

Matrices and Transformations, by Anthony J. Pettofrezzo.  
Prentice Hall, 1966. x + 133 pages. \$3.95.

This book is intended primarily as a text for a one semester course for teachers in the elements of matrix theory, or, for use with an in-service institute, though it may also be useful with advanced high school students. With very few exceptions, attention is restricted to  $2 \times 2$  and  $3 \times 3$  matrices with real entries. Geometric interpretations and applications are emphasized throughout. In general, new topics are presented with painstaking care and with an abundance of well-chosen numerical examples. Rings and isomorphisms of rings are mentioned very briefly early in the book and are not used again. Because of the relaxed pace and the number of examples and exercises (together with answers to the odd-numbered problems) this book might be particularly suitable for independent reading.

Martin Pearl, University of Maryland

Linear Transformations and Matrices, by F.A. Ficken. Prentice Hall, Inc., 1967. xiii + 398 pages. \$10.50.

This is a fairly large book and contains considerably more than the title promises. In addition to sufficient material for a one semester course in linear algebra, the first four chapters (consisting of 118 pages and comprising about one third of the text) present much of the material often found in a fundamentals course which usually precedes the calculus sequence. For this reason, and because the difficulties are introduced very gradually, it is possible to use this book for a one year course for students who have not had (and perhaps will never have) any calculus. In Chapter 1, the language of set theory and functions is presented. Chapter 2 is devoted to a development of the real number system. The vector geometry of three dimensional Euclidean space is treated in Chapter 3 and serves to prepare the student for the more general  $n$  dimensional geometry of later chapters. In addition, there are topics (e.g., the cross product of vectors) which are applicable primarily to three-dimensional spaces. Chapter 4 gives the barest essentials of groups, rings and fields. Permutations and polynomials are introduced, primarily because they will be needed later, as are partially ordered sets and lattices. Equivalence relations and the partitions they determine, are mentioned and are used later in describing canonical forms.

The remaining two thirds of the book is given over to a conventional (according to the author) basis - free presentation of real and complex finite dimensional vector spaces. Matrices are not emphasized and the determinant first appears in Chapter 11. Geometry is used throughout whenever possible and often provides insight into what would otherwise be tough sledding (e.g., projections).

A number of unusual topics are covered - most of them too briefly. These include: the pseudo-inverse of a matrix, the general form of the Laplace expansion (rather than just expansions by a single row or column),

pairs of Hermitian and quadratic forms, some extremal properties of eigenvalues, etc. The way is paved for a discussion of tensors by the instructor if he so desires, but tensors themselves are not mentioned.

There is an unusually extensive bibliography devoted almost exclusively to books and arranged by subject matter. Answers are provided to most of the problems, including some theoretical ones. There is an index of symbols in addition to the regular index.

Professor Ficken has provided a usable and unusually versatile text.

Martin H. Pearl, University of Maryland

Notes on Logic, by Roger C. Lyndon. Van Nostrand Mathematical Studies No. 6. vi + 97 pages. New York, 1966. \$2.50.

This book is a remarkable example of selection and distillation. Assuming only a slight acquaintance with Zorn's Lemma, cosets and equivalence classes, definition by induction, homomorphism and infinite cardinals, the author presents in 90 pages a highly readable and self-contained account, complete with proofs (reliance on intuitive plausibility does not appear until page 84) and a valuable set of exercises, of a closely knit and highly significant portion of mathematical logic. In fact, a strong case could be made for claiming that the totality of what these 90 pages leave out is less important than what they contain.

The central theme is "the semantic connection between a formal language and a mathematical system serving as a model", and the topics treated include the Deduction Theorem, The Consistency Theorem, The Adequacy Theorem and Compactness Theorem, The Löwenheim-Solem Theorem, Gentzen's Natural Inference, The Herbrand Gentzen Theorem, Tarski's Theorem (on the inexpressibility of the property of being a true formula), Gödel's Incompleteness Theorem and Church's Theorem. Other topics are the usual preliminaries to these.

To pack so much into so small a space something obviously has to be sacrificed: the sacrifice is astonishingly small. It is assumed that the reader can handle routine chores on his own and does not need the ministrations of pedagogical virtuosity in designing carefully graduated approaches to everything. Toward the end, some of the really long proofs are of course omitted, but the reader is succinctly made aware of what is missing. The reward of such a lean, but careful, treatment is a sense of pace and purpose, and clarity of structure which is very difficult to achieve otherwise. This book should be an invaluable aid to the student as a complement to most of the classics in this field.

Excellence notwithstanding, the reviewer feels that some further polishing would be well worth the trouble.

A slightly fuller exposition is desirable in a few places. On page