THEORETICAL EVOLUTION SEQUENCES FOR HOMOGENEOUS AND TWO-ZONE MODELS OF DELTA SCUTI VARIABLES

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ABSTRACT

Radial pulsation periods are calculated for models of variables in the lower Cepheid instability strip. Evolution tracks in several diagrams are constructed and used for comparisons with observed periods, period ratios, and effective temperatures. These comparisons are simpler and more direct than in earlier studies with separate evolution and pulsation calculations.

1. INTRODUCTION

The purpose of the present report is to present selected results of new theoretical stellar evolution sequences for models of variables in the lower part of the Cepheid instability strip.

The observed period ratios of (especially high amplitude) δ Sct variables have been discussed in numereous papers (Petersen and Jørgensen, 1972; Fitch, 1976; Petersen, 1976, 1978, 1979; Simon, 1979; Stellingwerf, 1979; Cox et al., 1979; and references therein). Cox et al. concluded that their models with content of elements heavier than helium Z = 0.01 (Population I) to 0.001 (Pop. II) predict the observed period ratios very well. They used homogeneous standard models, except for SX Phe where a helium depleted surface zone was introduced. Petersen and Jørgensen (1972) used Z = 0.03 for their Population I composition, and the disagreement between their period ratios for Population I models and those of Cox et al. seems to be entirely due to the difference in assumed Z values.

Cox et al. (1979) emphasized the importance of an accurate treatment of the central region of δ Sct models for accurate calculations of period ratios. In the present investigation we calculate pulsation periods for full stellar models in evolution sequences. Therefore, any uncertainty, arising from the structure of the central region, is removed. Because we have theoretical periods for all models, we can construct

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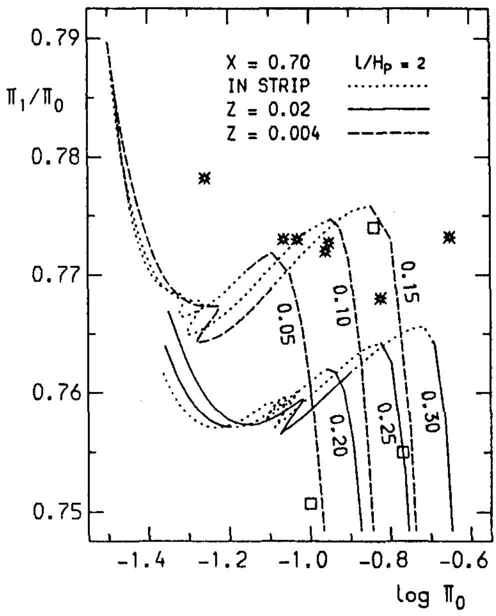

Fig. 1. Standard evolution tracks on the (log Π_0, Π_1/Π_0) plane for a Population I and an intermediate Population II composition. Numbers at tracks give model masses by log $M/M_\odot$. Asterisks: high amplitude variables, squares: low amplitude stars.

evolution tracks in many useful diagrams. In this way we can now perform simpler and more direct comparisons with observational data than in earlier studies, where evolution and pulsation calculations were carried out separately. Besides standard model series with homogeneous envelopes we have calculated series with helium depleted outer zones, as downwards helium diffusion in the surface layers is believed to be important in the lower Cepheid instability strip. We restrict the discussion in the present preliminary report to analysis of periods, period ratios, and resonances.

2. PERIODS AND PERIOD RATIOS

Standard Evolution Models

We first consider standard model sequences with homogeneous envelopes. All model sequences are calculated by means of the stellar evolution programme developed by Hejlesen (1980), in which we include the pulsation programme described by Petersen (1978, 1979), rewritten by Andreasen (1980). Fig. 1 compares evolution tracks in the (log Π_0, Π_1/Π_0) diagram calculated for our typical Population I composition with Z = 0.02 with similar tracks for Pop. II. The stars shown in Fig. 1 are besides the

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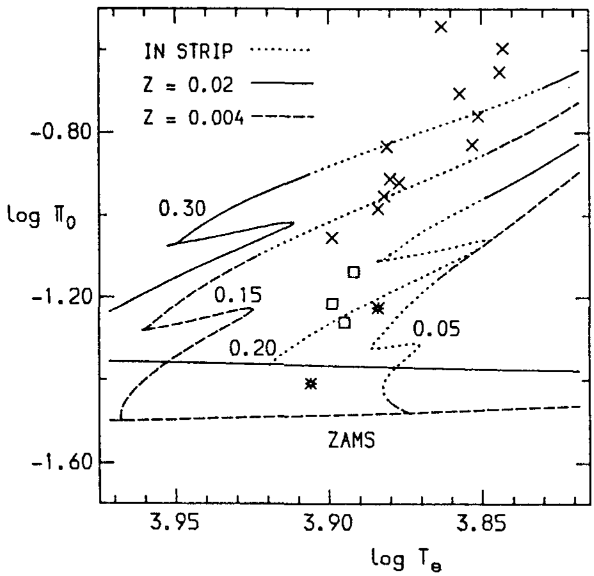

Fig. 2. Standard evolution tracks marked by log $M/M_\odot$ on the (log T_e, log Π_0) plane. The observed high amplitude stars' approximate metal content is indicated by the symbols. Asterisks: very metal poor, squares: intermediate Pop. II, crosses: normal Pop. I.

well established 7 high amplitude variables with Π_1/Π_0 = 0.768 - 0.778 (taken from Fitch, 1976) the 3 low amplitude stars 21 Mon (log Π_0 = -1.00, Π_1/Π_0 = 0.7507; Fitch, 1976), 44 Tau (-0.84, 0.774; Wizinowich and Percy, 1979), and 38 Cnc (-0.77, 0.755; Breger, 1980). The well known decrease in period ratio with increasing Z is clearly seen. While two of the low amplitude δ Sct stars require extreme Pop. I composition (Z > 0.02) all high amplitude variables and 44 Tau require Z somewhat smaller than 0.02. This is in agreement with Petersen and Jørgensen (1972) and Cox et al. (1979).

In Fig. 2 we show the high amplitude variables from Table IV of McNamara and Feltz (1978) and HD 94033 from Przybylski and Bessell (1979) on a (log T_e, log Π_0) plane with standard evolution tracks. We devide the stars into three groups with metal abundance [Fe/H] ≅ -1.5 -0.7, and 0.0, roughly corresponding to the Z values for our models Z = 0.0004, 0.004, and 0.02, respectively. Note that both theoretical and observational data can be plotted very accurately in Fig. 2. In agreement with several earlier studies (see McNamara and Feltz) Fig. 2 shows that most or all of these stars have evolved to the hydrogen shell burning stage. If a reliable Z value is known for a variable, an accurate mass can be derived (assuming X = 0.70).

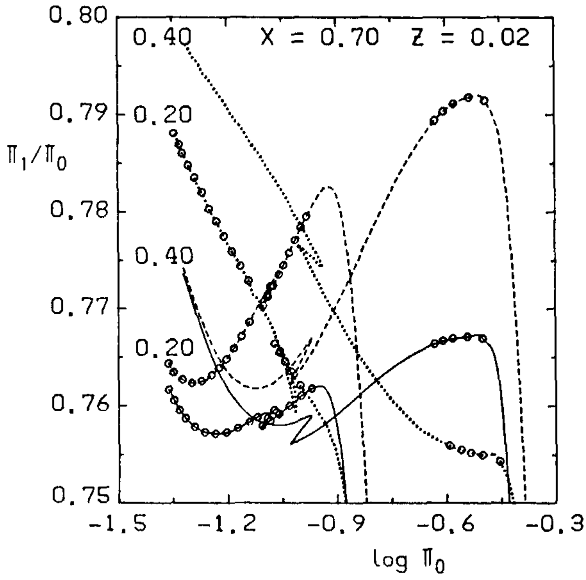

Fig. 3. Comparison of standard Pop. I evolution tracks (full curves) in the (log $\Pi_0, \Pi_1/\Pi_0$) diagram with tracks for models with an outer helium poor zone of depth given by log T = 5.5 (dashed curves) or 6.0 (dotted curves). Numbers at tracks give log $M/M_\odot$. Models inside the instability strip are marked.

Two-zone Envelope Models

Fig. 3 shows the effect of introducing a helium depleted surface layer in Pop. I models. In order to show the effect very clearly we have chosen depletion parameters more extreme than those studied by Cox et al. (1979). Our models have surface composition (X, Y, Z) = (0.95, 0.03, 0.02) down to a temperature given by log T = 5.5 or 6.0.

It is seen from Fig. 3 that the effect from helium depletion is very significant. Compared with standard models, periods are changed a few per cent which gives period ratio changes that are sometimes higher than those resulting from a composition change from Pop. I to extreme Pop. II. Therefore, this effect must be taken into account when period ratios are discussed, if even a small helium depletion occurs in δ Sct variables. Clearly, surface helium depletion is a possible alternative to (mild) Pop. II composition for the explanation of the typical period ratios $\Pi_1/\Pi_0 \cong 0.77$ of the high amplitude δ Sct stars. However, for simplicity we should stick to the homogeneous standard models as far as they are in agreement with observational data.

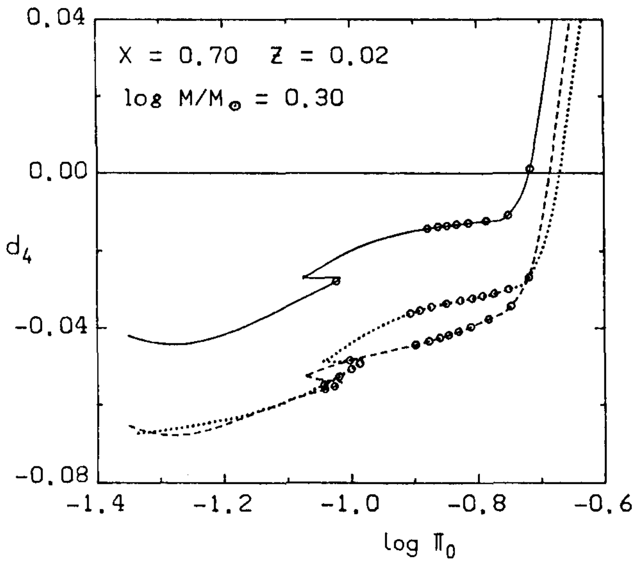

Fig. 4. Resonance distance as function of fundamental period in selected evolution tracks. Coding as in Fig. 3.

3. RESONANCES

Simon (1979) proposed that the maintenance of stable double mode pulsation involves a resonant interaction between the two modes. For the high amplitude δ Sct variables the resonance $\omega_0 + \omega_1 = \omega_4$ with the normalized resonance distance

$$d_4 = 1 - \Pi_4/\Pi_0 - \Pi_4/\Pi_1$$

was suggested. Simon concluded that the resonance idea seems to rule out normal Pop. I masses for the δ Sct variables. However, observational evidence to-day strongly favours normal masses, and more recent theoretical studies by Cox et al. (1979) and Petersen (1979) give pulsation periods and resonance distances that disagree with those of Simon. These studies are all based upon envelope models.

Fig. 4 shows three evolution tracks on the (log Π_0, d_4) plane. Typical resonance distances are - 0.06 to - 0.02 in the δ Sct phase. Homogeneous models generally have small resonance distances (but do not give precise resonance), while two-zone models are farther from resonance. This is in agreement with Petersen (1979). A survey of alternative simple resonance possibilities (cf. Petersen, 1979) show no attractive case.

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DISCUSSION

BREGER: Could you say more about the resonance distance d_4?

PETERSEN: Yes. These resonance distances were introduced by Simon. You can construct an evolutionary sequence and get the resonance distance for each model. It was hoped, originally, that these resonances could explain the double mode phenomenon. I think we must say today that this is an open question, but if you take these values literally, then the resonance argument favors homogeneous models rather than the two zone models. Perhaps Dr. Simon would like to comment on this point.

SIMON: The resonance was only applied to seven or eight double mode stars with stable behavior. My understanding of my calculations is that if you use standard population I δ Scuti models you just can't get the resonance. If those models are correct, then the resonance hypothesis is wrong.

A. COX: I would just like to mention that you do have a resonance at half a day. Do I understand that?

PETERSEN: But that is just because you reach the right boundary of the instability strip and the stars are no longer variable. I should have explained in more detail. The circles represent models inside the instability strip which we have chosen from diagrams like that shown by Dr. Breger. The stars to the right are not variables, but, of course, when you calculate the evolutionary tracks you get the resonance distance for all models.

J. COX: Are your pulsation calculations adiabatic or nonadiabatic?

PETERSEN: My pulsation calculations are very simple, linear adiabatic ones, but I think it has been investigated and for δ Scuti stars the differences are extremely small.

STOBIE: My question is related to that. If you make helium poor models, are they unstable?

PETERSEN: I have not investigated the question of stability, but other people have studied this question.

A. COX: With regard to the low helium I don't remember the answer, but it seems to me we got pulsations with $Y = 0.1$.