , but a larger C'_L . On the other hand, in the GB regime, an increasing m^* enhances A^* , f_r , \bar{C}_D , C'_L and C'_{Lf} , but reduces St , C'_{La0} , f_s/f_o and its gradient with U_r .

The effective added-mass ratio (m_{ae}^*) characterizes different vibration branches, being contingent on U_r and m^* . The value of m_{ae}^* in IB is positive and declines parabolically with U_r , which results from the fact that f_r being <1.0 approaches 1.0. The value of m_{ae}^* in LB is negative and further declines because f_r being >1.0 shifts away from 1.0. A larger m^* yields a greater m_{ae}^* magnitude in both IB and LB. In GB, m_{ae}^* is positive and decreases with increasing U_r . On the other hand, m_{aes}^* is negative, yet decreasing with U_r . A larger m^* corresponds to a smaller m_{ae}^* and m_{aes}^* while the effect of m^* on m_{aes}^* is greater than that on m_{ae}^* .

It is challenging to distinguish the different vibration branches. Here, several techniques are remarked upon for identifying the borders between different branches. The IB–LB boundary can be identified from the change in m_{ae}^* from +ve to –ve, jumps in St , ϕ and f_r or a dramatic drop in C'_L . On the other hand, the LB–GB boundary can be pinpointed from the change in m_{ae}^* from –ve to +ve, and a jump in C'_L or a drop in f_r .

Vibration responses undergo beating, which is linked to the difference between f_s and $3f_o$ or $5f_o$, with a beating frequency of $|f_s - 3f_o|$ or $|f_s - 5f_o|$. The beating amplitude in Y^* or C_L grows when f_s/f_o approaches $f_s/f_o = 3$ or 5 but declines when f_s/f_o departs from $f_s/f_o = 3$ or 5. The beating phenomenon of a square cylinder in GB is derived from the modulation between f_s and $(2n + 1)f_o$, where $n = 1, 2, \dots$, not directly caused by the difference between f_s and f_o .

Effects of m^* , ζ , $m^*\zeta$, $(m^* + m_{a0}^*)\zeta$ and $(m^* + m_{ae}^*)\zeta$ on U_{rc} and ϕ_{low} for galloping onset are revealed at $U_r = 1-80$. Parameter m^* has a negligible influence on U_{rc} for $\zeta \leq 0.01$. This is, nevertheless, not the case for $\zeta > 0.01$ where U_{rc} adjourns with increasing m^* and/or ζ . The competition between vortex shedding associated with low-frequency vibration (SM) and high-frequency vibration (WM) determines U_{rc} at low m^* ($=3$) as does ϕ_{low} at a large $m^* \geq 5$. Term $(m^* + m_{a0}^*)\zeta$ successfully collapses all ϕ_{low} data on a line for all m^* and ζ values, with a quadratic polynomial relationship between ϕ_{low} and $(m^* + m_{a0}^*)\zeta$. Similarly, U_{rc} data fit well on $(m^* + m_{ae}^*)\zeta$ at $m^* \geq 5$, where U_{rc} grows linearly with increasing $(m^* + m_{ae}^*)\zeta$. Term $m^*\zeta$ as well as $(m^* + m_{a0}^*)\zeta$ or $(m^* + C_A)\zeta$ does not serve well to characterize a fluid–structure system. We introduce a combined mass-damping parameter $(m^* + m_{ae}^*)\zeta$ that serves as the unique criterion to predict the galloping onset. This indicates that $(m^* + m_{ae}^*)\zeta$ works well to represent the property of a fluid–structure system.

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