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ELECTROPHYSIOLOGY OF A LEAKY CABLE MODEL FOR COUPLED NEURONS

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Abstract

An analytical expression for the voltage response to current stimulation at relatively short and long times is used to obtain estimates of the passive electrical constants of a neuron that is electrotonically coupled at the soma and dendritic terminals to other neurons in a neural network.

1. Introduction

There are various types of neurons in the central nervous system (including the brain) that are electrotonically coupled through gap junctions (Spray and Dermietzel [17]). If neurons are electrotonically coupled then it is more difficult to obtain their passive electrical constants since current leaks through the gap-junctions. Earlier modelling using simple 'lumped parameter' neurons has shown that the electrical constants of these neurons were modified in the presence of electrotonic coupling ([3]). As a result there needs to be a theoretical basis by which to interpret electrophysiological experiments which measure membrane potential (that is, voltage) changes between the exterior and interior of coupled neurons in a neural network.

A leaky cable model of a neuron represented by a one-dimensional uniform equivalent cylinder coupled to a lumped (isopotential) soma, is developed in order to incorporate the leakage of current in the dendrites and the soma, that occurs for electrotonically (that is, electrically) coupled neurons. The novelty in the model is that a leak resistance at both the soma (R_L) and at the terminal-end of the equivalent cylinder (R_C) is included, to represent the effects of somatic and dendritic coupling between neurons respectively.

Cable models with leaky boundary conditions started with Rall [12] and Jack and

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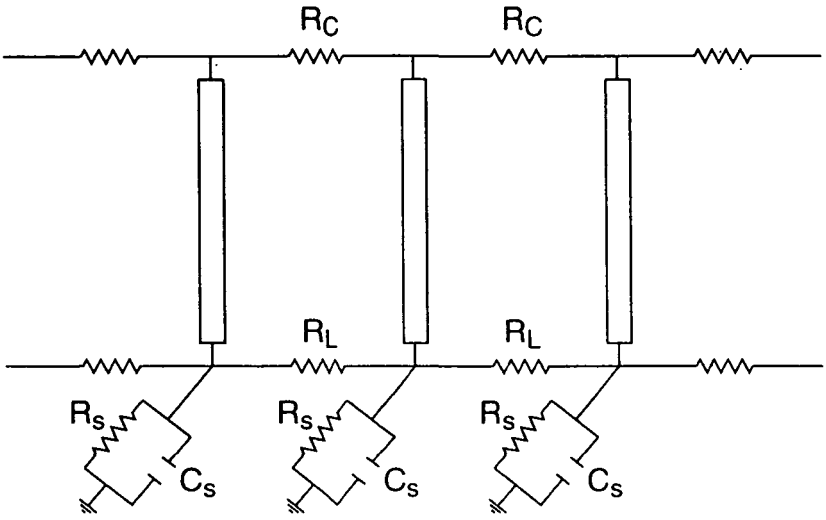

FIGURE 1. A network of electronically coupled neurons each represented by an equivalent cylinder for the dendritic tree (and axon), and a RC-circuit for the cell body (soma). Each neuron is coupled to another neuron at the soma by a resistance (R_L) and in the dendrites by a resistance (R_C).

Redman [5], and continued with advances in new experimental techniques of recording the voltage responses from single neurons ([14], [19]).

A mathematical model of a network of coupled neurons (shown in Figure 1) is based on the assumption that the soma of each neuron is isopotential; that the neuronal membrane is passive; that the dendritic and somatic membrane resistivity are both equal in all neurons; that the cytoplasmic resistivity is constant in all the neurons; that the axon and dendritic tree of each neuron can be reduced to an equivalent cylinder; and that the somatic membrane resistivity of each neuron is equal.

Our main goal will be to determine the effect of electrotonic coupling on the passive cable parameter estimates of single neurons using the leaky cable model approach of Figure 1 as an approximation to the network of coupled neurons. The reader not familiar with mathematical modelling of neurons may wish to consult [18] for an introduction to the techniques, and [10] for a more recent overview of techniques, methods and applications in neuroscience modeling.

2. Voltage response to be used at relatively short times

At relatively short times the current has not as yet 'seen' the terminal-end of the axon and dendrite and thus the equivalent cylinder of the neuron is taken to be infinitely long. This approach was advocated in single neuron cable modelling by Redman. The

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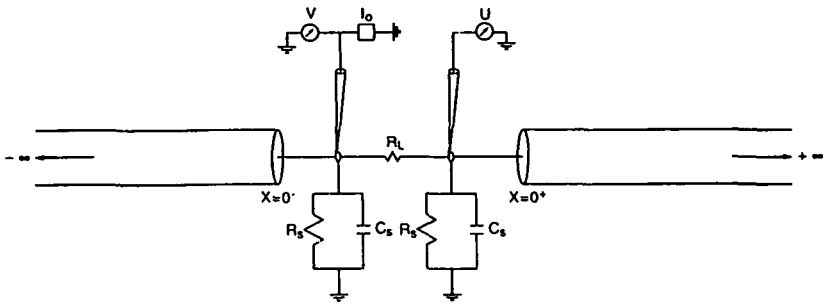

FIGURE 2. Electrophysiological experiment consisting of a pulse of current injected into the soma of a single neuron and a measure of the voltage response (at early times) in the coupled neuron, recorded in a system of two electronically coupled neurons. An emphasis on the somatic coupling between each neuron is given, since the dendritic terminals are assumed to be at an infinite electronic distance from the soma.

same assumption holds for the neighbouring neuron that is coupled via a gap-junction at the cell body. Therefore since the voltage response to a current step applied at the soma at early times is assumed to be influenced relatively little by the effects of current 'reflecting' from the terminal-end of the equivalent cylinder, we may replace the finite length of the equivalent cylinder shown in Figure 1 by an infinite equivalent cylinder attached to a lumped-soma (RC-circuit), coupled to another neuron of the same representation, as shown in Figure 2.

In the model depicted in Figure 2, the voltage in the neuron with the current step injection (I_0) is denoted by V, while the voltage response in the adjacent neuron is denoted by U. Both neurons are assumed to have the same somatic resistance value (R_s), and the resistivity of the dendrites in both neurons is assumed to be equal to the somatic resistivity.

The aim will be to obtain an exact expression for the time course of V, that has only the leakage resistance (R_L) as the unknown parameter. By matching this theoretical voltage response for different R_L values to the actual experimental voltage response, an estimate of R_L is obtained.

The initial range of the different R_L values to be selected could be determined from a morphological estimate of R_L under the assumption that it corresponds to the resistance for current flow through the somatic gap-junction. Such a morphological estimate of the junctional resistance between coupled neurons can be expressed by

$$R_L \simeq (r_{ch}/\epsilon_{ch})\,[\ell_{ch}/(\epsilon_{ch}\pi) + 1/2]\,/N \tag{2.1}$$

where N is the number of open-channels in the gap-junction, ℓ_{ch} and ϵ_{ch} are the length and radius of a single channel or pore respectively, and r_{ch} is the resistivity of the single channel or pore.

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Consider the following boundary-value problem depicting the voltage response of a neuron to current injection at the soma:

$$V_{XX} - V = \tau_m V_t, \tag{2.2a}$$

$$(1 + R)V(0, t) + \gamma V_X(0, t) + \tau_m V_t(0, t) = I_0 R_S + RU(0, t), \tag{2.2b}$$

$$V(-\infty, t) = 0, \qquad V(X, 0) = 0, \tag{2.2c}$$

together with the following boundary-value problem depicting the voltage in the coupled neuron:

$$U_{XX} - U = \tau_m U_t, \tag{2.3a}$$

$$(1 + R)U(0, t) - \gamma U_X(0, t) + \tau_m U_t(0, t) = RV(0, t), \tag{2.3b}$$

$$U(\infty, t) = 0, \qquad U(X, 0) = 0, \tag{2.3c}$$

where $X = x/\lambda$ is the dimensionless position variable normalised in terms of the length constant; t is the time; $V = V(X, t)$ is the electrotonic potential in the activated (injected) neuron, and $U = U(X, t)$ is the electrotonic potential in the coupled neuron; $\tau_m = R_m C_m = R_S C_S$ is the membrane time-constant; I_0 is the magnitude of the applied current step; R_S is the somatic input resistance; $\gamma = R_S/R_{D\infty}$ is the dendritic to somatic conductance ratio for a semi-infinite equivalent cylinder; and $R = R_S/R_L$ is the ratio between the somatic and leakage input resistance. (Note that subscripts t and X indicate partial derivatives with respect to these variables.)

The solution of the first boundary-value problem can be written in terms of the Green's function G:

$$V(X, t) = I_0 R_S \int_0^t G(X, t - \eta)\, d\eta + R \int_0^t U(0, \eta) G(X, t - \eta)\, d\eta \tag{2.4}$$

where $G^* = G \exp(t/\tau_m)$ satisfies

$$\tau_m G_t^* = G_{XX}^*, \tag{2.5a}$$

$$RG^*(0, t) + \gamma G_X^*(0, t) + \tau_m G_t^*(0, t) = \delta(t) \exp(t/\tau_m), \tag{2.5b}$$

$$G^*(-\infty, t) = 0, \qquad G^*(X, 0) = 0. \tag{2.5c}$$

Define $G_L^*(X, s)$ to be the Laplace transform of $G^*(X, t)$. Then the subsidiary equation corresponding to the above boundary-value problem is

$$G_L^{*\prime\prime} - s\tau_m G_L^* = 0, \tag{2.6a}$$

to be solved with

$$RG_L^*(0, s) + \gamma G_L^{*\prime}(0, s) + s\tau_m G_L^*(0, s) = 1, \tag{2.6b}$$

$$G_L^*(-\infty, s) = 0, \tag{2.6c}$$

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where s is the Laplace transform variable and the prime denotes differentiation with respect to X. The solution in the Laplace transform space is readily found to be

$$G_L^*(X, s) = \frac{\exp\left(\sqrt{s\tau_m}X\right)}{\left(R + \gamma\sqrt{s\tau_m} + s\tau_m\right)}. \tag{2.7}$$

The solution of the second boundary-value problem can be written in terms of another Green's function H:

$$U(X, t) = R\int_0^t V(0, \eta)H(X, t - \eta)\,d\eta \tag{2.8}$$

where $H^* = H\exp(t/\tau_m)$ satisfies

$$\tau_m H_t^* = H_{XX}^*, \tag{2.9a}$$

$$RH^*(0, t) - \gamma H_X^*(0, t) + \tau_m H_t^*(0, t) = \delta(t)\exp(t/\tau_m), \tag{2.9b}$$

$$H^*(\infty, t) = 0, \qquad H^*(X, 0) = 0. \tag{2.9c}$$

Define $H_L^*(X, s)$ to be the Laplace transform of $H^*(X, t)$. Then the subsidiary equation corresponding to the above boundary-value problem is

$$H_L^{*\prime\prime} - s\tau_m H_L^* = 0, \tag{2.10a}$$

to be solved with

$$RH_L^*(0, s) - \gamma H_L^{*\prime}(0, s) + s\tau_m H_L^*(0, s) = 1, \tag{2.10b}$$

$$H_L^*(\infty, s) = 0, \tag{2.10c}$$

where s is the Laplace transform variable and the prime denotes differentiation with respect to X. Consequently the solution in the Laplace transform space is readily found to be

$$H_L^*(X, s) = \frac{\exp\left(-\sqrt{s\tau_m}X\right)}{\left(R + \gamma\sqrt{s\tau_m} + s\tau_m\right)}. \tag{2.11}$$

Now taking the Laplace transform of the convolution integral between V and G we have

$$V_L(X, s) = I_0 R_S G_L^*(X, s + 1/\tau_m)/s + RU_L(0, s)G_L^*(X, s + 1/\tau_m), \tag{2.12}$$

where U_L and V_L represent the Laplace transforms of U and V respectively. Similarly, by taking the Laplace transform of the convolution integral between U and H we have

$$U_L(X, s) = RV_L(0, s)H_L^*(X, s + 1/\tau_m), \tag{2.13}$$

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and substituting (2.13) into (2.12) yields

$$V_L(0, s) = \frac{I_0 R_S G_L^*(0, s + 1/\tau_m)}{s\left\{1 - R^2 H_L^*(0, s + 1/\tau_m)\, G_L^*(0, s + 1/\tau_m)\right\}}. \tag{2.14}$$

But since $G_L^*(0, s + 1/\tau_m) = H_L^*(0, s + 1/\tau_m)$ the above expression reduces to

$$V_L(0, s) = \frac{I_0 R_S\left[R + \gamma\,(\tau_m s + 1)^{1/2} + (\tau_m + 1)\right]}{s\left\{\left[\gamma\,(\tau_m s+1)^{1/2}+(\tau_m s+1)\right]\left[2R+\gamma\,(\tau_m s+1)^{1/2}+(\tau_m s+1)\right]\right\}}. \tag{2.15}$$

The denominator of $V_L(0, s)$ will be zero when $s = 0, s = -1/\tau_m, s = [-1+\frac{1}{2}\gamma^2+\frac{1}{2}\gamma(\gamma^2 - 8R)^{1/2} - 2R]/\tau_m$, and $s = [-1 + \frac{1}{2}\gamma^2 - \frac{1}{2}\gamma(\gamma^2 - 8R)^{1/2} - 2R]/\tau_m$. Hence, by the use of the residue theorem from complex variable theory (see for example, McLachlan [8]) it can readily be shown that $V(0, t)$ is given by

$$\begin{aligned} V(0, t) = I_0 R_S \Bigg\{ & \frac{1 + \gamma + R}{1 + 2\gamma(1 + R) + 2R + \gamma^2} - \left(\frac{R}{\gamma^2 + 2R}\right) \exp(-t/\tau_m) \\ & + \left(R + \gamma a_1^{1/2} + a_2\right) / f(a_1) \exp[-\{1 - a_1\}(t/\tau_m)] \\ & + \left(R + \gamma a_2^{1/2} + a_2\right) / f(a_2) \exp[-\{1 - a_2\}(t/\tau_m)] \Bigg\} \end{aligned} \tag{2.16}$$

where $a_1 = \frac{1}{2}\gamma^2 + \frac{1}{2}\gamma(\gamma^2 - 8R)^{1/2} - 2R$, $a_2 = \frac{1}{2}\gamma^2 - \frac{1}{2}\gamma(\gamma^2 - 8R)^{1/2} - 2R$, and

$$\begin{aligned} f(a) = & \left[a^2 + 2\gamma a^{3/2} + (2R + \gamma^2)a + 2\gamma R a^{1/2}\right] \\ & + (a - 1)\left[2a + 3\gamma a^{1/2} + 2R + \gamma^2 + \gamma R a^{-1/2}\right]/\tau_m. \end{aligned}$$

It is not difficult to show that the time-course of the voltage response measured in the coupled neuron $U(X, t)$ satisfies

$$\begin{aligned} U(0, t) = I_0 R_S R \Bigg\{ & \frac{1}{1 + 2\gamma(1 + R) + 2R + \gamma^2} - \left(\frac{1}{\gamma^2 + 2R}\right) \exp(-t/\tau_m) \\ & + \left(\frac{1}{f(a_1)}\right) \exp[-\{1 - a_1\}(t/\tau_m)] \\ & + \left(\frac{1}{f(a_2)}\right) \exp[-\{1 - a_2\}(t/\tau_m)] \Bigg\}. \end{aligned} \tag{2.17}$$

Analytical solutions for the voltage time-course is more difficult to obtain when both neurons are assumed to be biophysically non-identical. This is because it is not clear when the denominator of $V_L(0, s)$ will be zero, apart from the obvious

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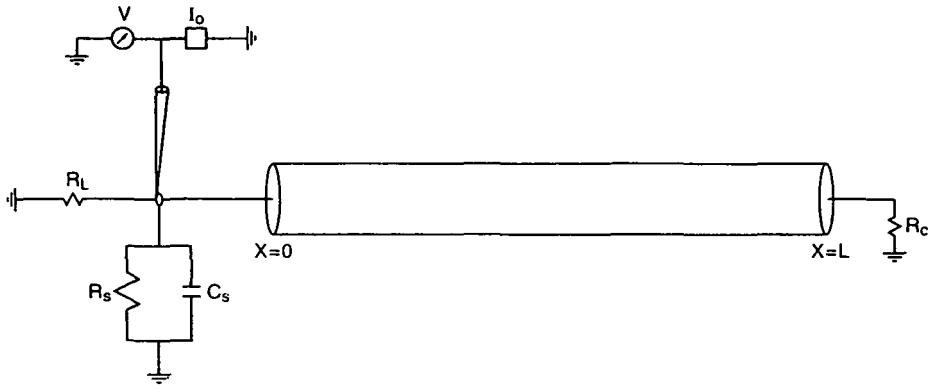

FIGURE 3. Electrophysiological experiment consisting of a pulse of current injected into the soma of a single neuron and a measure of the voltage response recorded. The effects of current leakage due to somatic and dendritic gap-junctions have been approximated with resistances R_L and R_c respectively.

choice of $s = 0$ from which the residue theorem yields an expression that is time independent and therefore not useful in the estimation of R_L. However in a recent series of papers by Major *et al.* [7], detailed analytical solutions for the voltage time-course in a multiple equivalent cylinder model of a single neuron, were found. Each cylinder corresponded to a biophysically non-identical branch segment of the neuron's dendritic tree. Thus it may be possible to extend this type of analysis to the coupled neuron problem (J. D. Evans, personal communication).

3. Voltage response to be used at relatively long times

The voltage response to a current step applied at the soma of the neuron represented by the leaky cable model (see Figure 3) can be depicted by the following initial-boundary value problem:

$$V_{XX} - V = V_T \qquad (0 < X < L, T > 0), \tag{3.1a}$$

$$(1 + R)V(0, T) + V_T - \gamma V_X = I_0 R_S, \tag{3.1b}$$

$$V_X(L, T) + \beta V(L, T) = 0, \tag{3.1c}$$

$$V(X, 0) = 0, \tag{3.1d}$$

where L is the electrotonic length of the equivalent cylinder, $T = t/\tau_m$ is the normalised time, and $\beta = R_{D\infty}/R_C$ is the ratio of the leakage conductance at the terminal end of the equivalent cylinder to the input conductance of an infinite equivalent cylinder.

The solution of the above boundary-value problem can be found by the classical separation of variables method and is similar to the solution obtained by Poznanski [9]

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and Redman *et al.* [14], excepting that the resistivity of the dendrites is assumed to be equal to the somatic resistivity:

$$V(X,T) = H(X) - \sum_{n=1}^{\infty} B_n \phi_n (X;\xi_n) \exp\left[-\left(1+\xi_n^2\right) T\right] \tag{3.2}$$

where the steady-state component satisfies

$$H(X) = I_0 R_S \left\{ \frac{\cosh(L-X) + \beta \sinh(L-X)}{(1+R+\gamma\beta)\cosh(L) + [\beta(1+R)+\gamma]\sinh(L)} \right\}, \tag{3.3}$$

the eigenfunctions are given by

$$\phi_n (X;\xi_n) = \left[\gamma\xi_n / \left(R-\xi_n^2\right)\right] \cos(\xi_n X) + \sin(\xi_n X), \qquad \xi_n \neq 0 \tag{3.4}$$

with the eigenvalues $\xi_n (n = 1, 2, \ldots)$ representing the roots of the transcendental equation

$$\tan(\xi_n L) = -\xi_n \left\{ \frac{R - \xi_n^2 + \gamma\beta}{(R-\xi_n^2)\beta - \gamma\xi_n^2} \right\}, \tag{3.5}$$

and the Fourier coefficients are given by

$$B_n = I_0 R_S \left(\frac{\gamma\xi_n\psi_1 + \beta\gamma\xi_n\psi_2 + \left(R-\xi_n^2\right)\psi_3 + \xi_n\left(1+\xi_n^2\right)\psi_4}{\left(1+\xi_n^2\right)\left(R-\xi_n^2\right)\psi_5(\psi_6+\psi_7)} \right), \tag{3.6}$$

where

$$\psi_1 = \xi_n \sin(\xi_n L) + \sinh(L), \tag{3.7}$$

$$\psi_2 = \cosh(L) - \cos(\xi_n L), \tag{3.8}$$

$$\psi_3 = \xi_n \cosh(L) - \xi_n \cos(\xi_n L) + \beta[\xi_n \sinh(L) - \sin(\xi_n L)], \tag{3.9}$$

$$\psi_4 = \cosh(L) + \beta \sinh(L), \tag{3.10}$$

$$\psi_5 = \cosh(L)[1 + R + \gamma\beta] + \sinh(L)[\beta + \beta R + \gamma], \tag{3.11}$$

$$\psi_6 = \left(\frac{\gamma\xi_n}{R-\xi_n^2}\right)^2 \left(\frac{L}{2} + \frac{1}{\gamma} + \frac{\sin(2\xi_n L)}{4\xi_n}\right), \tag{3.12}$$

$$\psi_7 = \frac{\gamma \sin^2(\xi_n L)}{\left(R-\xi_n^2\right)} - \frac{\sin(2\xi_n L)}{4\xi_n} + \frac{L}{2}. \tag{3.13}$$

Solution (3.2) consists of an infinite summation of exponential terms of which the largest time-constant of exponential decay is smaller than the membrane time-constant (τ_m) due to the presence of current leakage through the gap-junction. The solution exhibits characteristics of the somatic-shunt cable model (*cf.* [9]). Strictly

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speaking, R_L could also reflect the leakage of current caused by the sharp electrode penetrating the somatic membrane, which may in fact be larger than the leak associated with the gap-junction. To overcome this caveat, the somatic-shunt parameter should be incorporated into the equations, or alternatively, if tighter seal patch electrodes are used, then an access resistance of the pipette under the patch-clamp recording technique should be included in the model [6]. A theory has recently been developed for neurons with dendro-dendritic gap-junctions under the patch-clamp recording technique [10].

4. Estimating the passive electrical constants

Apart from the classical work of Jack and Redman [5] there appears to be no quantitative method of estimating parameters for cable models of neurons with leaky terminations.

The leaky cable model shown in Figure 3 is characterized by the following parameters:

(i) input resistance at the soma (R_N),
(ii) input leak resistance at the soma (R_L),
(iii) membrane time-constant (τ_m),
(iv) input coupling resistance (R_C),
(v) equivalent electrotonic length (L), and
(vi) dendritic to somatic input conductance (ρ).

The input resistance (R_N) is measured for intrasomatic penetrations and can be written as a combination of the somatic, dendritic and leakage resistances in parallel, that is, $1/R_N = 1/R_S + 1/R_D + 1/R_L$. The input resistance of the cell at the impalement site (assumed to be the soma) is given by the experimentally measured input resistance and is expressed by the relation

$$R_N = \frac{R_L R_S R_D}{R_S R_D + R_L(R_S + R_D)}. \tag{4.1}$$

For the present purpose the specific membrane resistivity (R_m) and hence the membrane time-constant (τ_m) can be obtained from the experimental values of R_N using the following approximation to the above formula:

$$R_N \simeq \frac{R_L R_S R_{D\infty}}{R_S R_{D\infty} + R_L(R_S + R_{D\infty})} \tag{4.2}$$

where $R_{D\infty} = (\frac{2}{\pi D^{3/2}})(R_i R_m)^{1/2}$ is the input resistance of a semi-infinite equivalent cylinder ([4]), R_i is the resistivity of the cytoplasm (interior fluid), and D is the

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diameter of the equivalent cylinder. The above relation provides an estimate of R_m from the solution of a quadratic equation in $R_m^{1/2}$,

$$R_m = \left(\frac{R_L R_N}{R_L - R_N}\right)^2 \frac{\pi^2 D^3}{16 R_i} \left(1 + \sqrt{1 + \frac{16 R_i (R_L - R_N) d_{\text{soma}}^2}{\pi D^3 R_N R_L}}\right)^2, \tag{4.3}$$

and hence the corresponding values for $R_{D\infty}$ and R_S may be obtained:

$$R_{D\infty} = \left(\frac{2 R_i^{1/2}}{\pi D^{3/2}}\right) R_m^{1/2} \tag{4.4}$$

and

$$R_S = \frac{R_m}{\pi d_{\text{soma}}^2}, \tag{4.5}$$

where R_S is the input resistance of the soma and d_{soma} is the diameter of the soma.

It should be noted that all the above parameters (R_m, R_S, $R_{D\infty}$) are dependent on the unknown leakage resistance at the soma (R_L) which can be found from the experimentally observed voltage transients at relatively short times (see Section 2). For example, using the determined values of R_s and $R_{D\infty}$, the relations (2.16) and (2.17) can be used to determine the leakage resistance (R_L) by observing the magnitude of the somatic potential and scaling the response in terms of the input current magnitude (I_0).

The cytoplasmic resistivity (R_i) is a function of internal electrolyte composition and in most instances is difficult to estimate (see, for example, [16]). The value may be increased by the presence of a large concentration of organic solutes and by binding of some of the intracellular organic anions.

This method of estimating the specific membrane resistivity from the input resistance of a neuron was first developed for the electrical constants of the motoneuron membrane by Coombs *et al.* [2].

It should be noted that the standard procedure of estimating the membrane time-constant from the slope of the semi-logarithmic plot of the voltage transients at the soma, cannot apply if the end-termination of the equivalent cylinder is not sealed (see [12]) or if the cable length exceeds two length-constants (see [3]). Thus, if the specific capacitance of the membrane (C_m) is a biological constant taken to equal $1 \mu F/cm^2$ [1], then the membrane time-constant (τ_m) can be found from $\tau_m = R_m C_m$ (see [13]). If the neuron showed evidence of membrane folding or any other abnormal membrane morphology, this would justify assuming the capacitance was other than $1 \mu F/cm^2$.

The decay of voltage response from the steady-state value at the soma has been shown in Section 3 to be governed by an infinite series of exponential terms:

$$V(T) = C_1 \exp(-t/\tau_1) + C_2 \exp(-t/\tau_2) + C_3 \exp(-t/\tau_3) + \ldots, \tag{4.6}$$

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where $C_1 = B_1\gamma\xi_1/(R - \xi_1^2)$, $C_2 = B_2\gamma\xi_2/(R - \xi_2^2)$, $C_3 = B_3\gamma\xi_3/(R - \xi_3^2)$ are constants representing each component's amplitude (independent of time), and $\tau_1, \tau_2, \tau_3, \ldots$ represent time-constants whose relation to the membrane time-constant (τ_m) can be shown to be

$$\tau_n = \tau_m / \left(1 + \xi_n^2\right). \tag{4.7}$$

On plotting the experimentally derived voltage transients against time on a logarithmic scale and fitting a regression line to averaged voltage records, a straight line would indicate that the charging curve is well described by a single exponential. The slope of this line yields an estimate of the time-constant (τ_1) that is shorter than the membrane time-constant (τ_m) and is expressed by the relation

$$\tau_1 = \tau_m / \left(1 + \xi_1^2\right). \tag{4.8}$$

The electrotonic length parameter (L) can now be estimated from the transcendental equation for the eigenvalues using the above relation (since the ratio (τ_m/τ_1) is known). Unfortunately the exact value of the electrotonic length (L) can only be found if the coupling resistance (β) is known. Hence, we follow the procedure adapted by Jack and Redman [5] and obtain an estimate of the electrotonic length parameter (L^*) from the assumption that the end-termination is short-circuited, and by putting $\beta = 0$ in the transcendental equation for the eigenvalues to yield

$$L^* = (1/\xi_1)\tan^{-1}\left\{\left(R - \xi_1^2\right)/(\gamma\xi_1)\right\}. \tag{4.9}$$

The coupling resistance (R_C) can now be determined from (3.5) with the electrotonic length parameter (L^*) used as an initial approximation for the electrotonic length parameter of the neuron (L):

$$R_C \simeq R_{D\infty}\left(\frac{\left(R - \xi_1^2\right)\tan(\xi_1 L^*) + \xi_1\gamma}{\gamma\xi_1^2\tan(\xi_1 L^*) - \xi_1\left(R - \xi_1^2\right)}\right). \tag{4.10}$$

In a recursive manner the electrotonic length parameter (L) can be estimated more accurately from (3.5) given that the value of $\beta = R_{D\infty}/R_C$ is known, and at the same time a better estimate of R_C can be found by replacing L^* with L.

Finally, the dendritic to somatic conductance ratio parameter ($\rho = R_S/R_D$) can be obtained from the following relation for R_D (*cf.* [14]):

$$R_D = R_{D\infty}\left(\frac{[1 + \exp(-2L)] + \beta[1 - \exp(-2L)]}{[1 - \exp(-2L)] + \beta[1 + \exp(-2L)]}\right). \tag{4.11}$$

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5. Summary and conclusions

In this paper a theoretical analysis was presented to interpret cable parameter estimates for neurons that are electrotonically coupled. It provides the experimenter with a way of interpreting the electrophysiological data that is obtained in situations when the neuron in question is known to be coupled to other neurons. The present theory can also be extended to incorporate neurons with dendritic trees that are not of the equivalent cylinder class (see [11]). Of course, with any theory, its usefulness lies in its correct validation of the experimental data.

The effect of electrotonic coupling on passive membrane parameters (R_N, τ_m, L) was recently examined for immature rat neocortical-pyramidal neurons by Rorig *et al.* [15]. It was shown that the influence of gap-junction coupling decreases the input resistance (R_N), decreases the membrane time-constant (τ_m), and increases the electrotonic length (L). Unfortunately, Rorig *et al.* [15] used a simple expression for L, originally derived by Rall [12] which applies only to uncoupled neurons with sealed-ends. However, the influence of gap-junction coupling on L can be shown from the theory presented herein to produce a less compact neuron due to an increase of L for coupled neurons, in agreement with the experimental results of [15].

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