31. LINKAGE OF SEVEN APPARITIONS OF PERIODIC COMET FAYE 1925–1970 AND INVESTIGATION OF THE ORBITAL EVOLUTION DURING 1660–2060

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Abstract. Seven apparitions of this comet, from 1925 to 1970, have been linked, with account taken of the perturbations by Venus to Neptune; 202 observations have been represented with mean square error ±2″15. The resulting orbit was integrated from 1947 back to 1843 and compared with Möller's results. The result of the comparison after 104 years was found to be very good. The comet's orbital evolution was studied altogether over a period of 400 yr (1660 to 2060). There were nine approaches within 1.5 AU of Jupiter, the comet on one occasion passing through Jupiter's sphere of action. The orbital evolution was free from catastrophic changes, the comet remaining in the Jupiter family throughout the interval. The problem of the comet's discovery was considered.

Periodic comet Faye, under observation for more than 125 yr, is one of the few short-period comets with well-studied motion. Such comets are of particular interest for cometary astronomy. On the one hand, there is a wealth of observational material on which to build a reliable orbital theory; 15 or more apparitions can be linked and the results checked over many decades. On the other hand, the changes in their outward appearance, as well as unexpected variations in and systematic reduction of their brightness, render such comets highly valuable for exploring the problem of cometary ageing and disintegration.

Such comets have the added interest of offering a possibility for checking integration methods and computer programmes. All our calculations have been done on a BESM-4 computer, using the complex of programmes described by Belyaev (1972).

Extensive accounts of the history of P/Faye and its motion have been given by Khanina and Barteneva (1959, 1961), Khanina (1966) and Kazimirchak-Polonskaya (1961). After its discovery in 1843, the comet has returned to perihelion as many as 18 times, escaping observation but twice. It was last at perihelion 1969 October 7; the comet was recovered on 1969 May 17 and observed at several observatories until 1970 March 8. Forty precise positions at this apparition have been published to date.

Many scientists have investigated the motion of P/Faye. Its motion between 1843 and 1881 was studied in great detail by Möller, who devoted 40 years of his life to this comet. The period from 1881 to 1932 was studied by Kreutz, Strömgren, Fayet, Ristenpart, Prager, and others. Studies of the comet's motion since 1932 have been conducted at the Institute for Theoretical Astronomy, Leningrad (Zheverzheev, 1952; Khanina and Barteneva, 1959, 1961; Khanina, 1966; Khanina and Belyaev, 1968). The comet's motion prior to discovery was investigated by Möller (1873) and Zhdanov (1885a, 1885b).

Our own objective was to do the following:

(1) Link all the apparitions between 1925 and 1970, so that the last seven apparitions of the comet might be represented, if possible, by one set of elements;

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(2) Check to see how the new set of elements represents all the other apparitions of the comet, as far back as 1843;

(3) If this representation proves to be satisfactory, study the comet's motion back to 1660 and forward to 2060, so that a picture of the comet's orbital evolution over 400 years might be obtained.

We started from Khanina's set of elements for 1947, obtained by linking the five apparitions of the comet 1932–1962. It represented 21 normal places with a mean square error of $\pm 2''.3$. Linkage of the seven apparitions 1925–1970 was accomplished by varying the elements of the cometary orbit (by $0^\circ.000001$ for μ and $0^\circ.1$ for the other elements) and by successive sevenfold integration, taking account of the perturbations by the seven planets Venus to Neptune. This method was necessary because in 1959 P/Faye approached Jupiter within 0.60 AU, which resulted in large perturbations, as much as 7° in the line of nodes. This improvement and all our subsequent investigations were made without consideration of the effects of nongravitational forces.

The new set of elements below represents 202 observations 1925–1970 with mean square error $\pm 2''.15$. Initially, 286 observations were assembled, but 84 were rejected as erroneous. In the future the observations will be revised.

$$\begin{array}{rl@{\qquad}rl}
\text{Epoch} = & \text{1947 October 22.0 ET} & T = & \text{1947 September 28.4072 ET} \\
M_0 = & 3^\circ.1238 & \omega = & \left.\begin{array}{l} 200^\circ.5266 \\ 206.3504 \\ 10.5337 \end{array}\right\} \; 1950.0 \\
\varphi = & 34.3144 & \Omega = & \\
\mu = & 0.132400 & i = &
\end{array}$$

For the latest apparition this system gives $T = $ 1969 October 7.5918 ET, compared with the published value of 1969 October 7.639 ET (Khanina and Belyaev, 1968). For the next return we obtain $T = $ 1977 February 27.8418 ET.

Starting from the above elements the equations of motion were integrated back for 104 yr from 1947, again taking into account perturbations by Venus to Neptune. This was done to enable comparison with the observations and calculations by earlier investigators. Cometary astronomy offers very few opportunities for such comparisons, and this chance was used for checking both the starting elements and the operation of the programme over considerable time periods, and also for revealing possible nongravitational effects.

Table I compares the perihelion passages given by observation (Porter, 1961) and our calculations for the apparitions between 1910 and 1843. It is to be noted that ΔT exceeds $0^{d}.35$ only once; and in any case, the observed perihelion times from 1881 onwards are somewhat uncertain.

Table II compares our integrated elements for an epoch in 1843 with Möller's elements (Porter, 1961), the latter having been obtained by linking the four apparitions 1843–1866. The largest discrepancy is $0^\circ.017$ in ω; in Ω and i the discrepancies are several seconds of arc, while in q and e they are practically zero. We remark that in 1899 the comet approached Jupiter to within 0.51 AU and that the total number of integration steps from 1947 to 1843 was 2350. The results obtained testify to the high

TABLE I

Comparison of observed and calculated perihelion times

Year	T_{obs}	T_{calc}	$\Delta T = T_{\mathrm{obs}} - T_{\mathrm{calc}}$
1910	Nov. 1.96	Nov. 2.19	$-0^{\mathrm{d}}.23$
1896	Mar. 19.76	Mar. 19.30	$+0.46$
1881	Jan. 23.17	Jan. 22.96	$+0.21$
1873	July 18.99	July 18.74	$+0.25$
1866	Feb. 14.47	Feb. 14.22	$+0.25$
1858	Sept. 13.37	Sept. 13.11	$+0.26$
1851	Apr. 2.44	Apr. 2.14	$+0.30$
1843	Oct. 17.63	Oct. 17.30	$+0.33$

TABLE II

Comparison of Möller's elements for 1843 with the results of our integration

	Möller	Belyaev-Khanina	Difference
T (UT)	1843 Oct. 17.63	1843 Oct. 17.30	$+0^{\mathrm{d}}.33$
ω } 1950.0	$200^{\circ}.024$	$200^{\circ}.007$	$+0^{\circ}.017$
☊ } 1950.0	211.013	211.012	$+0.001$
i } 1950.0	11.364	11.367	-0.003
q (AU)	1.69223	1.69222	$+0.00001$
e	0.55583	0.55582	$+0.00001$
P (yr)	7.4364	7.4366	-0.0002

quality of the orbital elements obtained by Möller for 1843 and by ourselves for 1947. Besides, they confirm the possibility of utilizing our programme for this type of investigation and justify the supposition that all 16 apparitions of the comet might be linked without taking into account nongravitational effects, although this latter question will be settled finally only after a careful analysis of all the observational material.

The satisfactory results back to 1843 made it worthwhile to run the integration further back to 1660 and ahead to 2060. The evolution of the orbit of P/Faye is presented in Table III, which also lists the comet's nine approaches within 1.5 AU of Jupiter during these 400 yr. The approach in 1816 was inside the sphere of action. (For the known approaches during 1816–1959 the present work yielded more accurate data than has previously been published.) The effect of these approaches is a continuous retrograde rotation of the line of nodes through $50^{\circ}.6$ in the 400 yr, and since the argument of perihelion advanced by $39^{\circ}.5$, the total motion of the line of apsides was $\Delta\pi = -11^{\circ}.1$. The orbital inclination varied from $6^{\circ}.5$ to $13^{\circ}.1$. No significant changes occurred in the shape or size of the orbit, so the comet quite definitely remains in the Jupiter family during the entire period in question.

We have also integrated the motion of the comet from 1843 to 1813, starting from Möller's elements in Table II, and again taking into account the perturbations by Venus to Neptune. This was done in order to examine how the evolution of the comet's orbit through the approaches to Jupiter in 1841 and 1816 would be affected

TABLE III

Evolution of the orbit of P/Faye during 1660–2060

Epoch, t_J	Δ_J (AU)	ω	☊	$\pi = \omega +$ ☊	i	e	q (AU)	Q (AU)	P(yr)
1660 Feb. 28		168°.9	242°.1	51°.0	6°.5	0.554	1.74	6.05	7.69
1673 Aug. 10.71	0.57								
1679 Feb. 28		180.1	232.6	52.7	8.1	0.528	1.91	6.18	8.13
1734 Oct. 14.07	1.34								
1736 Aug. 19		182.9	231.4	54.3	8.1	0.533	1.87	6.13	8.01
1758 Aug. 8.78	0.66								
1760 July 13		190.7	226.1	56.9	7.5	0.559	1.69	5.99	7.53
1816 Apr. 5.21	0.249								
1820 Nov. 16		193.4	219.2	52.5	13.1	0.534	1.81	5.94	7.63
1841 Mar. 5.58	0.63								
1843 Aug. 28		200.0	211.0	51.5	11.4	0.556	1.69	5.93	7.44
1875 July 7.49	1.49								
1880 Nov. 21		201.2	210.6	51.8	11.3	0.549	1.74	5.97	7.57
1899 June 26.78	0.51								
1903 Mar. 27		199.4	206.8	46.3	10.6	0.566	1.65	5.94	7.39
1959 Feb. 17.15	0.60								
1962 June 25		203.6	199.1	42.7	9.1	0.576	1.61	5.97	7.38
2018 Mar. 7.32	0.63								
2021 Sept. 23		207.0	192.2	39.2	8.0	0.577	1.62	6.03	7.48
2059 Nov. 3		208.4	191.5	39.9	8.0	0.579	1.60	6.02	7.44

The short lines give the dates t_J of approaches to Jupiter and the minimum separations Δ_J.
The other lines give the comet's orbital elements for various epochs, for the beginning and end of the integration interval and for dates near the perihelion dates following the approaches to Jupiter.

by errors in the initial data (which are particularly large in T). The circumstances of the approaches to Jupiter, as given by these two integrations, are compared in Table IV, the final line giving Möller's own results, obtained by an approximate method. The two integrations do not differ significantly, either in time or minimum distance. Consequently, the elements for the 1813 perihelion passage differ but slightly. Notwithstanding the comet's passage through Jupiter's sphere of action and the discrepancy of 0.33 day in the 1843 perihelion times, the difference in perihelion times in 1813 increased only to 1.07 days.

TABLE IV

Circumstances of the approaches of P/Faye to Jupiter in 1841 and 1816

	t_J	Δ_J (AU)	t_J	Δ_J (AU)
Integration 1947–1813	1841 Mar. 5.58	0.63	1816 Apr. 4.92	0.246
Integration 1843–1813 from Möller's elements	1841 Mar. 5.68	0.63	1816 Apr. 5.21	0.249
Möller	1841 Mar. 5	0.6	1816 Mar.	0.3

Of particular interest in connection with orbital evolution studies is the question of comet discovery. It is necessary to trace the changes in perihelion distance prior to discovery and investigate the comet's encounters with the Earth, considering that the most favourable conditions for discovery occur when the distances of the comet from the Sun and the Earth are minimal and the date of perihelion passage close to that of the comet's approach to the Earth. Table III shows that after the 1841 approach to Jupiter the comet's perihelion distance decreased from 1.81 to 1.69 AU. When the comet was next at perihelion (1843 October 17) it was 0.92 AU away from the Earth. On 1843 November 24 it approached the Earth within 0.79 AU; and it was discovered two days before.

Table V lists all the instances between 1660 and discovery when the comet came within 1.1 AU of the Earth or the time interval between closest approach to the Earth and perihelion passage was less than three months. It shows that the comet was discovered under the most favourable conditions since 1660, although such conditions are not the only ones responsible for the comet's discovery in 1843.

TABLE V

Approaches of P/Faye to the Earth between 1660 and 1843

t_E	Δ_E (AU)	T	q (AU)	Δ_E^T (AU)	$T-t_E$ (dy)
1703 Nov. 19	0.98	1703 Oct. 28	1.95	1.02	−22
1736 Dec. 13	1.10	1736 Sept. 12	1.87	1.61	−91
1744 Dec. 7	1.04	1744 Sept. 24	1.88	1.45	−74
1752 Dec. 4	1.00	1752 Oct. 1	1.87	1.31	−64
1775 Dec. 14	0.92	1775 Sept. 30	1.70	1.28	−75
1790 Dec. 4	0.80	1790 Oct. 17	1.68	0.96	−48
1805 Oct. 18	0.96	1806 Jan. 11	1.74	1.40	+86
1843 Nov. 25	0.79	1843 Oct. 17	1.69	0.92	−38

t_E are the dates of closest approach to the Earth, the separation being Δ_E.
Δ_E^T are the separations at perihelion times T.

We plan to link all the apparitions of P/Faye, in order that a continuous numerical theory may be developed to account for the comet's motion from 1843 to 1970. On completion of that, we can obtain a more precise picture of the comet's orbital evolution over 1660–2060. We hope that the definitive version will not differ considerably from that described in this paper, considering that the orbit obtained to date forms a very reliable foundation for this kind of study.

References

Belyaev, N. A.: 1972, this Symposium, p. 90.
Kazimirchak-Polonskaya, E. I.: 1961, *Trudy Inst. Teor. Astron.* **7**, 131.
Khanina, F. B.: 1966, *Problemy Dvizheniya Malykh Tel Solnechnoj Sistemy*, Baku, p. 69.
Khanina, F. B. and Barteneva, O. N.: 1959, *Byull. Inst. Teor. Astron.* **7**, 466.
Khanina, F. B. and Barteneva, O. N.: 1961, *Byull. Inst. Teor. Astron.* **8**, 229.
Khanina, F. B. and Belyaev, N. A.: 1968, *IAU Circ.* No. 2062.

Möller, A.: 1873, *Ofvers. Kongl. Vetensk. Akad. Stockholm Förhandl.* **1**, 3.
Porter, J. G.: 1961, *Mem. Br. Astron. Assoc.* **39**, No. 3.
Zhdanov, A.: 1885a, *Mem. Acad. Imp. Sci. St. Petersb.* **33**, 1.
Zhdanov, A.: 1885b, *Astron. Nachr.* **111**, 113.
Zheverzheev, V. F.: 1952, *Byull. Inst. Teor. Astron.* **5**, 97.

Discussion

S. K. Vsekhsvyatskij: P/Faye is famous for its rapid secular decrease in brightness. Have you tried to identify it with any comets of the seventeenth or eighteenth centuries?

N. A. Belyaev: Not yet, but we plan to do so in the near future.

W. J. Klepczynski: In his study, Möller determined a correction to the mass of Jupiter. Do you plan to solve for such a correction?

N. A. Belyaev: No. Requirements for comets that may be utilized for determining corrections to the mass of Jupiter have been formulated and will be reported by Kazimirchak-Polonskaya in Paper 40. P/Faye does not meet those requirements.