OBITUARY

JOHN LESLIE BRITTON

John Leslie Britton died in a climbing accident on the Isle of Skye on 13 June 1994 at the age of 66. He became a member of the London Mathematical Society in 1957, served the society as Meetings and Membership Secretary from 1973 to 1976, and at the same time was Editor of the Society's Newsletter.

John Britton was born on 18 November 1927 in Cottingham, near Hull, to a family of modest means who ran a grocer's shop and off-licence. His ability obviously showed itself early, for he gained admission to Beverley Grammar School at the age of 12 in 1939, and was remembered 50 years later by his school contemporaries as a 'brainbox'. Upon leaving school in 1946, he went to what was then the University College of Hull, which in those days awarded London external degrees, so that when he graduated in 1949 his degree was from London. At that time Hull enjoyed the privilege of counting both Bernhard and Hanna Neumann as members of staff, and they had already spotted John's gift for algebra during his second year as an undergraduate. In the summer of 1950, when Bernhard was home from his post at Manchester where he had gone in 1948, the two Neumanns organised a small research seminar at their house in Hull, in which John was a participant. From the discussion at the seminar came John's first published work, the paper [**1**] written jointly with J. A. C. Shepherd. In 1951, John obtained an MSc degree, again a London external degree, for which he was supervised by Hanna Neumann, in group theory and in complex analysis (for the latter they worked their way together through Whitaker and Watson).

Bernhard Neumann recounts the circumstances of the MSc examining process. Candidates were anonymous, being identified only by numbers. Bernhard, as external examiner for group theory, had the difficult task of assessing two candidates, knowing both personally *and* by their handwriting. Both candidates in fact went on to PhD degrees, John with Bernhard as his supervisor at Manchester. John was considering Cambridge as well as Manchester for his PhD, and at one point sought Bernhard Neumann's advice. According to John's sister, Bernhard told John that if he wanted trappings and show, then he should go to Cambridge, but if his interest was mathematics, then he should go to Manchester. To John, who was never unduly concerned with external appearances (his wife had once been known to buy him a suit the day before a wedding), Manchester was the place to go.

John spent only two years as a PhD student, and in that time wrote a thesis of considerable technical virtuosity, extending to a more general setting work of the Russian group theorist Tartakovskii on what is known as the word problem in group theory—these results appeared in the papers [**2**] and [**3**]. This was a considerable achievement, all the more so since it was done with very little direct guidance from his PhD supervisor. Formally, his PhD was awarded in 1954, by which time he was finally doing his National Service, serving at a radar station on Anglesey, and he turned up for the oral examination looking very smart in Sergeant's uniform. In 1956 he married Catherine Stewart.

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John's first academic post was at the University of Glasgow, where he began as a Research Fellow, having turned down a lucrative offer to work at Harwell Research Laboratory, and stayed until 1966 as Lecturer. During his time at Glasgow he continued his work on the word problem, producing in 1958 an example of a finitely presented group with unsolvable word problem which appeared in the paper [**4**]. Although not the first example of such a presentation, it attracted the interest of the American logician W. W. (Bill) Boone, who invited John to spend the academic year 1960–61 at the University of Illinois. This was a critical year in John's career, for it moved him onto the international stage as a mathematician, with Bill Boone as a tireless supporter—and at the same time came the birth of his eldest daughter. By approaching Boone's own example of a negative solution to the word problem in a style more group-theoretic than the traditional logician's approach, John was able to shorten and simplify the argument decisively, and his paper [**5**] became a seminal influence throughout combinatorial group theory.

John was highly regarded by his colleagues in Glasgow, but he eventually felt that the Scottish system imposed a heavy marking and examining burden, and in 1966 moved to the just-founded University of Kent as Reader. By this time he had embarked on his attempt to solve the Burnside Problem (can a finitely generated group of bounded exponent be infinite?), which was to occupy him for virtually a decade and which, sadly, was to turn out to be flawed. However, his reputation was such that in 1973 he obtained a Chair in Pure Mathematics at Queen Elizabeth College, moving on in 1985 to King's College when the two colleges merged, although he kept his home in Canterbury. Alan Pears, who was his contemporary in London throughout this period, recalls that John was an excellent teacher and colleague, highly regarded by his students and always willing to do his part in departmental activities. During his time in London he was active in a number of ways in the mathematical community, both within the University of London and for the London Mathematical Society.

His later mathematical research (in [**9**], [**10**] and [**11**]) was more low key, but he made a substantial scholarly contribution in editing the volume on pure mathematics in the Collected Works of A. M. Turing [**12**]. He retired in 1988, although continuing to teach part-time until 1991. In his retirement he maintained his mathematical interests and had more time for the hill-walking which was a great source of pleasure to him.

John Britton was a quiet man, shy but at the same time independent and self-contained. He had a gentle sense of humour, and the writer recalls how he once began his lecture at a British Mathematical Colloquium, having come straight off a transatlantic jet, with the remark that he hoped this would not be the first occasion when the lecturer fell asleep instead of the audience. Jens Mennicke recounts how kind John was to Jens' son when the young man was having difficulty with the German authorities about his military service and needed help when he came to England for several months. John's professional life was marked by substantial achievements, but also by stressful pressures which he bore with uncomplaining stoicism. In the early 1960s his mathematical career was flourishing when he started work on the Burnside Problem. However, rivals in Moscow, who had started work earlier than John, published a proof in 1968 [**R6**] before John had finished his argument. In his independent way, John determined he would finish his own proof without consulting their published proof and it must have been a very bitter blow when Sergei Adian pointed out that at one place John had been too hasty and some inequalities needed for a step in the proof were in fact inconsistent. Mathematically John never entirely recovered from this, and a further burden was imposed upon him in the form of a personal tragedy when his wife Catherine contracted cancer and died after a long illness in 1980. He is survived by their three daughters Anne, Christine and Mary.

Mathematical work

An indication has already been given above of John Britton's mathematical work, but some more detailed comments should be made. His first published paper [**1**] with J. A. C. Shepherd is outside the line of the main thrust of his work, and is of interest mainly for its role in John's development as a young mathematician. His thesis, published as [**2**] and [**3**], is, on the other hand, central to his career.

The setting for these papers is that of a group G given by relators as the quotient of a free product $E = \bigstar E_\alpha$ of a specified family of groups, and the aim is to solve the word problem, that is, to produce an algorithm which will determine of any word in the generators whether or not that word represents the identity element. The key result obtained is that if the relators used to form the quotient, when viewed as elements of the ambient free product E, have only a relatively small interaction with one another—the precise technical condition is that a common part of distinct relators has length less than one sixth of the length of each of the two relators concerned—then one can prove that if a given element w of the free product E represents the identity element of the quotient G, then w has a subword which consists of more than half a relator. The argument is an elaborate case by case analysis of how the given word w is transformed and cancelled step by step into the empty word. The whole argument is made much more complex than the parallel situation for the quotient of a free group by the interactions within the individual factors E_α of the free product. From this result, it follows immediately that if the word problem for each factor E_α is solvable, then so is the word problem for the quotient group G. (In the 1960s, Jim McCool, John's most distinguished research student, was to prove in his PhD thesis, published as [**R3**], that these techniques could be used to classify the torsion elements in a group G constructed in the above way.) Subsequent developments using planar diagrams to represent the cancellation process have greatly simplified the proof, but these do not diminish the value of John's work. Interestingly, he once said that while the diagrammatic methods were clearly slicker, he felt that the more primitive word cancellation arguments were ultimately more powerful.

Having provided a positive solution to the word problem for a wide class of groups, John set about trying to see how to give a negative solution by constructing an example of a finitely presented group with unsolvable word problem. His starting point was the example of P. S. Novikov [**R4**] which he had reviewed for *Mathematical Reviews*. However, John's approach was to use the techniques he had already developed, and in this he was successful—although it has to be admitted that [**4**] is not at all easy to read.

As noted earlier, this first attempt to give a more group-theoretic proof of the existence of a negative solution to the word problem was followed by the paper [**5**], published five years later. The key idea is to view the construction in Boone's paper [**R1**] as a sequence of what are now called HNN-extensions, after their creators [**R2**], and to use what is effectively a normal form theorem for the necessary analysis. This normal form theorem is not considered in [**R2**], and it is justifiably referred to in [**5**]

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as the Principal Lemma. The result is now standard and, in its basic form, can be stated as follows.

Proposition. *Let G be a group presented as* $\langle S \mid D \rangle$, *and let H and K be subgroups which are isomorphic via a given isomorphism* $\phi : H \to K$. *Let* G^* *be the group given by the presentation* $\langle S, p : D, p^{-1}hp = \phi(h), h \in H \rangle$. *If the word w in the elements of S and p represents the identity element of* G^*, *then w has a subword of the form* $p^{-\varepsilon}up^{\varepsilon}$ *where* $\varepsilon = \pm 1$ *and u is a word over S which represents an element of H or K according as* $\varepsilon = 1$ *or* $\varepsilon = -1$.

Although the name is no longer in such common use, for a number of years this result was universally known as Britton's Lemma. Moreover, the group-theoretic point of view employed throughout [**5**] represents a major advance over earlier arguments, and it is the introduction of this point of view that makes [**5**] of such importance both in the area of decision problems and in combinatorial group theory as a whole. Stimulated by this incisive new approach, Boone and his pupils and associates helped to develop the field of decision problems in group theory throughout the later 1960s and early 1970s.

John, however, had not followed them down this route. Instead he started to investigate the Burnside Problem. This was in some ways a risky thing to do for, already in 1959, P. S. Novikov [**R5**] had announced a claim that for every $n \geqslant 72$, there was a finitely generated infinite group of exponent n. However, time went by with no complete argument forthcoming, and John was sufficiently tempted by the challenge to start considering how the kind of cancellation arguments he had used in his early work could be adapted to tackle the problem. Although the basic situation is far from identical, there is a similarity between the interactions of relators for a free Burnside group and the 'small cancellation' interactions of the kind John had dealt with in his thesis. At the international group theory meeting in Canberra in 1964, he had indicated in private that he was following what he thought was a hopeful line of argument, and he allowed himself to be persuaded to give a talk on his ideas. Regrettably, this led to differences with Sergei Adian who had by this time joined forces with Novikov in the effort to complete the proof that Novikov had outlined in [**R5**]. As noted earlier, the Adian–Novikov proof appeared in 1968, but John soldiered on, believing finally in 1970 that he had completed the proof which was published in [**6**].

However, some time after publication of the proof, Adian pointed out that, while individual lemmas were correct, in order to apply them all simultaneously the inequalities needed to make their hypotheses valid were inconsistent. For John the matter was exceptionally unfortunate for he had spent a number of his prime mathematical years on the problem. The mistake is acknowledged in [**8**] where, in the introduction to the volume, it is also acknowledged that the claim in Novikov's announcement [**R5**] must be taken as unproved.

John's mathematical work for the remainder of his career consisted largely of occasional pieces, but he took particular pleasure in being given the honour of editing part of the collected works of A. M. Turing. John had known Turing slightly when at Manchester, but the grounds for the choice were much more John's position as Turing's most distinguished successor in the field of decision problems in algebra in the United Kingdom.

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