Vector Potential Magnetic Null Points

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Recent observations (Tarbell *et al.*, 1990; see also Ruzmaikin *et al.* these Proceedings) show that the surface distribution of magnetic fields on the solar surface is selfsimilar, with an approximately constant fractal dimension over a large range of horizontal scales. Also, recent ideas about flare energy release assumes a "fibrous" corona, with the energy release consisting of many small scale events (Vlahoz, 1989) Prompted partly by these observations, we investigate the properties of self-similar three-dimensional magnetic fields.

1. Magnetic null points

The global connectivity of non-trivial 3-D magnetic fields is strongly influenced by the presence of critical points (null points) where all components of the magnetic field vanish. In a neighbourhood of such points, the magnetic field is characterized by the magnetic field gradient matrix $\underline{\underline{\mathbf{G}}} = B_{i,j}$;

$$\mathbf{B}(\mathbf{r}) = \sum_j B_{i,j}\delta r_j = \underline{\underline{\mathbf{G}}} \cdot \delta\mathbf{r} \,. \tag{1}$$

where $\underline{\underline{\mathbf{G}}}$, again, is completely characterized by its eigenvalues and eigenvectors. Since $div(\mathbf{B}) = 0$, the sum of the eigenvalues ($= trace(\underline{\underline{\mathbf{G}}})$) vanishes. Thus there are only two, non-trivially different, combinations of eigenvalues; those with all eigenvalues real (one with the sign differing from the other two), and those with one real and two complex conjugate eigenvalues. Corresponding to these two cases, there are two different classes of field structure near magnetic null points; non-spiralling field lines corresponding to the case with all real eigenvalues, and spiraling field lines corresponding to the case with complex eigenvalues. In both cases, the field lines in one direction asymptotically approach an axis along the eigenvector belonging to the largest (real) eigenvalue, while in the other direction the field lines asymptotically approach the plane containing the other two eigenvectors (Fukao *et al.*, 1975; Green, 1988).

2. Field representation

We construct schematic, but reasonably complicated ("non-textbook") magnetic fields, by representing the field by the real part of a complex exponential series. Specifically, here, we write $\mathbf{B}$ in terms of a vector potential $\mathbf{A}$

$$\mathbf{B} = \nabla \times \mathbf{A} \,, \tag{2}$$

where

$$\mathbf{B}(\mathbf{r}) = \mathrm{Re}\left\{ \sum_i \sum_j \mathbf{f}_i \mathbf{A}_j e^{\mathbf{g}_i \mathbf{k}_j \cdot \mathbf{r}} \right\} . \tag{3}$$

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j indexes groups of components with similar wave numbers $\mathbf{k}_j$, and i indexes several sets of these groups, scaled with wavenumber scaling factors g_i and amplitude scaling factors f_i.
For a force-free field, the current is proportional to the magnetic field

$$\mathbf{I} = (\nabla \times \mathbf{B}) = \alpha \mathbf{B} \,. \tag{4}$$

For the complex exponential series, this may be shown to imply that the size of α must be equal to the length of the wavevector, $\alpha = |\mathbf{k}|$. A more detailed analysis shows that $\mathbf{A}$, in the force free case, must lie in the plane orthogonal to $\mathbf{k}$, and must consist of a certain combination of two linearly independent vectors in that plane.

3. Examples of 3-D topologies

To construct force-free fields with single (periodic) null points, we use groups of three terms with different $\mathbf{k}$ directions. Sets of such groups, with increasing wavevector and decreasing amplitude, where each group fulfills the force-free condition, produce fields that are not exactly force-free, but must be in a rather low energy state.

The panels in Figure 1. show two examples of such fields. The first column is a sequence where one null point from each group remains in the same location. The second column is a sequence where the null points for the $i >= 2$ groups are displaced by 1/8 period with respect to the null points of the $i = 1$ group. The plots are made by first finding all null points in the region, then placing a small number of starting points at a small distance from the null point, around the main axis, and trace these in both directions. As illustrated by the figure, both these selfsimilar fields exhibit a complicated field structure with many null points. Whether or not superpositions of fluctuations on several scales results in many null points depends on the relative amplitudes of the fluctuations. If the field gradients near a null point of a large scale component are smaller than the gradients of the smaller scale components, then in a neighbourhood of the null point of the large scale fluctuation the small scale fluctuations are able to create a number of null points. These form a cluster of "satellite" null-points around the null point of the large scale fluctuation (cf. Galsgaard and Nordlund, 1990ab). Presumably, for a field with a continuous spectrum of fluctuations, there is a critical spectrum slope, below which the field topology is vastly more complicated than above.

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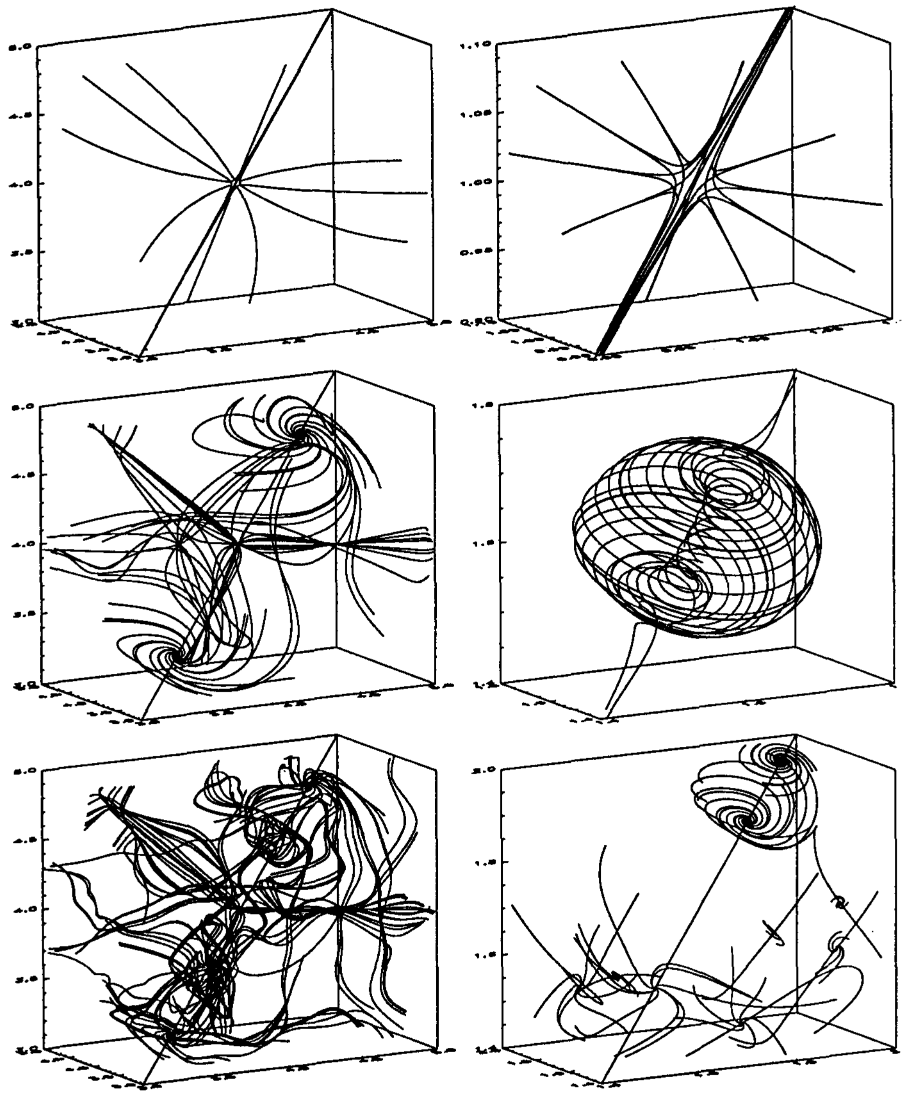

Fig. 1. The panels show two series of magnetic fields generated from groups of three vector potentials with increasing wave number and decreasing amplitude. The left column shows a sequence of magnetic field line plots where one null point from each group with $i >= 2$ is located at the same position as a null point from $i = 1$ group. The field is defined by the following wavevectors, vector potentials and scaling constants:
$\mathbf{k}_1$=[(0,1),(0,0),(0,0)], $\mathbf{A}_1 = [(0,0),(1,1),(-1,1)]$,
$\mathbf{k}_2$=[(0,0),(0,1),(0,0)], $\mathbf{A}_2 = [(-1,1),(0,0),(1,1)]$,
$\mathbf{k}_3$=[(0,0),(0,0),(0,1)], $\mathbf{A}_3 = [(1,1),(-1,1),(0,0)]$,
g_i=(1,4,16), f_i=(1,.13,.0169), i=1,3.
The plots in the right column are made from the same wavevectors $\mathbf{k}$, and scaling constants g_i, f_i but with these vector potentials:
$\mathbf{A}_1$=[(0,0),(1,0),(0,1)], $\mathbf{A}_2$=[(0,1),(0,0),(1,0)], $\mathbf{A}_3$=[(1,0),(0,1),(0,0)].
The last plot in this series has been simplified, by following the field lines for a shorter distance than for the other plots.

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