13

Coincidence experiments (e, e′ X)

With the advent of high-energy, high-intensity, high-resolution electron accelerators with continuous beams (c.w.), a whole new class of coincidence reactions becomes accessible. It is important to have a detailed understanding of such processes. In this section, a covariant analysis of the amplitude and cross section for the coincidence reaction (e, e′ X) will be developed. The results will be exact with one photon exchange, that is, to order α^2 in the cross section. The particle X can be anything. The kinematic situation is illustrated in Fig. 13.1. The four-momentum transfer from the electron is now consistently denoted by

$$\begin{aligned} k &\equiv k_1 - k_2 \\ k^2 &= -2k_1 \cdot k_2 \\ &= 4\varepsilon_1\varepsilon_2 \sin^2 \frac{\theta}{2} \qquad ; \text{lab frame} \end{aligned} \tag{13.1}$$

The second relation holds for relativistic (massless) electrons, and the third relation holds in the laboratory frame. The four-momentum of the emitted particle X will be consistently denoted by $q = (\mathbf{q}, i\omega_q)$, and conservation

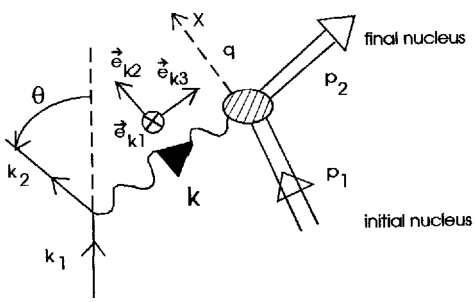

Fig. 13.1. Kinematic situation for a (e, e′ X) coincidence experiment.

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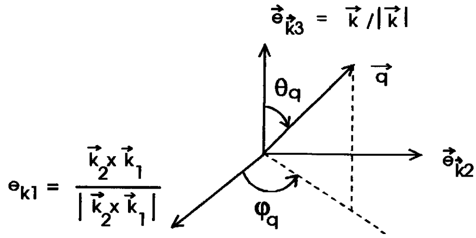

Fig. 13.2. Angles for particle X in the C-M system.

of four-momentum states that

$$k + p_1 = q + p_2 \tag{13.2}$$

Here $p_1(p_2)$ are the four-momenta of the initial (final) nucleus or nucleon.

Two distinct Lorentz frames are of primary interest. The center of momentum (C-M) frame is defined by the relation

$$q + p_2 = k + p_1 = (\mathbf{0}, iW) \qquad ; \text{C-M frame} \tag{13.3}$$

Note that W is the total energy in the C-M frame. The laboratory frame is defined by

$$p_1 = (\mathbf{0}, iM_1) \qquad ; \text{lab frame} \tag{13.4}$$

The C-M frame is reached from the laboratory frame by making a Lorentz transformation along the direction of the three-momentum transfer **k**.

Introduce the orthonormal system of unit vectors in the laboratory (lab), as defined in Fig. 13.1

$$\mathbf{e}_{k3} \equiv \frac{\mathbf{k}}{|\mathbf{k}|} \qquad ; \mathbf{e}_{k1} \equiv \frac{\mathbf{k}_2 \times \mathbf{k}_1}{|\mathbf{k}_2 \times \mathbf{k}_1|} \qquad ; \mathbf{e}_{k2} = \mathbf{e}_{k3} \times \mathbf{e}_{k1} \tag{13.5}$$

It is important to note that since $\mathbf{e}_{k1}$ and $\mathbf{e}_{k2}$ are transverse to **k**, they are *unchanged* under the Lorentz transformation along **k** from the lab to the C-M system. $\mathbf{e}_{k3}$, defined as the third unit vector in this orthonormal system, is thus also uniquely defined in the C-M system (it lies along the direction of **k**).

In addition, we define the angles (θ_q, ϕ_q) that the particle X makes with respect to this orthonormal basis as seen in the C-M frame; this is indicated in Fig. 13.2.

From the general discussion of electron scattering in chapter 11,

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one has

$$
\begin{aligned}
d\sigma &= \frac{4\alpha^2}{k^4}\frac{d^3k_2}{2\varepsilon_2}\frac{1}{\sqrt{(k_1\cdot p_1)^2}}\eta_{\mu\nu}W_{\mu\nu} \\
\eta_{\mu\nu} &= k_{1\mu}k_{2\nu}+k_{1\nu}k_{2\mu}-(k_1\cdot k_2)\,\delta_{\mu\nu} \\
W_{\mu\nu} &= (2\pi)^3\overline{\sum_i}\sum_f \delta^{(4)}(p'-p_1-k)\langle i|J_\nu(0)|f\rangle\langle f|J_\mu(0)|i\rangle(\Omega E_1)
\end{aligned} \tag{13.6}
$$

For definiteness and clarity, specify to a two-particle final state of particle X plus a second nucleus or nucleon (denoted with subscript 2)[1]

$$
\langle f|J_\mu(0)|i\rangle = \langle p_2q^{(-)}|J_\mu(0)|p_1\rangle \tag{13.7}
$$

Here $|p_2q^{(-)}\rangle$ is an exact eigenstate of the total hamiltonian; it is a two-particle scattering state with incoming wave boundary conditions. To go to states with Lorentz invariant norm, one defines (c.f. chapter 12)

$$
J_\mu \equiv \left(\frac{2\omega_q E_1E_2\Omega^3}{M_1M_2}\right)^{1/2}\langle p_2q^{(-)}|J_\mu(0)|p_1\rangle \tag{13.8}
$$

Here $J_\mu = (\mathbf{J}, iJ_0)$, and this quantity now properly transforms as a four-vector under Lorentz transformations. The hadronic response tensor then takes the form

$$
\begin{aligned}
W_{\mu\nu} &= (2\pi)^3\overline{\sum_i}{\sum_f}' \frac{\Omega d^3q}{(2\pi)^3}\frac{\Omega d^3p_2}{(2\pi)^3}\delta^{(4)}(p_2+q-p_1-k) \\
&\quad\times\frac{M_1M_2}{2\omega_qE_1E_2\Omega^3}(\Omega E_1)J^\star_\nu J_\mu
\end{aligned} \tag{13.9}
$$

Here $\sum'$ indicates a sum over all the remaining variables. The complex four-vector $J^\star_\nu$ is defined by

$$
J^\star_\nu \equiv (\mathbf{J}^\star, iJ^\star_0) \tag{13.10}
$$

Thus

$$
W_{\mu\nu} = \frac{2M_1M_2}{(2\pi)^3}\overline{\sum_i}{\sum_f}'\delta^{(4)}(p_2+q-p_1-k)\frac{d^3q}{2\omega_q}\frac{d^3p_2}{2E_2}J^\star_\nu J_\mu \tag{13.11}
$$

This expression is now manifestly Lorentz covariant.

[1] As long as one sums over everything else in $\sum_f$ the subsequent results for the general form of the coincidence cross section hold for arbitrary nuclear final states.

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Consider next the Lorentz invariant combination $\eta_{\mu\nu}J_\nu^\star J_\mu$. It follows from Eq. (13.6) that

$$\eta_{\mu\nu}J_\nu^\star J_\mu = (k_1\cdot J^\star)(k_2\cdot J)+(k_2\cdot J^\star)(k_1\cdot J)-(k_1\cdot k_2)(J^\star\cdot J) \tag{13.12}$$

Current conservation states that

$$\begin{aligned} k\cdot J &= 0 \\ k_1\cdot J &= k_2\cdot J \end{aligned} \tag{13.13}$$

Hence

$$\eta_{\mu\nu}J_\nu^\star J_\mu = 2(k_1\cdot J^\star)(k_1\cdot J)+\frac{k^2}{2}J^\star\cdot J \tag{13.14}$$

This expression is explicitly Lorentz invariant. Let us proceed to evaluate it in the C-M frame. Since $\mathbf{k}_1$ has no projection on $\mathbf{e}_{k1}$, which is perpendicular to the electron scattering plane, one can write in the C-M system (recall $k_1^2=0$)

$$k_{1\mu} = [(\mathbf{k}_1\cdot\mathbf{e}_2)\mathbf{e}_2+(\mathbf{k}_1\cdot\mathbf{e}_3)\mathbf{e}_3, ik_1] \tag{13.15}$$

Now use current conservation

$$\mathbf{e}_3\cdot\mathbf{J} = \frac{\mathbf{k}\cdot\mathbf{J}}{|\mathbf{k}|} = \frac{\omega_k J_0}{|\mathbf{k}|} \tag{13.16}$$

Thus

$$k_1\cdot J = (\mathbf{k}_1\cdot\mathbf{e}_2)(\mathbf{e}_2\cdot\mathbf{J})+\left[\frac{\omega_k}{\mathbf{k}^2}(\mathbf{k}_1\cdot\mathbf{k})-k_1\right]J_0 \tag{13.17}$$

The Coulomb amplitude is defined by $J_0\equiv J_C$, hence

$$\begin{aligned} J_\mu &= (\mathbf{J}, iJ_C) \\ J^\star\cdot J &= |\mathbf{J}_\perp|^2+|\mathbf{J}\cdot\mathbf{e}_3|^2-|J_C|^2 \\ |\mathbf{J}_\perp|^2 &\equiv |\mathbf{J}\cdot\mathbf{e}_1|^2+|\mathbf{J}\cdot\mathbf{e}_2|^2 \end{aligned} \tag{13.18}$$

Use current conservation again

$$\begin{aligned} J^\star\cdot J &= |\mathbf{J}_\perp|^2+\left(\frac{\omega_k^2}{\mathbf{k}^2}-1\right)|J_C|^2 \\ &= |\mathbf{J}_\perp|^2-\frac{k^2}{\mathbf{k}^2}|J_C|^2 \end{aligned} \tag{13.19}$$

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A combination of these results yields the following expression in the C-M system

$$
\begin{aligned}
\eta_{\mu\nu} J_\nu^\star J_\mu &= 2\left\{ \frac{1}{4}k^2 |\mathbf{J}_\perp|^2 + (\mathbf{k}_1 \cdot \mathbf{e}_2)^2 |\mathbf{e}_2 \cdot \mathbf{J}|^2 \right. \\
&\quad + |J_C|^2 \left[\left(\frac{\omega_k}{\mathbf{k}^2}(\mathbf{k}_1 \cdot \mathbf{k}) - k_1 \right)^2 - \frac{k^4}{4\mathbf{k}^2} \right] \\
&\quad \left. + (\mathbf{k}_1 \cdot \mathbf{e}_2) \left[\frac{\omega_k}{\mathbf{k}^2}(\mathbf{k}_1 \cdot \mathbf{k}) - k_1 \right] 2\,\mathrm{Re}\,[(\mathbf{e}_2 \cdot \mathbf{J})\, J_C^\star] \right\}_{\mathrm{CM}} \qquad (13.20)
\end{aligned}
$$

The next step is to re-express the electron variables appearing in this expression in the laboratory frame. Start by observing that the combination $\mathbf{k}_1 \cdot \mathbf{e}_2$ is transverse and hence unaffected by the Lorentz transformation from the lab to the C-M system

$$
\begin{aligned}
\mathbf{k}_1 \cdot \mathbf{e}_2 &= \mathbf{k}_1 \cdot (\mathbf{e}_3 \times \mathbf{e}_1) = \mathbf{k}_1 \cdot \left[\frac{\mathbf{k}}{|\mathbf{k}|} \times \frac{(\mathbf{k}_2 \times \mathbf{k}_1)}{|\mathbf{k}_2 \times \mathbf{k}_1|} \right] \\
&= \frac{1}{|\mathbf{k}| k_1 k_2 \sin\theta} [(\mathbf{k}_1 \cdot \mathbf{k}_2)(\mathbf{k}_1 \cdot \mathbf{k}) - \mathbf{k}_1^2 (\mathbf{k}_2 \cdot \mathbf{k})] \\
&= \frac{1}{|\mathbf{k}| k_1 k_2 \sin\theta} [(\mathbf{k}_1 \cdot \mathbf{k}_2)(\mathbf{k}_1^2 - \mathbf{k}_1 \cdot \mathbf{k}_2) - \mathbf{k}_1^2 (\mathbf{k}_1 \cdot \mathbf{k}_2 - \mathbf{k}_2^2)] \\
&= \frac{1}{|\mathbf{k}| k_1 k_2 \sin\theta} \varepsilon_1^2 \varepsilon_2^2 \sin^2\theta \\
\{\mathbf{k}_1 \cdot \mathbf{e}_2\}_{\mathrm{CM}} &= \frac{\varepsilon_1 \varepsilon_2 \sin\theta}{\kappa} \qquad ;\ \text{lab variables} \qquad (13.21)
\end{aligned}
$$

Here κ is now the three-momentum transfer in the lab frame

$$
\begin{aligned}
\kappa &\equiv |\mathbf{k}|_{\mathrm{lab}} \\
&= \sqrt{\varepsilon_1^2 + \varepsilon_2^2 - 2\varepsilon_1 \varepsilon_2 \cos\theta} \qquad (13.22)
\end{aligned}
$$

To distinguish C-M variables, the four-momentum transfer as seen in the C-M system will be written in the final expressions as

$$
k_\mu \equiv (\mathbf{k}^\star, i\omega_k^\star) \qquad ;\ \text{C-M frame} \qquad (13.23)
$$

Then with the aid of Eq. (13.3), which defines the C-M frame, one can write

$$
\begin{aligned}
\mathbf{k}^{\star 2} &= k^2 + \omega_k^{\star 2} = k^2 - \frac{[k \cdot (p_1 + k)]^2}{(p_1 + k)^2} \\
&= \frac{1}{(p_1 + k)^2} [k^2 p_1^2 + 2k^2 (p_1 \cdot k) + k^4 - (k \cdot p_1)^2 - 2k^2 (p_1 \cdot k) - k^4] \\
&= \frac{1}{(p_1 + k)^2} [k^2 p_1^2 - (k \cdot p_1)^2] \qquad (13.24)
\end{aligned}
$$

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This expression is now in invariant form and can be evaluated in the lab frame defined by Eq. (13.4) to give

$$k^\star = \frac{M_1}{W}\kappa \qquad ; \text{ lab variables} \qquad (13.25)$$

Note that W is expressed in terms of lab variables by

$$\begin{aligned} W^2 &= -(p_1+k)^2 \\ &= M_1^2 - 2p_1\cdot k - k^2 \\ &= M_1^2 + 2M_1(\varepsilon_1-\varepsilon_2) - 4\varepsilon_1\varepsilon_2 \sin^2\frac{\theta}{2} \quad ; \text{ lab variables} \end{aligned} \qquad (13.26)$$

Next use (for massless electrons)

$$k_1\cdot k = -k_1\cdot k_2 = \frac{1}{2}k^2 \qquad (13.27)$$

to work out in the C-M system

$$\begin{aligned} \left[\frac{\omega_k}{\mathbf{k}^2}(\mathbf{k}_1\cdot\mathbf{k}) - k_1\right]^2_{\text{CM}} &= \frac{1}{\mathbf{k}^4}[\omega_k(k_1\cdot k + k_1\omega_k) - k_1(k^2+\omega_k^2)]^2 \\ &= \frac{k^4}{\mathbf{k}^4}\left[\frac{1}{2}\omega_k - k_1\right]^2 \\ &= \frac{k^4}{\mathbf{k}^4}\left[\frac{-1}{(p_1+k)^2}\right]\left\{-\frac{1}{2}k\cdot(p_1+k) + k_1\cdot(p_1+k)\right\}^2 \\ &= -\frac{k^4}{4k^{\star 4}}\frac{1}{(p_1+k)^2}[p_1\cdot(k_1+k_2)]^2 \end{aligned} \qquad (13.28)$$

This is also now in invariant form [note Eq. (13.24)] and can be evaluated in the lab frame to yield

$$\left\{\left[\frac{\omega_k}{\mathbf{k}^2}(\mathbf{k}_1\cdot\mathbf{k}) - k_1\right]^2\right\}_{\text{CM}} = \frac{k^4}{k^{\star 4}}\frac{M_1^2}{4W^2}(\varepsilon_1+\varepsilon_2)^2 \quad ; \text{ lab variables} \qquad (13.29)$$

Now in the lab

$$k^2 = \kappa^2 - (\varepsilon_1-\varepsilon_2)^2 \qquad (13.30)$$

Thus

$$\begin{aligned} (\varepsilon_1+\varepsilon_2)^2 &= \kappa^2 - k^2 + 4\varepsilon_1\varepsilon_2 \\ &= \kappa^2 - 4\varepsilon_1\varepsilon_2\sin^2\frac{\theta}{2} + 4\varepsilon_1\varepsilon_2 \\ &= \kappa^2 + 4\varepsilon_1\varepsilon_2\cos^2\frac{\theta}{2} \end{aligned}$$

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$$= 4\varepsilon_1\varepsilon_2\cos^2\frac{\theta}{2}\left(1+\frac{\kappa^2}{k^2}\tan^2\frac{\theta}{2}\right)$$

$$= 4\varepsilon_1\varepsilon_2\cos^2\frac{\theta}{2}\left(\frac{k^{\star 2}}{k^2}\right)\left(\frac{k^2}{k^{\star 2}}+\frac{W^2}{M_1^2}\tan^2\frac{\theta}{2}\right) \quad (13.31)$$

Also, since $\kappa^2 = \varepsilon_1^2+\varepsilon_2^2-2\varepsilon_1\varepsilon_2\cos\theta$,

$$\left\{\left[\frac{\omega_k}{\mathbf{k}^2}(\mathbf{k}_1\cdot\mathbf{k})-k_1\right]^2-\frac{k^4}{4\mathbf{k}^2}\right\}_{\text{CM}} = \frac{k^4}{k^{\star 4}}\left[\frac{M_1^2}{4W^2}(\varepsilon_1+\varepsilon_2)^2 - \frac{1}{4}\frac{M_1^2}{W^2}(\varepsilon_1^2+\varepsilon_2^2-2\varepsilon_1\varepsilon_2\cos\theta)\right]$$

$$= \frac{k^4}{k^{\star 4}}\frac{M_1^2}{W^2}\varepsilon_1\varepsilon_2\cos^2\frac{\theta}{2}\ \ ;\ \text{lab variables} \quad (13.32)$$

Note that since $k^2 = \mathbf{k}^2-\omega_k^2 \geq 0$ in electron scattering, one can determine the sign of the quantity in square brackets in Eq. (13.29) as

$$k_1 - \frac{\omega_k(\mathbf{k}\cdot\mathbf{k}_1)}{\mathbf{k}^2} \geq 0 \quad (13.33)$$

In *summary* the expressions involving the electron variables in the cross section are Lorentz transformed from the C-M to the laboratory frame according to

$$\{(\mathbf{k}_1\cdot\mathbf{e}_2)^2\}_{\text{CM}} = \varepsilon_1\varepsilon_2\cos^2\frac{\theta}{2}\left(\frac{M_1^2}{W^2}\frac{k^2}{k^{\star 2}}\right) \quad (13.34)$$

$$\left\{\left[\frac{\omega_k}{\mathbf{k}^2}(\mathbf{k}_1\cdot\mathbf{k})-k_1\right]^2\right\}_{\text{CM}} = \varepsilon_1\varepsilon_2\cos^2\frac{\theta}{2}\left(\frac{M_1^2}{W^2}\frac{k^2}{k^{\star 2}}\right)\times\left(\frac{k^2}{k^{\star 2}}+\frac{W^2}{M_1^2}\tan^2\frac{\theta}{2}\right)$$

$$\left\{\left[\frac{\omega_k}{\mathbf{k}^2}(\mathbf{k}_1\cdot\mathbf{k})-k_1\right]^2-\frac{k^4}{4\mathbf{k}^2}\right\}_{\text{CM}} = \varepsilon_1\varepsilon_2\cos^2\frac{\theta}{2}\left(\frac{M_1^2}{W^2}\frac{k^2}{k^{\star 2}}\right)\frac{k^2}{k^{\star 2}}$$

Here

$$W^2 = -(p_1+k)^2 = M_1^2+2M_1(\varepsilon_1-\varepsilon_2)-4\varepsilon_1\varepsilon_2\sin^2\frac{\theta}{2}$$

$$\mathbf{k}^{\star 2} = k^2-\frac{[k\cdot(p_1+k)]^2}{(p_1+k)^2} = \frac{M_1^2}{W^2}|\mathbf{k}_{\text{lab}}|^2 \quad (13.35)$$

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are respectively the squares of the total energy and three-momentum transfer in the C-M system. The quantities $(\varepsilon_1, \varepsilon_2, \theta)$ with $\mathbf{k}_{\text{lab}}^2 = \varepsilon_1^2 + \varepsilon_2^2 - 2\varepsilon_1\varepsilon_2 \cos\theta$ are the electron scattering variables in the lab.

The remaining task is to work out the phase space integral. The Lorentz invariant expression is

$$\Phi \equiv \int \frac{d^3q}{2\omega_q} \int \frac{d^3p_2}{2E_2}\, \delta^{(4)}(k_1 + p_1 - k_2 - p_2 - q) \tag{13.36}$$

We choose to evaluate this in the C-M frame. The $\int d^3p_2$ can be immediately evaluated with the aid of the $\delta^{(3)}$ to give

$$\begin{aligned} \Phi &= \int \frac{q^2 d\Omega_q}{4\omega_q E_2} \left(\frac{\partial q}{\partial W_f}\right) \delta(W_f - W_i) dW_f \\ &= \frac{q^2}{4\omega_q E_2} \left(\frac{\partial q}{\partial W_f}\right) d\Omega_q \end{aligned} \tag{13.37}$$

Next use

$$\begin{aligned} W_f &= \sqrt{\mathbf{q}^2 + m_X^2} + \sqrt{\mathbf{q}^2 + M_2^2} & ;\ W_i \equiv W \\ \frac{\partial W_f}{\partial q} &= \frac{q}{\omega_q} + \frac{q}{E_2} = \frac{qW}{\omega_q E_2} \end{aligned} \tag{13.38}$$

One has finally

$$\int \frac{d^3q}{2\omega_q} \int \frac{d^3p_2}{2E_2}\, \delta^{(4)}(k_1 + p_1 - k_2 - p_2 - q) = \frac{q}{4W} d\Omega_q \ ; \text{C-M frame} \tag{13.39}$$

Note that the first of Eqs. (13.38) allows a determination of $q(W)$.

The above results are now combined to yield the laboratory cross section

$$\begin{aligned} d\sigma &= \frac{4\alpha^2}{k^4} \frac{d^3k_2}{2\varepsilon_2} \frac{1}{\sqrt{(k_1 \cdot p_1)^2}} \eta_{\mu\nu} W_{\mu\nu} \\ &= \frac{4\alpha^2}{k^4} \frac{\varepsilon_2^2 d\varepsilon_2 d\Omega_2}{2\varepsilon_2} \frac{1}{M_1\varepsilon_1} \frac{2M_1M_2}{(2\pi)^3} \left(\frac{q}{4W} d\Omega_q\right) 2 \left\{ \left(\frac{M_1^2}{W^2}\right) \varepsilon_1\varepsilon_2 \cos^2\frac{\theta}{2} \right. \\ &\quad \times \left[\frac{W^2}{M_1^2} \tan^2\frac{\theta}{2} |\mathbf{J}_\perp|^2 + \frac{k^2}{k^{\star 2}} |\mathbf{J}\cdot\mathbf{e}_2|^2 + \frac{k^4}{k^{\star 4}} |J_C|^2 \right. \\ &\quad \left.\left. - \frac{k^2}{k^{\star 2}} \left(\frac{k^2}{k^{\star 2}} + \frac{W^2}{M_1^2} \tan^2\frac{\theta}{2} \right)^{1/2} 2\,\mathrm{Re}\, J_C^\star (\mathbf{J}\cdot\mathbf{e}_2) \right] \right\} \end{aligned} \tag{13.40}$$

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Define[2]

$$\mathscr{J}_\mu = \frac{\sqrt{M_1 M_2}}{4\pi W} J_\mu = \frac{\sqrt{M_1 M_2}}{4\pi W}\left(\frac{2\omega_q E_1 E_2 \Omega^3}{M_1 M_2}\right)^{1/2} \langle q p_2^{(-)} | J_\mu(0) | p_1 \rangle \quad (13.41)$$

The differential cross section in the lab is then given by

$$\frac{d^5\sigma}{d\varepsilon_2 d\Omega_2 d\Omega_q} = \sigma_M \left(\frac{qM_1}{\pi W}\right) \left\{ \frac{k^4}{k^{\star 4}} |\mathscr{J}_C|^2 + \frac{k^2}{k^{\star 2}} |\mathscr{J}\cdot \mathbf{e}_2|^2 + \frac{W^2}{M_1^2}\tan^2\frac{\theta}{2}\,|\mathscr{J}_\perp|^2 \right.$$
$$\left. - \frac{k^2}{k^{\star 2}}\left(\frac{k^2}{k^{\star 2}} + \frac{W^2}{M_1^2}\tan^2\frac{\theta}{2}\right)^{1/2} 2\,\mathrm{Re}\,[\mathscr{J}_C^\star(\mathscr{J}\cdot\mathbf{e}_2)] \right\} \quad (13.42)$$

Here $(\varepsilon_1, \varepsilon_2, \theta)$ are electron scattering variables in the lab, and $[W, q(W), k^\star, \theta_q, \phi_q]$ are C-M variables, the first three of which can be calculated in terms of electron lab variables by utilizing the Lorentz invariant expressions in Eqs. (13.35). The current is evaluated in the C-M system.

It is useful to rewrite this cross section in terms of helicity polarization vectors for the virtual photon.[3] Define helicity unit vectors (see Fig. 13.1) according to

$$\mathbf{e}_{\mathbf{k}\pm 1} = \mp\frac{1}{\sqrt{2}}(\mathbf{e}_{\mathbf{k}1} \pm i\mathbf{e}_{\mathbf{k}2}) \quad (13.43)$$

Since these are still transverse, they are also unchanged under the Lorentz transformation from the lab to the C-M system. Inversion of the definition gives (we again suppress the **k** subscript)

$$\begin{aligned} \mathbf{e}_2 &= \frac{i}{\sqrt{2}}(\mathbf{e}_{+1} + \mathbf{e}_{-1}) \\ \mathbf{e}_1 &= \frac{1}{\sqrt{2}}(\mathbf{e}_{-1} - \mathbf{e}_{+1}) \end{aligned} \quad (13.44)$$

Define

$$\mathscr{J}^\lambda \equiv \mathbf{e}_\lambda \cdot \mathscr{J} \quad (13.45)$$

It follows that

$$\begin{aligned} |\mathscr{J}_\perp|^2 &= |\mathscr{J}\cdot\mathbf{e}_1|^2 + |\mathscr{J}\cdot\mathbf{e}_2|^2 = |\mathscr{J}^{+1}|^2 + |\mathscr{J}^{-1}|^2 \\ |\mathscr{J}\cdot\mathbf{e}_2|^2 &= \frac{1}{2}|\mathscr{J}_\perp|^2 + \mathrm{Re}\,(\mathscr{J}^{+1})^\star(\mathscr{J}^{-1}) \\ 2\,\mathrm{Re}\,\mathscr{J}_C^\star(\mathscr{J}\cdot\mathbf{e}_2) &= -\sqrt{2}\,\mathrm{Im}\,\mathscr{J}_C^\star(\mathscr{J}^{+1} + \mathscr{J}^{-1}) \end{aligned} \quad (13.46)$$

[2] By looking at a simple example for the matrix element, the reader can establish that this expression still has dimensions $[M]^{-1}$.
[3] Think of this as the *annihilation* of a photon.

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Thus one arrives at the basic result for the (e, e′ X) coincidence cross section in the laboratory frame

$$\frac{d^5\sigma}{d\varepsilon_2 d\Omega_2 d\Omega_q} = \sigma_M \left(\frac{qM_1}{\pi W}\right) \left[\frac{k^4}{k^{\star 4}}|\mathscr{J}_C|^2 + \left(\frac{k^2}{2k^{\star 2}} + \frac{W^2}{M_1^2}\tan^2\frac{\theta}{2}\right)|\mathscr{J}_\perp|^2 \right.$$
$$+\frac{k^2}{2k^{\star 2}} 2\,\mathrm{Re}\,(\mathscr{J}^{+1})^\star(\mathscr{J}^{-1})$$
$$\left. +\frac{k^2}{k^{\star 2}}\left(\frac{k^2}{k^{\star 2}} + \frac{W^2}{M_1^2}\tan^2\frac{\theta}{2}\right)^{1/2} \sqrt{2}\,\mathrm{Im}\,\mathscr{J}_C^\star(\mathscr{J}^{+1}+\mathscr{J}^{-1})\right] \quad (13.47)$$

In this expression $k^\star$ is the three-momentum transfer, W is the total energy, and $q = |\mathbf{q}|$ and $d\Omega_q$ refer to the momentum of particle X, all in the C-M system. The electron variables $(k^2, k^\star, W, \theta)$ appearing in the cross section are functions of $(k^2, k\cdot p_1, \theta)$ where θ is the electron scattering angle in the laboratory frame. The appropriate relations for $(k^\star, W)$ as functions of $(k^2, k\cdot p_1)$ are given in Eqs. (13.35). There are three independent electron scattering variables in the lab, $(\varepsilon_1, \varepsilon_2, \theta)$; hence it is possible to fix $(k^2, k\cdot p_1)$ and vary θ. The current is evaluated in the C-M system.

There are four target responses appearing in the cross section expressed as bilinear combinations of current matrix elements where the current is defined by Eq. (13.41) with $\mathscr{J}_\mu \equiv (\boldsymbol{\mathscr{J}}, i\mathscr{J}_C)$ and $\mathscr{J}^\lambda \equiv \mathbf{e}_\lambda \cdot \boldsymbol{\mathscr{J}}$. These four responses are functions of the variables $(k^2, W, \theta_q, \phi_q)$ or $(k^2, k\cdot p_1, \theta_q, \phi_q)$. The dependence on the angle variables will be made explicit in the subsequent analysis. The dependence on the "out-of-plane" angle ϕ_q, whose content must be transmitted through the virtual photon, turns out to be particularly simple. It is explicitly exhibited as

$$\begin{array}{rcl} |\mathscr{J}_C|^2 & & \\ |\mathscr{J}^{+1}|^2 + |\mathscr{J}^{-1}|^2 & & \\ 2\,\mathrm{Re}\,(\mathscr{J}^{+1})^\star(\mathscr{J}^{-1}) & \propto & \cos 2\phi_q \\ \sqrt{2}\,\mathrm{Im}\,\mathscr{J}_C^\star(\mathscr{J}^{+1}+\mathscr{J}^{-1}) & \propto & \sin\phi_q \end{array} \quad (13.48)$$

The dependence on (θ, ϕ_q) in Eqs. (13.47, 13.48) now allows a complete kinematic separation of the four target response functions at fixed $(k^2, k\cdot p_1, \theta_q)$, or equivalently fixed (k^2, W, θ_q). Since the term in $\cos 2\phi_q$ takes the same value at $\phi_q = \pi/2$ and $\phi_q = 3\pi/2$, for which the reaction plane and electron scattering plane in Fig. 13.1 coincide, an out-of-plane measurement is needed to separate its contribution. Conversely, the term in $\sin\phi_q$ can be isolated with two in-plane measurements at these two values.

This derivation is from appendix C of [Pr69]. Other work on coincidence experiments is contained in [de67, Wa79, Kl83]. Work on coinci-

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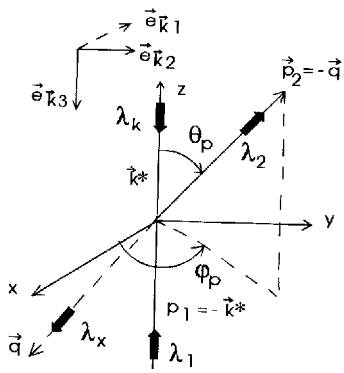

Fig. 13.3. Configuration for helicity analysis of current matrix elements in the C-M system. Here everything is referred to the incoming and outgoing target states with momenta and helicities $(\mathbf{p}_1\lambda_1)$ and $(\mathbf{p}_2\lambda_2)$ respectively. Note how the (x, y, z) coordinate system is related to the original system defined by $\mathbf{e}_i$ with $i = 1, 2, 3$. Note in particular the relations $\theta_q = \theta_p$ and $\phi_q + \phi_p = 2\pi$.

dence experiments in pion electroproduction is contained in [Be66, Pr70]. Coincidence experiments with a polarized electron beam are discussed in [Ad68, Ra89] [4] and with both a polarized electron beam and polarized target in [Ra89].

The next step is to demonstrate the angular dependence in the nuclear matrix elements. This will be done through the use of a *helicity analysis* of the current matrix elements in the C-M system. Let us go back to the form of the cross section before the $\overline{\sum_i} \sum_f$ has been carried out. The cross section is then being calculated for given initial and final helicities of all the particles in the C-M system, and of the virtual photon. The situation is illustrated in Fig. 13.3. The analysis parallels that of Jacob and Wick [Ja59]. First, recall some of the basic results from that work

For two-particles in the C-M system, the transformation from a state where the relative momentum is directed at an angle (θ, ϕ), to a state of definite angular momentum (J, M) is given by

$$\langle JM\lambda_1'\lambda_2'|\theta\phi\lambda_1\lambda_2\rangle = \delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'}\left(\frac{2J+1}{4\pi}\right)^{1/2}\mathscr{D}^J_{M,\lambda}(-\phi, -\theta, \phi) \qquad (13.49)$$

Here $\lambda \equiv \lambda_1 - \lambda_2$ is the net helicity of the state. We have seen this expression before as the "photon wave function" in Eq. (9.35). The transformation in Eq. (13.49) is unitary.

[4] In a coincidence reaction with a polarized electron beam $(\vec{e}, e'X)$ there is an additional, *fifth response function*, sensitive to final-state interactions, which can only be accessed with out-of-plane measurements [Ra89].

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The S-matrix for an arbitrary two-particle process in the C-M system can be written as

$$\langle \mathbf{p}_c\mathbf{p}_d\lambda_c\lambda_d|S|\mathbf{p}_a\mathbf{p}_b\lambda_a\lambda_b\rangle = \frac{(2\pi)^4}{\Omega^2}\delta^{(4)}(P_\mu - P'_\mu)\left[\frac{(2\pi)^2\sqrt{v'v}}{p'p}\right] \times\langle\theta'\phi'\lambda_c\lambda_d|S(P_\mu)|\theta\phi\lambda_a\lambda_b\rangle \tag{13.50}$$

Here (p, v) are relative momenta and velocity in the C-M system and P is the total four-momentum in that frame. For transitions, the S-matrix is related to the T-matrix by $S = 1 + iT$.

With the aid of completeness, one then establishes the following relation for the required S-matrix in the C-M system

$$\langle\theta\phi\lambda_c\lambda_d|S(W)|0\,0\lambda_a\lambda_b\rangle = \sum_{JM}\sum_{J'M'}\langle\theta\phi\lambda_c\lambda_d|JM\lambda_c\lambda_d\rangle \times\langle JM\lambda_c\lambda_d|S(W)|J'M'\lambda_a\lambda_b\rangle\langle J'M'\lambda_a\lambda_b|00\lambda_a\lambda_b\rangle \tag{13.51}$$

The scattering operator S is a scalar under rotations; it commutes with the angular momentum operator **J**. The Wigner–Eckart theorem then implies that the matrix element of S must be diagonal in J and independent of M. Use

$$\left(\frac{2J+1}{4\pi}\right)^{1/2}\mathscr{D}^J_{M,\lambda}(0,0,0) = \left(\frac{2J+1}{4\pi}\right)^{1/2}\delta_{M\lambda} \tag{13.52}$$

Here the initial angular momentum along the z-axis is $M = \lambda = \lambda_a - \lambda_b$. A combination of the above results then yields the expression

$$\langle\theta\phi\lambda_c\lambda_d|S(W)|0\,0\lambda_a\lambda_b\rangle = \sum_J\left(\frac{2J+1}{4\pi}\right)\mathscr{D}^J_{\lambda_i,\lambda_f}(-\phi,-\theta,\phi)^\star\langle\lambda_c\lambda_d|S^J(W)|\lambda_a\lambda_b\rangle$$
$$;\ \lambda_i = \lambda_a - \lambda_b \qquad ;\ \lambda_f = \lambda_c - \lambda_d \tag{13.53}$$

There are various conditions on the helicity matrix elements of the scattering operator that follow from unitarity and symmetry properties of the strong interactions. Parity invariance essentially cuts the number of independent matrix elements in half. The parity operator reflects momentum and leaves particle spins unchanged; hence it reflects the helicity. It leaves the angular momentum and z-component of the angular momentum unchanged. The parity operator thus has the following effect on a two-particle state [Ja59]

$$P|JM\lambda_1\lambda_2\rangle = (-1)^{J-S_1-S_2}\eta_1\eta_2|JM - \lambda_1 - \lambda_2\rangle \tag{13.54}$$

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Here the η_i are intrinsic parities and the overall phase is conventional.[5] If the scattering operator is invariant under the parity transformation, i.e. if it commutes with P, then [Ja59]

$$\langle -\lambda_c - \lambda_d | S^J(W) | -\lambda_a - \lambda_b \rangle = \eta_a \eta_b \eta_c^\star \eta_d^\star \langle \lambda_c \lambda_d | S^J(W) | \lambda_a \lambda_b \rangle \tag{13.55}$$

Now the electroproduction process (e, e′ X) in the C-M system presents *exactly the same problem* as discussed above.[6] The behavior under rotation of all quantities is exactly the same. The only new feature is that k^2, the mass of the virtual photon, provides an additional kinematic variable in the C-M system. To make the analogy more explicit, recall Low's first reduction of the S-matrix [Lo55]. For non-forward pion–nucleon scattering it takes the form

$$\frac{\langle p'q'|S|pq\rangle}{\langle 0|S|0\rangle} = -(2\pi)^4 i\delta^{(4)}(p'+q'-p-q)\frac{1}{\sqrt{2\omega_q\Omega}}\langle p'q'^{(-)}|J(0)|p\rangle \tag{13.56}$$

Here $J(0)$ is the pion current (the isospin label is suppressed). This expression now has exactly the same form in terms of target matrix elements of the current as that we have been studying. The only difference is that in our case it is the matrix element of the electromagnetic current that is required. With the Low reduction, one shifts the transformation properties from the state vector (which we *do not* have for a virtual photon) to those of the current (which we *do* have).

With the electromagnetic current, one can use current conservation to relate the Coulomb and longitudinal matrix elements

$$\mathbf{e}_{\mathbf{k}3} \cdot \mathscr{J} \quad \equiv \quad \mathscr{J}^{(0)} = \frac{\omega_k^\star}{k^\star} \mathscr{J}_C \tag{13.57}$$

This reduces the problem to the study of one or the other of these.

As a result of the above discussion, the helicity matrix elements of the electromagnetic current for the hadronic target in the C-M system, required for the cross section in Eq. (13.47), must have the following angular dependence

$$\begin{aligned}
(\mathscr{J}_C)_{\lambda_f,\lambda_i} \quad &= \quad \frac{k^\star}{\omega_k^\star}\frac{1}{\sqrt{4k^\star q}}\sum_J (2J+1)\,\mathscr{D}^J_{\lambda_i,\lambda_f}(-\phi_p,-\theta_p,\phi_p)^\star \\
&\quad \times \langle \lambda_2 \lambda_X | T^J(W,k^2) | \lambda_1 \lambda_k \rangle \qquad ; \ \lambda_k = 0 \\
\left(\mathscr{J}^{\lambda_k}\right)_{\lambda_f,\lambda_i} \quad &= \quad \frac{1}{\sqrt{4k^\star q}}\sum_J (2J+1)\,\mathscr{D}^J_{\lambda_i,\lambda_f}(-\phi_p,-\theta_p,\phi_p)^\star \\
&\quad \times \langle \lambda_2 \lambda_X | T^J(W,k^2) | \lambda_1 \lambda_k \rangle \qquad ; \ \lambda_k = \pm 1
\end{aligned} \tag{13.58}$$

[5] For the photon $(-1)^{S_\gamma}\eta_\gamma = 1$.

[6] The particular coordinate system chosen in Fig. 13.3, which might appear somewhat perverse to the reader, was chosen to make this analogy explicit.

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The normalization is conventional. Here

$$\lambda_i = \lambda_1 - \lambda_k \qquad ; \lambda_f = \lambda_2 - \lambda_X \tag{13.59}$$

All the angular dependence is now explicit. From Fig. 13.3

$$\begin{aligned} \theta_p &= \theta_q \\ \phi_p &= 2\pi - \phi_q \end{aligned} \tag{13.60}$$

The angular dependence with respect to the angles of Fig. 13.2 is then given by the relation

$$\mathscr{D}^J_{\lambda_i,\lambda_f}(-\phi_p, -\theta_p, \phi_p)^\star = \mathscr{D}^J_{\lambda_f,\lambda_i}(\phi_q, \theta_q, -\phi_q) \tag{13.61}$$

The proof follows from [Ed74] and the fact that $\lambda_f - \lambda_i$ is an integer

$$\begin{aligned} \mathscr{D}^J_{\lambda_i,\lambda_f}(-\phi_p, -\theta_p, \phi_p)^\star &= e^{i\lambda_i\phi_p} d^J_{\lambda_i,\lambda_f}(-\theta_p) e^{-i\lambda_f\phi_p} \\ &= e^{-i\lambda_i\phi_q} d^J_{\lambda_f,\lambda_i}(\theta_q) e^{i\lambda_f\phi_q} \\ &= \mathscr{D}^J_{\lambda_f,\lambda_i}(\phi_q, \theta_q, -\phi_q) \end{aligned} \tag{13.62}$$

In the expression for the (e, e′ X) cross section, for a given set of particle helicities, one needs the bilinear expression

$$\begin{aligned} \left(\mathscr{J}^{\lambda_k}\right)^\star_{\lambda_f,\lambda_i} \left(\mathscr{J}^{\lambda_k'}\right)_{\lambda_f,\lambda_i'} &= \frac{1}{4k^\star q} \sum_J \sum_{J'} (2J+1)(2J'+1) \langle \lambda_2 \lambda_X | T^J | \lambda_1 \lambda_k \rangle^\star \\ &\times \langle \lambda_2 \lambda_X | T^{J'} | \lambda_1 \lambda_k' \rangle \mathscr{D}^J_{\lambda_i,\lambda_f}(-\phi_p, -\theta_p, \phi_p) \mathscr{D}^{J'}_{\lambda_i',\lambda_f}(-\phi_p, -\theta_p, \phi_p)^\star \end{aligned} \tag{13.63}$$

Here

$$\lambda_f = \lambda_2 - \lambda_X \qquad ; \lambda_i = \lambda_1 - \lambda_k \qquad ; \lambda_i' = \lambda_1 - \lambda_k' \tag{13.64}$$

This expression is required for values of λ_k and λ_k' of 0 and ± 1. With the aid of Eq. (13.62) and formulas in [Ed74], the angular functions appearing in these bilinear combinations can be written as

$$\begin{aligned} &\mathscr{D}^J_{\lambda_f,\lambda_i}(\phi_q, \theta_q, -\phi_q)^\star \, \mathscr{D}^{J'}_{\lambda_f,\lambda_i'}(\phi_q, \theta_q, -\phi_q) = \\ &\qquad (-1)^{\lambda_f - \lambda_i} \mathscr{D}^J_{-\lambda_f,-\lambda_i}(\phi_q, \theta_q, -\phi_q) \, \mathscr{D}^{J'}_{\lambda_f,\lambda_i'}(\phi_q, \theta_q, -\phi_q) \end{aligned} \tag{13.65}$$

Now use the composition law for rotation matrices [Ed74] to rewrite the r.h.s. of this expression as

$$\begin{aligned} \text{r.h.s.} &= (-1)^{\lambda_f - \lambda_i} \sum_{lmm'} (2l+1) \begin{pmatrix} J & J' & l \\ -\lambda_f & \lambda_f & m \end{pmatrix} \mathscr{D}^l_{m,m'}(\phi_q, \theta_q, -\phi_q)^\star \\ &\times \begin{pmatrix} J & J' & l \\ -\lambda_i & \lambda_i' & m' \end{pmatrix} \end{aligned} \tag{13.66}$$

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Since m must vanish by the properties of the 3-j symbols, use

$$\mathscr{D}^{l}_{0,m'}(\phi_q, \theta_q, -\phi_q)^{\star} = \left(\frac{4\pi}{2l+1}\right)^{1/2} Y_{l,m'}(\theta_q, \phi_q) \tag{13.67}$$

Since $\lambda_i - \lambda'_i = \lambda'_k - \lambda_k$, one finally has

$$\begin{aligned}\left(\mathscr{J}^{\lambda_k}\right)^{\star}_{\lambda_f,\lambda_i}\left(\mathscr{J}^{\lambda'_k}\right)_{\lambda_f,\lambda'_i} &= \frac{1}{4k^{\star}q}(-1)^{\lambda_i-\lambda_f}\sum_J\sum_{J'}(2J+1)(2J'+1)\\ &\times\langle\lambda_2\lambda_X|T^J|\lambda_1\lambda_k\rangle^{\star}\langle\lambda_2\lambda_X|T^{J'}|\lambda_1\lambda'_k\rangle\sum_l\sqrt{4\pi(2l+1)}\\ &\times\begin{pmatrix} J & J' & l\\ \lambda_f & -\lambda_f & 0\end{pmatrix} Y_{l,\lambda'_k-\lambda_k}(\theta_q,\phi_q)\begin{pmatrix} J & J' & l\\ \lambda_i & -\lambda'_i & \lambda_k-\lambda'_k\end{pmatrix}\end{aligned} \tag{13.68}$$

This formula gives the general angular dependence of the bilinear forms of the current appearing in the cross section for an arbitrary set of helicities of the reaction participants.[7] As such, it can be used to calculate the angular distributions in the C-M system for any polarization of the initial and final systems. *It is a central result.*

If the target is unpolarized, and the final particles are unobserved, one must average over initial helicities and sum over final helicities. We denote these sums with a bar over the bilinear combinations of currents

$$\overline{\mathscr{J}^{\star}\mathscr{J}} \equiv \overline{\sum_{\lambda_1}}\sum_{\lambda_2}\sum_{\lambda_X}\mathscr{J}^{\star}\mathscr{J} \tag{13.69}$$

The transition matrix elements are functions of (W, k^2). Parity invariance of the strong and electromagnetic interactions implies

$$\begin{aligned}\langle -\lambda_2-\lambda_X|\, T^J(W,k^2)|-\lambda_1-\lambda_k\rangle &=\\ \eta_2^{\star}\eta_X^{\star}\eta_1(-1)^{S_2+S_X-S_1}&\langle\lambda_2\lambda_X|\, T^J(W,k^2)|\lambda_1\lambda_k\rangle\end{aligned} \tag{13.70}$$

A change of dummy helicity sum values to their negatives, use of the parity relation, and use of the symmetry properties of the 3-j symbols allow us to write the bilinear products of current matrix elements required in the electron scattering cross section in Eq. (13.47) in the following form[8]

$$\overline{|\mathscr{J}_{\mathscr{C}}|^2} = \frac{1}{4k^{\star}q}\sum_l A_l P_l(\cos\theta_q) \tag{13.71}$$

[7] This includes, for example, the process of "virtual Compton scattering," now studied extensively through the coincidence reaction p(e, e′p)γ.

[8] The spherical harmonics are defined by $Y_{l,m} = (-1)^m\left[\frac{(2l+1)(l-m)!}{4\pi(l+m)!}\right]^{1/2}P_l^m(\cos\theta)e^{im\phi}$ for $m \geq 0$ while for $m < 0$ one has $Y^{\star}_{l,m} = (-1)^m Y_{l,-m}$. Here $P_l^m(\cos\theta)$ are the associated Legendre polynomials [Ed74], which for positive m are given by $P_l^m(x) = (1-x^2)^{m/2}d^m P_l(x)/dx^m$.

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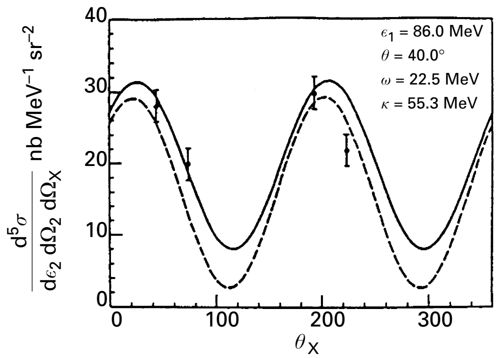

Fig. 13.4. In-plane angular distribution of protons in $^{12}_{6}$C(e, e′ p_0)$^{11}_{5}$B through the giant dipole resonance measured with the SCA at HEPL [Kl83]. Data from [Ca80].

$$\overline{|\mathscr{J}^{+1}|^2} + \overline{|\mathscr{J}^{-1}|^2} = \frac{1}{4k^{\star}q}\sum_l B_l P_l(\cos\theta_q)$$

$$\mathrm{Im}\,\overline{\mathscr{J}_{\mathscr{C}}^{\star}\left(\mathscr{J}^{+1} + \mathscr{J}^{-1}\right)} = \frac{1}{4k^{\star}q}\sum_l C_l P_l^{(1)}(\cos\theta_q)\,\sin\phi_q$$

$$\mathrm{Re}\,\overline{\left(\mathscr{J}^{+1}\right)^{\star}\left(\mathscr{J}^{-1}\right)} = \frac{1}{4k^{\star}q}\sum_l \eta\, D_l P_l^{(2)}(\cos\theta_q)\,\cos 2\phi_q$$

These expressions provide the general angular distributions in the C-M system for (e, e′ X) for any target particles and any X. The coefficients (A_l, B_l, C_l, D_l) are bilinear combinations of helicity amplitudes; they are functions of (W, k^2). They are developed in detail in appendix F. The quantity $\eta = \eta_1\eta_2^{\star}\eta_X^{\star}$ is the real combination of intrinsic parities. These expressions are further analyzed and tabulated in [Kl83].

The claim made in exhibiting the dependence on the out-of-plane angle ϕ_q in Eqs. (13.48) has now been established.

To give the reader some feel for coincident electron scattering, we present three brief examples. First, consider Fig. 13.4 which shows the coincidence cross section for $^{12}_{6}$C(e, e′ p_0)$^{11}_{5}$B [Ca80]. This is the first coincidence experiment done with the superconducting accelerator (SCA) at the Stanford High Energy Physics Laboratory (HEPL), a machine that proved to be the prototype for CEBAF . The energy transfer is controlled so that ^{12}C is excited to the giant dipole resonance. The in-plane angular distribution of the emitted proton leading to the ground state of ^{11}B is then measured with respect to the momentum transfer $\boldsymbol{\kappa}$. This is an example of the angular correlation measurement discussed above, where the inelastic scattering of the electron first aligns the target along the direction of the momentum transfer. Notice the very nice dipole pattern

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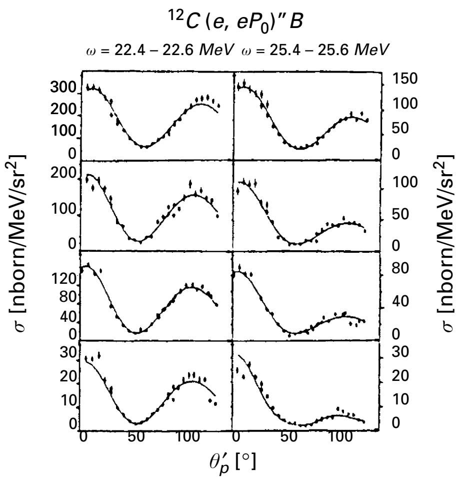

Fig. 13.5. Same reaction as in Fig. 13.4 with subsequent data from Mainz [De86, Ca94]. Here $\kappa = 0.25, 0.34, 0.41, 0.59\,\mathrm{fm}^{-1}$.

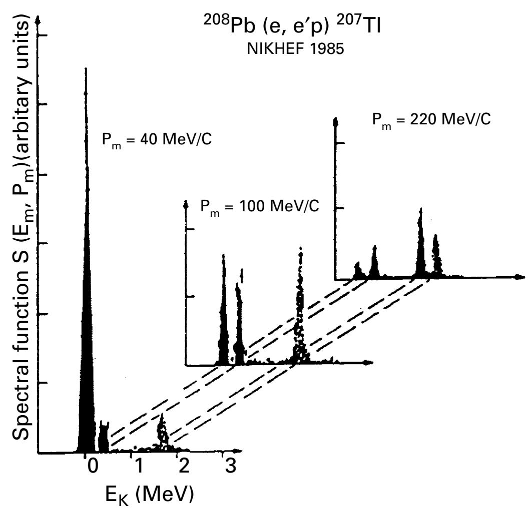

Fig. 13.6. Nuclear response for the reaction ${}^{208}_{82}\mathrm{Pb}(\mathrm{e},\mathrm{e}'\,\mathrm{p})^{207}_{81}\mathrm{Tl}$ measured at NIKHEF [de86].

of the subsequently emitted proton.[9] The two theoretical curves in Fig. 13.4 are calculations carried out within the particle–hole model of the giant dipole resonance in ^{12}C [Kl83]. Now one may well say that the four points do not determine an angular distribution, and it is hard to disagree; however, Fig.13.5 shows the quality of the data one can now

[9] The initial aligned ^{12}C nucleus has $J^{\pi} = 1^{-}$ (hence the phrase "dipole pattern"). The final ground state of ^{11}B is $(3/2)^{-}$ and the emitted proton conserves angular momentum.

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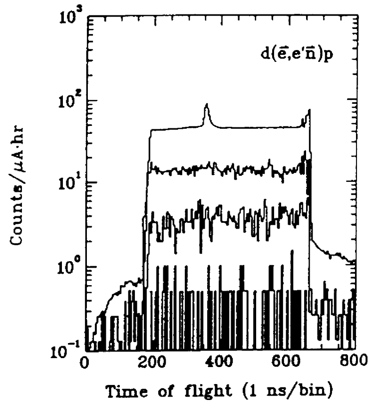

Fig. 13.7. Triple coincidence signal from $^2_1$H($\vec{\text{e}}$, e′ $\vec{\text{n}}$) experiment done at Bates [Ma92, Wa93].

obtain using the new generation of c.w. electron accelerators on the same reaction — this data is from Mainz [De86, Ca94]. The dipole pattern is now beautifully displayed.

As a second example, Fig.13.6 shows the nuclear response function for the reaction $^{208}_{82}$Pb(e, e′ p)$^{207}_{81}$Tl measured at NIKHEF [de86]. This example illustrates the discussion of (e, e′ p) in chapter 6.[10] One sees the ground state ($E_x = 0$), and then several excited hole states of $^{207}_{81}$Tl. Consider first the ground state. As $\boldsymbol{\kappa} - \mathbf{q}$ is increased, the data exhibit the fall-off of the Fourier transform of the $(3s_{1/2})_\pi^{-1}$ wave function. The growth and fall-off of the Fourier transform of the $(2d_{3/2})_\pi^{-1}$ first excited state is then seen. At somewhat larger E_x, the high-multipolarity transition to the $(1h_{11/2})_\pi^{-1}$ appears from nowhere until it *dominates* the spectrum at the highest $\boldsymbol{\kappa} - \mathbf{q}$. Note that one requires good resolution at high momenta to resolve the states.

This class of experiments represents one of the most important results coming from NIKHEF. These data are even more impressive when one realizes that they were obtained with only a few percent duty factor (d.f.) — the new generation of c.w. accelerators provides a significant advance. With this reaction, one can take the nucleus apart layer by layer and probe the limits of the single-nucleon description of nuclei.

As a third example, Fig. 13.7 shows the timing signal from the polarization transfer experiment $^2_1$H($\vec{\text{e}}$, e′ $\vec{\text{n}}$) carried out at Bates [Ma92]. This experiment provides an excellent example of how one can use interference in coincidence experiments to measure small quantities, in this case the electric form factor of the neutron which interferes with the well-known

[10] Here $(\varepsilon_b, \vec{\kappa} - \vec{q}) \equiv (E_m, \vec{P}_m)$.

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magnetic form factor [Ar81]. This is really a triple coincidence experiment. The electron is detected, then the produced neutron, then the up or down scattering of the neutron to measure its polarization. The final signal is the small peak in the middle of the figure; the background consists of accidentals. The experiment was performed with an accelerator with $\sim 1\%$ d.f.. Now imagine that the signal forms a sea mount and the background an ocean. With a c.w. (100% d.f.) accelerator, one can lower the ocean level by over two orders of magnitude, and the small peak sticking up becomes a mountain. This is the most dramatic example, of which the author is aware, of what one gains with a c.w. machine.

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