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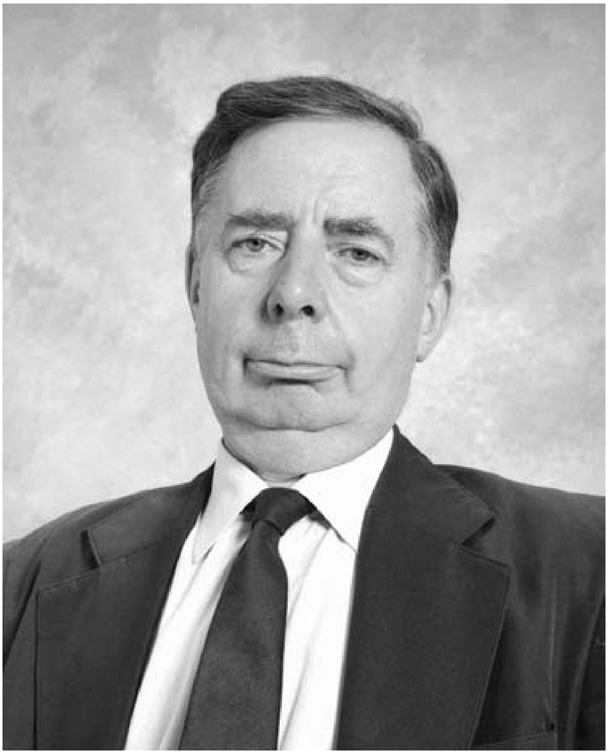
OBITUARY

CRISPIN ST J. A. NASH-WILLIAMS (1932–2001)



Crispin St John Alvah Nash-Williams was born in Cardiff on 19 December 1932. His father was Keeper of Archaeology at the National Museum of Wales and also Senior Lecturer in Archaeology at University College, Cardiff. War broke out when Crispin was six; his father joined the army and his consequent absence changed the family's life.

Crispin became a boarder at Christ Church Cathedral School, Oxford, where he was very happy. The headmaster, the Reverend Wilfrid Oldaker, was a formative influence, and the teaching and ethos were inspiring, although perhaps Crispin did not appreciate all the musical offerings of a choir school. At this time his mother and younger brother moved to Swaffham, Norfolk. She had read classics at Oxford and this enabled her to obtain a teaching post at a grammar school in Swaffham, and the two of them lived with the headmaster and his wife. It turned out to be rather a nomadic existence, as during the school holidays the family lived in a rented flat in Reading. More changes came when she moved to a girls' grammar school in London and the flat in Reading was replaced by one in Chelsea.

In 1945, Crispin went to Rugby School; he took School Certificate at the age of 14 and from then on concentrated on Mathematics. This came as no surprise to his family, for he had always been good at this subject. His brother, Piers, recalls riddles involving negative numbers, and there is a family story of a very young Crispin crying with rage because his count of the number of pieces of paper in a toilet roll was out by ten.

At Rugby, Crispin had been much influenced by his mathematics teacher, H. P. Sparling. It was he who directed Crispin to Trinity Hall, Cambridge where he was very happy, as Sparling knew he would be under the mathematical guidance of S. Wylie. During his first year he was active in the College Boat Club and coxed one of the College boats, but then gave it up to concentrate on his studies. Crispin was awarded a scholarship and graduated as Senior Wrangler in 1953. Before going up to Cambridge, Crispin spent three months living with a family in Grenoble whilst taking a course in French. He also later acquired a good reading knowledge of German.

Paying school fees for two boys made it a financial necessity for both parents to keep their respective jobs. As one was in Cardiff and the other in London, this led to a somewhat fragmented family life. His parents were looking forward to buying a cottage in Burford and spending their retirement together there, but two years before his retirement date Crispin's father died at the age of 58, and his mother later decided to move there alone on her retirement in 1963.

The lifestyle enjoyed by Crispin was essentially a simple one, with the exception of the extraordinarily large number of shirts and ties he owned. He never bought a car, although he did pass the Ontario driving test while in Waterloo – or rather, as he put it in a letter to his family, 'The Ontario Department of Transport passed my test of their competence in examining me.'

When a car was really necessary he would take a taxi, sometimes for long distances, but he preferred the train. Indeed, his holidays usually consisted of buying a Runabout Ticket and travelling on his beloved railways wherever the whim took him. Crispin was also noted for his punctuality. He was paranoid about being late for anything, which is hard to reconcile with his love of rail travel.

He also loved walking, and was a member of the Ramblers' Association. Many holidays were taken with his widowed mother, to whom he was devoted. I recall that several times after visiting Oxford he would continue his journey to Burford to spend time with her.

His mathematical precision carried over to his everyday life. His brother Piers recounts how at Rugby there was a book in which the boys had to write their music practice times. Instead of the usual 5.00–5.30 Crispin's entry would read 5.01 to 5.28! The piano lessons were an unexpected venture because, as Piers says, Crispin was tone deaf! Violin lessons at Christ Church had made his teeth go on edge and in conversations in later life he confessed that his concept of music was something akin to a 'row of notes', although he did admit to one or two pieces of music (particularly J. S. Bach's *Jesu, joy of man's desiring*) as being 'beyond this world'.

Throughout his time at the University of Reading he lived in a flat at St Patrick's Hall, where he was acting Warden for a short period. Most visitors were struck by the large array of filing cabinets where everything was meticulously stored, even down to staples under 'S'.

He took early retirement in September 1996, prompted by a great dislike of the administrative chores he had felt it was his duty to take on. These included being Head of Department for six years and President of the Reading Branch of the

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Association of University Teachers from 1987 to 1989. According to ⟨**24**⟩ he was once 'described by someone in high office as an out-and-out militant'. To those who knew him this seems a quite remarkable statement, as in everything he approached he was unfailingly polite. However, this chore had its lighter side, at least for other people. The story was told at his retirement party in Reading that he had been seen on one occasion marching down Whitehall carrying a placard that bore the legend, 'Rectify the Anomaly'!

Crispin's intention was to spend the rest of his life doing research. I recall well the last seminar he gave in Oxford in November 1997 on his work with David White. This really excited him, and it was a tragedy that in the summer of 2000 he became ill with what turned out to be terminal cancer. Shortly after a major operation at Hammersmith Hospital he became too ill to live alone, so he moved to a retirement home in Ascot to be near to Piers, who was then Rector of Ascot.

Since childhood Crispin had been a practising Christian, and throughout his time at Reading was a regular worshipper at Christ Church, Reading. At his funeral, held there soon after his death on 20 January 2001, David West, the vicar, spoke movingly, and with great insight, of Crispin's humility.

Returning to his mathematical career, after graduating in 1953 he spent the next two years at Cambridge with the support of an Amy Mary Preston Read Scholarship, where his research supervisors were D. Rees and S. Wylie. He then spent the year 1956–57 at Princeton on a J. S. K. Visiting Fellowship, where he met N. E. Steenrod. All three are acknowledged as being supervisors of his PhD thesis, 'Decomposition of graphs into infinite chains', which he submitted in 1958. His thesis was extremely long (more than 500 pages), and one of his examiners was the late Richard Rado. Richard's wife Luise, rather unjustly, never forgave Crispin for writing at such length, as apparently Richard took the thesis with him on a walking holiday and spent more time reading it than she would have liked.

After the year in Princeton, in October 1957 he became an Assistant Lecturer at Aberdeen University, where he stayed for ten years, being promoted first in 1958 to Lecturer, and later from 1964 to Senior Lecturer, before moving to the University of Waterloo when the Faculty of Mathematics was founded in 1967. He had already spent time in Waterloo as a visiting professor in 1965. At Waterloo he was one of three founding professors in the Department of Combinatorics and Optimisation, and was the first graduate officer in the department. His Waterloo colleague, Professor Daniel Younger, reports that:

> As graduate officer, he almost single-handedly set the rigorous comprehensive examinations given to graduate students in that period. PhD students for which he was the chief supervisor were V. Chvatal, J. A. Zimmer, A. K. Dewdney, V. Chungphaisan, J. Chvatalova, the last two completing their degrees after his departure. He was a most helpful and rigorous examiner of PhD theses, for example, those of N. Robertson and C. L. Lucchesi. Many other graduate students took his courses and learned a lot from him of how to make proper mathematical arguments. He was a strong, energetic and memorable presence throughout his tenure.

His death was reported in the *Mathematics Alumni Newsletter* of the University of Waterloo, from which the following quotation is taken:

> Professor Nash-Williams was a leading contributor to combinatorial mathematics for his entire academic career. Beginning at a time when the subject was relatively undeveloped, he led the way in identifying important research

> problems, developing new techniques and establishing the highest standards of rigour and integrity. In addition to the creation of new mathematics, an important and largely unrecognised part of the life of research mathematicians is the critical reading and critiquing of the work of others. The contributions of Crispin Nash-Williams in this regard were truly outstanding. A leading combinatorial mathematician recalls that a turning point in his career was the fifty-page commentary that Nash-Williams wrote on his hundred-page doctoral thesis!

He returned to Aberdeen as Professor of Pure Mathematics in 1972. While he was there, he and John Sheehan organised the Fifth British Combinatorial Conference, which was distinguished by a galaxy of star speakers. However, his stay in Aberdeen was brief, as in 1975 he moved to the chair at Reading as successor to Richard Rado. Apart from frequent short-term visits to Waterloo (with which he maintained strong connections as an adjunct professor) and a year at the University of West Virginia, he stayed there until he died.

At Reading he joined a group of combinatorically inclined mathematicians; apart from Richard Rado (very active, albeit retired) there were also David Daykin, Anthony Hilton and David White. Among his graduate students were K. Apostolidou, J. Caunter, D. Grant, A. King and D. Marušič. Testimony to his care as a research supervisor can be found in the tribute by Marušič in ⟨**16**⟩.

Nash-Williams' lectures were superbly organised. No detail was omitted and yet the effect was one of simplicity, with the main ideas clearly highlighted. The first lecture I heard him give, a seminar at Oxford in 1966, on the applications of matroids to other areas of combinatorics, had a huge effect on my mathematical interests.

As an examiner he was exceptionally careful. I recall one doctoral thesis for which he was the external examiner, where the candidate used to wait until the end of each week before collecting into one response his answer to perhaps three handwritten letters from Crispin seeking clarification on various points. Indeed, looking back, it was a brave person who nominated Crispin to be an external; his standards were so high.

It was not only in research and teaching that Nash-Williams was the embodiment of care and accuracy. His conscientiousness as a referee was very well known in the Mathematical community. Nothing either trivial or slipshod would get past him, yet as has been reported ⟨**24**⟩ by a departmental colleague, 'As a teacher he is remembered and loved for his consideration for the weak students as well as the strong.'

I have little knowledge of his administrative skills within his own university, but I do know that being Head of Department at Reading took a great deal out of him. I did, however, have quite a bit to do with him through the British Combinatorial Committee. He was one of a small group, which also included Norman Biggs, Hazel Perfect, Douglas Woodall and myself, that, at the 1972 Oxford Combinatorial Conference in Oxford, set up a committee to organise future regular combinatorial conferences in Britain. I believe that he was a member of the committee from that day until his death. He was always a rock of good sense and judgement, and an excellent Chairman from 1987 to 1992.

Crispin was elected a Fellow of the Royal Society of Edinburgh on 3 March 1969, and was awarded an Honorary Doctorate by the University of Waterloo in 1994 for his outstanding contributions to combinatorial mathematics.

Mathematical work

Although Crispin Nash-Williams' principal interests were in graph theory, he made significant contributions to the theory of random walks and Markov chains, transversal theory, and particularly the theory of quasi-orders.

His first two papers, both in terms of publication date and submission date, did not form part of his thesis and appear to have been submitted on the same day. The paper [**1**] is a classic. While he acknowledges helpful discussions with D. G. Kendall, R. Swan and H. Trotter, it is not clear where the impetus to work on this problem originated. Possibly, it arose out of his interest in infinite graphs, as one way of looking at the simplest version of the problem is as follows.

Suppose that G is connected and locally finite (in other words, each vertex has only a finite number of neighbours). Imagine a particle starting at a vertex v and performing a random walk on G, each step moving to a neighbour chosen at random. A standard theorem of Markov chains is that either the particle has probability one of returning to v (independently of the choice of v), in which case G is called *recurrent*, or the probability of return is strictly less than 1, in which case G is *transient*. The main result of [**1**] can be regarded as a precise characterisation of those infinite graphs that are recurrent. Intuitively, such a graph must not 'widen out too rapidly as it goes off to infinity'.

Although [**1**] is not an easy paper to read, it contains an important tool in Markov chain theory, which is now known as the Nash-Williams recurrence criterion for reversible Markov chains. A good example of its applications can be found in the paper of Griffeath and Liggett ⟨**9**⟩ on interactive particle systems. A shorter proof of the recurrence criterion has now been found by Lyons ⟨**15**⟩. The paper [**1**] is also of importance historically for, as pointed out in ⟨**6**⟩, it appears to have been the first to have applied Rayleigh's method from classical electrical network theory to random walks, and the full importance of this has still to be realised.

As an example of this, let G be d-regular connected on n vertices and consider a simple random walk on G, at each step moving to a neighbour with probability $1/d$. The mean *access* time t_{ij} is the expected number of steps required to reach vertex j starting from vertex i. The mean *commute* time $g_{ij} = t_{ij} + t_{ji}$.

Now consider G as an electrical network with each edge having unit resistance. If the overall or equivalent resistance between i and j is denoted by R_{ij} then Nash-Williams shows that the equivalent resistance R_{ij} and the mean commute time g_{ij} are related by

$$g_{ij} = ndR_{ij}.$$

For more on this and its applications, we refer to Lovász ⟨**14**⟩.

Nash-Williams' second published work [**2**] appears to have been work that he carried out while a student at Cambridge. It originated in an attempt to generalise work of Vázsonyi ⟨**27**⟩, who showed that the n-dimensional infinite grid L_n can be covered by both a one-way and a two-way infinite sequence of knight's moves so that each position is visited exactly once. In other words, the rather strange graph obtained by joining two vertices of the grid by an edge if and only if they are a knight's move apart has both a one-way and a two-way Hamilton path. The generalisation is obtained by considering the following much more general problem about infinite Abelian groups.

Suppose that A is a countably infinite Abelian group and S is a subset of its elements. A sequence of elements of A is called S-*admissible* if the difference between successive terms of the sequence all belong to $S \cup (-S)$. The main result of [**2**] is that if G is a set of generators of the group A, then the elements of A can be ordered (with no repetitions) in both a one-ended and an endless G-admissible sequence unless G is finite and A has rank 1, when the one-ended order is impossible. Applying this to Vázsonyi's original problem shows that the 'knight move' of his theorem can be replaced by any 'chess piece move', provided that the move has the properties (a) of being able to reach any vertex of L_n in some finite sequence of moves from the origin, and (b) that if it is possible to move from x to y in one move, then it is also possible to get from y to x in one move. Nash-Williams acknowledges that the connection between the chessboard problem of Vázsonyi and this very nice generalisation to groups was pointed out to him by his research supervisor D. Rees. Nevertheless, the fact that he could make all the arguments go through for this much more general situation is testimony to his mathematical ingenuity.

The two papers that can be regarded as direct successors of [**2**] are [**6**] and [**13**]. Ringel ⟨**23**⟩ had generalised Vázsonyi's earlier result ⟨**27**⟩ by showing that whenever n is a power of 2, the grid L_n has n edge disjoint two-way infinite Hamilton paths with the property that each edge of L_n is contained in exactly one of them. In [**6**] Nash-Williams proves the same result for all values of n. The connection with the main result of [**13**] comes from regarding the grid L_n as the product $L_r \times L_s$ for any positive integers r, s such that $r+s=n$. One can now see how an inductive proof that L_n has a one- or two-way Hamilton path leads to the main theorem of [**13**], which is that if T_1 and T_2 are countably infinite trees without endpoints, then the product $T_1 \times T_2$ has both a one-way and a two-way Hamilton path.

I turn now to Nash-Williams' contributions to the theory of graphs, and treat the different areas separately.

Decompositions

In his thesis, Nash-Williams had concentrated on issues of decomposability of graphs, both finite and infinite, into open and closed chains. Out of it came the substantial papers [**3**], [**5**], [**8**] and [**18**], the last of which was not submitted until April 1965 even though, as he observes in [**23**], he had obtained this result characterising infinite Eulerian digraphs as early as 1955. He goes on to explain this delay as being due in part to its 'somewhat complicated and specialised character'.

In his first purely graph-theoretic paper [**3**], Nash-Williams proved that a graph G is *decomposable* into cycles (that is, there is a collection of cycles in G such that each edge of G is in exactly one of them) if and only if it has no finite edge-cut of odd cardinality. For finite or countably infinite graphs this has a relatively easy proof. However, in the uncountable case Nash-Williams' proof is very complicated, and it is not surprising that the problem lay dormant for nearly thirty years until in 1987 Polat ⟨**20**⟩ was able to generalise it to *matchable set systems*. This is based on defining a *matching* in a family of sets to be a subfamily whose members are disjoint. In his obituary of Eric Milner [**70**], Nash-Williams gives a typically clear account of how this extends to a result about infinite binary matroids.

Eulerian trails

Erdös, Gallai and Vázsonyi ⟨**8**⟩ established necessary and sufficient conditions for a countably infinite graph to have a two-way infinite Eulerian trail. In his thesis, Nash-Williams considered the more general problem of deciding when a graph is decomposable into k but no fewer two-way infinite trails. The Eulerian case corresponds to taking $k = 1$.

Obvious necessary conditions for G to have such a decomposition are that the degree of each vertex in G is either even or infinite, and that G has no finite component. A slightly less obvious condition is that G is *p-limited* for some finite integer p, which means that G looks roughly like p infinite 'wings' branching out of a finite centre. The main result of [**8**] is that these three conditions, together with one technical condition:

$$1/2w_{\mathrm{o}} + w_{\mathrm{e}} = k,$$

where w_{o} and w_{e} denote the number of odd and even wings of G respectively, are necessary and sufficient.

The corresponding question for infinite directed graphs was also completely answered in Nash-Williams' thesis but, as he explains in [**18**], was not published until 1966. This is a difficult paper, and a useful explanation of the underlying ideas of the proof of the main theorem can be found in [**23**]. The paper [**18**] also contains (p. 712) a summary of the problems solved, or only partially solved, in Nash-Williams' thesis, which had not at that time been published.

The papers [**7**] and [**10**] concern decompositions into forests and spanning trees. It is curious that the main theorems of both were also discovered independently by W. T. Tutte ⟨**26**⟩. There is a typical observation on this in [**50**, p. 198], where Nash-Williams modestly observes, 'My own method of proof of these theorems was a fairly heavy-handed induction argument but Tutte found a more ingenious, and at first sight improbable, method of proof.' The main results of [**7**] and [**10**], giving necessary and sufficient conditions for a graph to have, respectively, k edge disjoint spanning trees or to be the union of k edge disjoint forests, have had a huge impact. This is due in no small measure to the fact that both results have a very natural extension to matroids and indeed the slickest proof of both is as corollaries of a general theorem about matroid union. This theorem has a much shorter proof than the original proofs, which were purely for graphs (see for example ⟨**19**⟩ or ⟨**28**⟩) and is based on Nash-Williams' elegant short note [**20**], in which the notion of matroid union first makes an explicit appearance.

Orientations

One of Nash-Williams' earliest results [**4**] was a generalisation of a classical (but easy) theorem of H. E. Robbins. A digraph is *strongly connected* if there is a directed path between any pair of vertices. It is *k-edge connected* if for any two vertices u, v, there are k edge disjoint paths joining u to v. By the classical theorem of Menger this is equivalent to the requirement that for every subset $U \subseteq V, U \neq \emptyset, U \neq V$, there are at least k edges whose terminal vertex belongs to U. Necessary and sufficient conditions for an undirected graph to be *k-edge connected* are that every cut contains at least k edges. Nash-Williams [**4**] proves that for every positive integer k an undirected graph $G = (V, E)$ has an orientation which makes it a k-edge

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connected digraph if and only if G is $2k$-edge connected. Taking $k=1$ gives the theorem of Robbins.

However, not content with this, Nash-Williams went on to prove the following much stronger result, which illustrates the detail and thoroughness that characterise his work.

Suppose that $\lambda(x, y)$ denotes the maximum number of edge disjoint paths joining two vertices x, y of an undirected graph G. It is clear that if D denotes any oriented version of G then there can be at most $\lfloor \lambda(x, y)/2 \rfloor$ edge disjoint paths joining each ordered pair of vertices of D. The striking result of [**4**] is that there always exists an orientation which simultaneously achieves this upper bound for all ordered pairs (x, y).

A far-reaching generalisation of this result concerning *mixed* graphs in which only some edges have an orientation can be found in [**68**].

Hamiltonian questions

Although Nash-Williams had clearly had a long-standing interest in Hamiltonian questions, his earlier papers on the topic, [**2**], [**6**] and [**13**], had been concerned with proving that a particular infinite graph, such as the 'knight-move grid' was decomposable into Hamilton paths. However, in a series of papers published between 1966 and 1974, he made significant improvements on the classical theorem of Dirac. Dirac's result is that if G is simple on n ($n \geqslant 3$) vertices with minimum degree $\delta \geqslant n/2$, then G contains a Hamilton circuit. Nash-Williams' first paper on this theme was a two-page note [**17**] which gives a new proof of Pósa's $\langle \mathbf{21} \rangle$ strengthening of Dirac's theorem.

In [**24**], which is part survey and part original, Nash-Williams considers for the first time what he calls *Dirac graphs*. These are simple graphs on $n \geqslant 3$ vertices, each of which has degree at least $n/2$. His first result is that for each integer k there exists an integer N_k such that any Dirac graph with more than N_k vertices has at least k edge-disjoint Hamilton circuits; for example, N_2 is approximately 10.

In [**29**] he announces two ways in which he has been able to extend Pósa's theorem $\langle \mathbf{21} \rangle$ to infinite graphs. Specifically, he gives new sets of sufficient conditions for a countably infinite graph to have (a) a two-way infinite Hamilton path and (b) a one-way infinite Hamilton path. Not surprisingly, both theorems demand the hypothesis that the graph does not have a shape consisting of too many infinite wings branching out of a finite central portion. The proofs are difficult and can be found in [**33**].

In the same paper [**29**] he also announces the very striking result of [**32**] that when G is simple of minimum degree δ on n vertices, $n \geqslant 3$, then if $\delta \geqslant n/2$, G contains at least $\lfloor 5n/224 \rfloor$ edge-disjoint Hamilton circuits. The proof of this result is long and not easy, and the constant $5/224$ is certainly not best possible, but, as far as I am aware, the exact constant is still not known.

Apart from Dirac-like theorems, there are now many other sufficient conditions for a graph to be Hamiltonian that do not demand that the vertices have very large degrees. As Bollobás $\langle \mathbf{2} \rangle$ points out, the first notable result of this kind was proved by Nash-Williams [**32**] when he showed that provided an n-vertex graph G is 2-connected and has minimum degree $\delta(G) \geqslant (n+2)/3$ then G is Hamiltonian, provided that $\delta(G)$ is at least as big as the stability number.

The reconstruction problem

It is now more than fifty years since the 'reconstruction conjecture' was proposed by S. M. Ulam and P. J. Kelly. The original vertex form can be stated as follows. Let G be a simple graph with n vertices, with $n > 2$, and suppose that one is given a deck of n cards, where each card depicts the subgraph of G obtained by deleting one vertex. The conjecture is that from this information it is possible to determine G up to isomorphism. In another form of the problem, known as the 'edge reconstruction conjecture', the cards contain the edge-deleted subgraphs instead. There is now a huge literature on these problems and it was a continuing long-term interest of Nash-Williams. Certainly he was interested in the problems in the late 1960s (see, for example, [**19**]) but it was not until 1978 that he produced his first contribution [**47**] to the subject. This contains what is described in ⟨**3**⟩ as 'the most powerful tool developed thus far in the assault on the Edge Reconstruction Conjecture'. This is now known as the Nash-Williams' lemma, and can be stated as follows.

LEMMA. *Let G be a graph and suppose that H is an edge reconstruction of G that is not isomorphic to G. Then*

$$|G \to G|_F - |G \to H|_F = (-1)^{|E(G)|-|E(F)|}|\mathrm{aut}(G)|,$$

where $|G \to H|_F$ denotes the number of permutations of the vertex set V such that (in an obvious notation) $\pi(G) \cap H = \pi(F)$.

One immediate consequence of this lemma is that G is edge reconstructible if there exists a spanning subgraph F of G such that $|G - H|_F$ is the same for all edge reconstructions of G. Very easy applications of this lemma yield as corollaries two of the strongest results on edge reconstruction, namely those of Lovász ⟨**13**⟩ and Müller ⟨**18**⟩, which imply that a graph on n vertices having m edges is edge reconstructible if either

$$\text{(a) } 2m \geqslant \binom{n}{2} \quad \text{or} \quad \text{(b) } 2^{m-1} \geqslant n!$$

It must be emphasised that it was in trying to understand the ideas of ⟨**13**⟩ and ⟨**18**⟩ that Nash-Williams came up with this lemma.

Several other sufficient conditions for G to be edge constructible, which follow easily from Nash-Williams' lemma, are contained in his unpublished manuscript with J. Caunter and are described in ⟨**3**⟩. For example, a graph of maximum degree Δ and average degree d is edge reconstructible if $2\log_2(2\Delta) \leqslant d$.

What is even more striking is the way in which the arguments used in the proofs of the Nash-Williams lemma have been amenable to a much more general treatment, allowing powerful algebraic methods to be used; for a much wider class of structures see, for example, Babai ⟨**1**⟩.

After this major incursion in 1978, it was not until 1987 that Nash-Williams returned to reconstruction with a series of five papers, spread over seven years, that were entirely about reconstruction in infinite graphs.

The main results of this investigation are that for $p \geqslant 2$, every p-coherent, connected, locally finite graph is reconstructible. (An infinite graph is *p-coherent* if it can be expressed as the union of a finite subgraph and p pairwise disjoint infinite subgraphs, but cannot be so expressed with $p+1$ pairwise disjoint infinite subgraphs.) His paper [**59**] proves the theorem for $p \geqslant 3$; the case $p = 2$, which is more difficult because of the lack of a unique recognisable centre, is proved in [**63**].

Detachments

Over the last ten years or so of his time at Reading, one of Nash-Williams' main graph-theoretical interests was the theory of detachments. This was a topic that he developed himself, though it is clearly motivated by his earlier work on Eulerian trails.

A *detachment* of a graph G is a graph D obtained by splitting each vertex v of G into one or more vertices called images and insisting that an edge that was incident with v in G is incident with exactly one of the images of v in D. If g is a function from the vertex set into the positive integers, D is a g-*detachment* if it is a detachment of G in which each vertex v has $g(v)$ images.

It is easy to frame several of the classical decomposition theorems in graph theory in terms of detachments; for example Euler's trail theorem and [**3**, Theorem 2], which gives necessary and sufficient conditions for a graph to have a 2-regular acyclic detachment (that is, only even and infinite valences are allowed). The main thrust of Nash-Williams' theory of detachments therefore is to see how much can be generalised to arbitrary g-detachments.

For example, suppose that G is finite and that g is a function from $V(G)$ to the positive integers. Then G has a *connected* g-*detachment* if and only if for every non-empty subset $U \subseteq V$,

$$g(U) + k(G \backslash U) \leqslant 1 + e(U),$$

where $e(U)$ denotes the number of edges having both endpoints in U, $g(U)$ denotes the sum of $g(v)$ over members of U and $k(G \backslash U)$ is the number of connected components of the graph obtained from G by deleting U and its incident edges.

I turn now to Nash-Williams' contributions in other areas of Mathematics.

Transversal theory

Transversal theory is one of the areas of set theory in which Nash-Williams' contributions have had a major impact. It was the subject of at least eight of his papers. A *transversal* of a family $(A_i : i \in I)$ of sets is a family $(s_i : i \in I)$ such that $s_i \in A_i$ $(i \in I)$, and $s_i \neq s_j$ when $i \neq j$. Philip Hall's classical 'marriage theorem' is the cornerstone of the subject. This gives necessary and sufficient conditions for a *finite family* to have a transversal, namely that for each k, the union of any k of the A_i contains at least k distinct elements.

Characterising when an infinite family has a transversal is a much harder problem. Nash-Williams' first written contribution to this problem was based on his talk [**36**] at the 1972 Oxford Combinatorial Conference. In the case of a countably infinite family the first characterisation was given by Damerell and Milner ⟨**4**⟩, thus proving a conjecture that Nash-Williams made in [**36**]. In 1978, Nash-Williams himself gave another answer to this problem in his paper [**45**]. Turning to the much harder case of an uncountable family, it was not until 1983 that in two papers [**53**, **54**] with Aharoni and Shelah, Nash-Williams gave definitive answers to this question. These are major contributions to the subject and it is a pity that the complete answer in the uncountable case is so complicated. More details on these and related problems can be found in ⟨**29**⟩.

I know that the late Eric Milner, who devoted a major portion of his mathematical research to this difficult area, had a huge respect for the insight and depth of Nash-Williams' contributions to transversal theory.

Quasi-orders

The area of set theory in which Nash-Williams' impact has been of even more significance is in the theory of quasi-orders. A *quasi-order* is a reflexive and transitive relation. A quasi-ordered set $(S, \leqslant)$ is *well-quasi-ordered* if for every infinite sequence $(x_1, x_2, \ldots)$ of elements of S there are indices $i < j$ such that $x_i \leqslant x_j$.

Nash-Williams became interested in quasi-orders in 1957 when he first learnt of Vázsonyi's conjecture that in any infinite sequence $T_1, T_2, \ldots$ of trees there are trees T_i and T_j such that $i < j$ and T_i is a *subdivision* of T_j. A *subdivision* of a graph G is a graph obtained from G by inserting into each edge some non-negative finite number of vertices of degree 2. In 1960, Kruskal ⟨**12**⟩ found a proof of the conjecture for *finite* trees and, as reported in [**51**], 'after working intensively on the problem for four years' Nash-Williams managed to prove the full conjecture in his paper [**15**]. In order to do this he needed to invent a whole new theory for, as observed by Robin Thomas in a recent NSF-CBMS lecture series, 'the Kruskal type argument breaks down completely'.

As Nash-Williams remarks in [**51**], 'the large amount of effort invested in this problem brought some additional "spin off".' This was because it led to his very short proof [**9**] of Kruskal's theorem for finite trees. This is the proof that Neil Robertson has said 'just cannot be improved'. It also led to his proof in [**14**] and [**22**] of conjectures of Rado and Milner respectively on the well-quasi-ordering of transfinite sequences. In order to explain these, suppose that $F(A)$ denotes the set of all finite sequences whose terms belong to the set A. Higman ⟨**10**⟩ proved that if $(Q, \leqslant)$ is well-quasi-ordered then $F(Q)$ is well-quasi-ordered under the *domination ordering* induced by the ordering on Q. In $F(Q)$, a sequence $a \leqslant b$ if there exists a subsequence b' of b such that each term a_i of a is dominated in Q by the corresponding term of b'. This theorem does not go through to infinite sequences in full generality. However, suppose that one calls a transfinite sequence *restricted* if it has only finitely many distinct terms. Rado ⟨**22**⟩ conjectured that if Q is well-quasi-ordered then the set of all restricted transfinite sequences on Q is well-quasi-ordered, and in the same paper he proved this to be true for transfinite sequences of length at most ω^3. This was extended by Erdős and Rado ⟨**7**⟩ to restricted transfinite sequences of length at most ω^ω and finally in [**14**] Nash-Williams proved the result in its full generality.

Again, this proof is quite complicated, but as a bonus Nash-Williams obtains a significant extension of the classical Ramsey theorem for infinite sets, which we now explain.

Suppose that $\mathscr{A}$ is a family of subsets of an infinite set E such that no two distinct members of $\mathscr{A}$ are contained in each other. Then for every finite partition of $\mathscr{A} = \mathscr{A}_1 \cup \ldots \cup \mathscr{A}_m$ there exists an infinite subset Y of E such that all members of $\mathscr{A}$ that are subsets of Y belong to the same class of the partition.

To obtain the classical infinite Ramsey theorem from this, just take p and m to be positive integers and $\mathscr{A}$ to be the set of all p-element subsets of E. Then we see that for every partition of $\mathscr{A}$ into m classes, one of the classes contains all p-element subsets of an infinite set Y.

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Nash-Williams' second paper on well-quasi-orders and transfinite sequences settled a problem raised by Milner in ⟨**17**⟩, where it had been shown that if the elements of Q are transfinite sequences of ordinal numbers and $q \leqslant q'$ means that some subsequence of q' dominates the sequence q, then $(Q, \leqslant)$ is well-quasi-ordered if every element of Q is a transfinite sequence of length less than ω^3. Milner conjectured that the result held without any restriction on length. Nash-Williams proved this conjecture in [**22**] using a quite complicated notion which he called *better-quasi-orders*, and for which he can prove that if Q is better-quasi-ordered then the set of transfinite sequences on Q is also better-quasi-ordered. The arguments are again long and difficult.

Apart from the intrinsic merits of these results on well-quasi-ordering, the work has also had a profound impact on the development of the subject. This is through its influence on the powerful theory of the graph minors programme developed over the last 15 years or so by Neil Robertson and Paul Seymour, which has clearly revolutionised not only the theory of graphs, but also its algorithmic aspects.

Rearrangements

Nash-Williams' last three published papers [**69**], [**71**] and [**72**], written jointly with his colleague David White, concern a completely new area of mathematics. The underlying problem in [**69**] is the study of how permutations f of the natural numbers N affect the convergence and/or sum of a series of real terms when f acts on the indices. They call a permutation f of N *subversive* if there is a real series $\sum a_n$ such that $\sum a_n$ and $\sum a_{f(n)}$ have unequal sums. To solve the problem of characterising such sequences, raised more than twenty years ago in ⟨**25**⟩, they introduce the concept of the *width* $w(f)$ of a permutation f of N. This is a non-negative integer, which they show to be a rough measure of the ability of f to upset convergence. For example, $w(f) = \infty$ if and only if there exists some convergent series $\sum a_n$ such that $\sum a_{f(n)}$ also converges, but to a different sum. At the other extreme $w(f) = 0$ if and only if $\sum a_{f(n)}$ converges to the same sum as $\sum a_n$ whenever this series converges.

If a rearrangement has positive but finite width, then it cannot change the sum of a series but may destroy its convergence. The papers [**71**] and [**72**] concern the same basic question except that now the series have terms in $\mathbf{R}^d$ with $d \geqslant 2$. As might be expected, the results are not so clear-cut and the proofs are more technical. What is particularly appealing about them is that in order to deal with this very natural problem of real analysis, the authors need to make significant use of the max-flow min-cut theorem of capacitated networks.

Surveys

Apart from the many pure research articles covered above, Nash-Williams was always in demand as a lecturer at combinatorial conferences. As a result of this, he produced a large number of 'survey articles' and ideas on future directions of graph theory. Of these, the highlight must be the papers [**50**] and [**51**], which are based on his series of lectures at the St Andrews Mathematical Colloquium in 1980. In the opening paragraphs of [**50**] he explains that 'one of my own reasons for becoming interested in graph theory was that I was intrigued by the possibility of developing non-trivial and fairly deep mathematics from a very simple initial concept.' He goes on to compare the subject with that of set theory, which he describes as the ultimate illustration of this.

The survey [**51**] also describes three of his own results that he confessed, 'I personally found fairly difficult to prove (whatever their degree of difficulty in an absolute sense may be).' These are the theme of his papers [**3**] on decompositions, two roughly equivalent results from [**4**] on orientations, and his theorem [**15**] on well quasi-ordering infinite trees.

Conclusion

In summing up, I must highlight the very high regard in which Nash-Williams was held by his contemporaries. The proceedings of both the 18th British and 4th Slovenian conferences have been dedicated to his memory, and the *Journal of Combinatorial Theory* is to publish a special issue in his honour, which will contain his unfinished joint paper [**73**]. I have attempted to highlight his mathematical achievements, which are much greater than his modest style of presentation would suggest. He was a very private and mildly eccentric person, but it is impossible to capture the affection towards him of those who knew him. To quote a former colleague, 'He was one of the nicest men I have ever met'.

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