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THE DARK NIGHT-SKY RIDDLE, "OLBERS'S PARADOX"

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ABSTRACT. The riddle of cosmic darkness, nowadays known as Olbers's paradox, asks, "Why is the sky at night dark?" The riddle is almost as old as the Copernican revolution and has inspired important contributions to the development of astronomy and cosmology. It climaxes in the twentieth century with the realization that the sky is covered not by unseen stars but by the infrared gloom of the big bang.

1. INTRODUCTION

The riddle of why the sky is dark at night emerged soon after the Copernican revolution in the sixteenth century. The riddle has since played an important role in the history of astronomy, and particularly in the history of cosmology, drawing the attention of astronomers and scientists to questions concerning the physical nature and large-scale structure of the universe. The riddle may be expressed in various ways: Why is the sky not covered by stars? Why is space not full of starlight? Why are luminous stars and interstellar space not in thermodynamic equilibrium? Thomas Digges in the sixteenth century originated the riddle in a rudimentary form; then Johannes Kepler, Otto von Guericke, Bernard de Fontenelle, and Christaan Huygens in the seventeenth century, and Edmund Halley in the early eighteenth century expressed the riddle in various forms [1].

The argument is as follows: In a universe consisting of unbounded space populated endlessly with luminous stars, a line of sight in any direction from the eye, extended into the depths of space, eventually intercepts the surface of a star. Visible stars should therefore cover the entire night sky without any separating gaps. Hence, the riddle of cosmic darkness: Why is the sky dark at night? Of course, these are not the words used by earlier astronomers, but they convey the sense of their groping thoughts. Instead of stars we may speak of galaxies. But then we lose contact with the history of the subject. We might also fail to realize that stars are still the primary sources—the sky still blazes with starlight—and the riddle of why stars fail to cover the sky is not a whit diminished or solved by the substitution of galaxies for stars.

The riddle of cosmic darkness is nowadays known as Olbers's paradox. The riddle has inspired poets and scientists, and it forms a fascinating theme in the history of scientific ideas that reaches a startling and even awe-inspiring climax in the twentieth century. In this brief essay on Olbers's paradox, in the true spirit of the history of science, I pay attention more to the underlying scientific ideas and less to anecdotal detail concerning the scientists involved.

S. Bowyer and C. Leinert (eds.), The Galactic and Extragalactic Background Radiation, 3–17.

2. CHESEAUX CALCULATES

Edmund Halley in 1721 discussed the "infinity of the sphere of stars" in two papers (Halley 1720–1721a,b). He divided space into concentric spherical shells of equal thickness, with the Sun located at the center, and assumed that stars are uniformly distributed (Figure 1). The number of stars in successive shells increases with the square of distance, and the light from each star decreases with the inverse square of distance. From this argument Halley should have concluded that all shells contribute equal amounts of starlight. But his argument was confused and his conclusion incorrect [2].

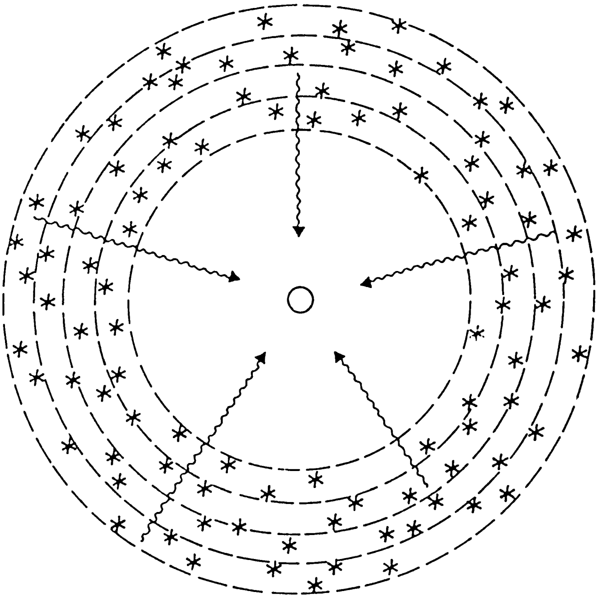

Figure 1. Halley's concentric shells of equal thickness in a universe uniformly populated with stars.

Jean-Phillipe Loys de Chéseaux of Lausanne saw to the heart of the problem in 1744. Previously, the riddle had been discussed in vague and imprecise terms, and Chéseaux, in his superb discussion [3], was the first to treat the problem in a lucid and quantitative manner (Chéseaux 1744). In an unbounded universe populated with sunlike stars, as envisaged in the world systems of René Descartes and Isaac Newton, the visible disks of stars should cover the entire sky, and every point of the sky should shine as brightly as any point on the Sun's disk. The sky is 180,000 times larger (in solid angle) than the Sun's disk, said Chéseaux, and therefore the starlight falling everywhere on Earth should be 180,000 times more intense than sunlight.

Chéseaux estimated that the nearest stars lie at a distance 240,000 astronomical units (roughly 4 light-years) and used this as the average separating interval D between stars. Stars, all assumed to have a solar radius $R_\odot$, stretch away, covering the sky, forming a continuous stellar background. Updating his terminology, we can say that the distance λ of the background stars (defined as the mean free path of a ray of starlight between emission and absorption by stars) is

$$\lambda = V/S \ , \tag{1}$$

where $V = D^3$ defines an average volume occupied by a single star, and $S = \pi R_\odot^2$ is the star's cross-sectional area. Chéseaux calculated that the background distance λ was $7.6\times10^{14}\,D$, or 1.8×10^{20} AU (3×10^{15} light-years). This result inspired the idea that most starlight, while covering the vast distances of interstellar space, is absorbed by a tenuous interstellar medium. We observe only the foreground stars, said Chéseaux, and an obscuring medium hides from our view hosts of distant stars [4].

3. OLBERS ELABORATES

Heinrich Wilhelm Olbers of Bremen performed similar calculations in 1823 and also concluded that interstellar absorption accounts for the missing starlight (Olbers 1823) [5]. Like Chéseaux, he used Halley's construction of concentric shells. He also introduced the line-of-sight argument, and the simplicity of this argument makes immediately clear in nontechnical language the essence of the problem. Most accounts of Olbers's paradox, even the most elementary, start with the assumption that the distribution density of stars is uniform, then follow Halley's construction of concentric shells of equal thickness, and conclude that the flux of starlight reaching the observer at the origin from any shell is independent of the shell's distance. This elementary treatment, which confused even Halley, often carries the reader and frequently the writer out of their depth. Strictly, one should show how this twentieth-century discussion of radiation relates to stars covering the sky as envisaged in earlier formulations of the riddle, and as far as I am aware, Kelvin is the only person who has attempted this. Also, one must allow for occultations and show why diffraction is irrelevant. Little wonder that stellar occultations are often omitted in elementary accounts, and the author arrives at the conclusion that the radiation density of starlight must be infinitely great in a universe of infinite extent [6].

Instead of summing increments of starlight, Olbers summed increments of area of visible stellar disks. His argument, with updated terminology, goes as follows: In a shell of radius q and thickness dq, the number of stars equals $4\pi nq^2dq$, where n is the number per unit volume. The fraction of the sky covered by the stars in the shell, seen by an observer at the origin $q = 0$, is this number multiplied by $S/4\pi q^2$, where S is the cross section of each star (assumed to equal that of the Sun). The fraction of the sky covered by stars in the shell is thus $d\alpha = dq/\lambda$, where $\lambda = 1/Sn$ from equation (1) with $n = D^{-3}$. Hence, integrating from $q = 0$ to $q = r$, we find that the fraction of the sky covered by stars out to distance r is

$$\alpha = r/\lambda \ . \tag{2}$$

According to this argument, stars occupying a sphere of radius $r = \lambda$ cover the entire sky, as seen by an observer at the origin, and the total number of observed stars is

$$N = 4\pi n \lambda^3/3 = 4\pi D^6/3S^3 \ . \tag{3}$$

The stars at distance $r > \lambda$ are obscured from view by those at $r < \lambda$.

Equations (2) and (3) are the essence of the analysis made by both Chéseaux and Olbers. The treatment, while illuminating in its use of a cut off distance, fails to allow properly for geometric occultation of distant stars by foreground stars. We have shown that the fraction of the sky covered by stars in a shell is dq/λ. To allow for occultations of stars in this shell by nearer stars, we must multiply this expression by $\exp(-q/\lambda)$. As before, λ is the mean free path of starlight between emission and absorption by stars. The fraction of the sky covered by stars in a shell thus becomes $d\alpha = \exp(-q/\lambda)dq/\lambda$. On integrating from $q = 0$ to $q = r$, we find that the fraction of the sky covered by stellar disks is

$$\alpha = 1 - \exp(-r/\lambda) , \tag{4}$$

in place of equation (2). Thus, $\alpha = r/\lambda$ for $r \ll \lambda$, in agreement with equation (2), and $\alpha = 1$ for $r \gg \lambda$. The total number of visible stars becomes

$$N = 8\pi n \lambda^3 = 8\pi D^6/S^2 , \tag{5}$$

instead of equation (3).

Olbers realized, by virtue of the line-of-sight argument, that the riddle remains little affected when n is nonuniform and the stars form clusters and even milky way systems or galaxies. Halley's and Chéseaux's assumption that stars uniformly populate space is much too restrictive. A forest presents a background of overlapping trees even when the trees are irregularly distributed. Olbers did not use the forest analogy; the first to use this analogy in connection with the riddle was Otto von Guericke in 1672. Olbers's claim that clustering of stars fails to solve the riddle went too far and inspired a few years later John Herschel's refutation (Herschel 1848).

Both Chéseaux and Olbers assumed that the sky is actually covered by stars, most of them unseen, and interpreted the riddle of why is the sky dark at night to mean: What has happened to the missing starlight? From the sixteenth to the twentieth century, most persons who showed interest in the riddle automatically adopted this interpretation. The alternative interpretation assumes that the observed dark gaps between the visible stars are devoid of stars, and remain devoid, or almost devoid, no matter how powerful our telescopes. This second interpretation has not received as much attention as the interpretation adopted by Chéseaux and Olbers. According to this second interpretation, the riddle of why the sky is dark at night means: What has happened to the missing stars?

John Herschel proposed a solution based on the second interpretation of a dark night sky. He pointed out that the dark gaps between the stars may actually be empty of stars even in an infinite universe containing an infinity of stars. By arranging the stars hierarchically, in larger and larger clusters, in a way similar to that proposed by Immanuel Kant, Herschel claimed that the sky will remain dark and uncovered by stars. Richard Proctor, Fournier d'Albe, Carl Charlier, and Hugo Seeliger were among those who sought with mixed success to express this hierarchical system in general theoretical terms. We now know that for self-similar clustering on all scales, the fractal dimension must be less than 2 for a dark night sky (Harrison 1987).

Olbers, apparently unaware of Chéseaux's discussion, proposed that interstellar absorption of starlight explains the darkness of the night sky. We can modernize the calculations performed by Chéseaux and Olbers in the following way. The number of stars in a shell of radius q and thickness dq is $4\pi n q^2 dq$, and the fraction of the sky covered by these stars is $\exp(-q/\lambda)dq/\lambda$, as shown previously. To include the effect of interstellar absorption, we multiply this expression by $\exp(-q/\mu)$, where μ is the absorption distance (in fact, q/μ is the optical depth of the medium out to distance q), and obtain

$$d\alpha = \exp[-q(1/\lambda + 1/\mu)]dq/\lambda .$$

Integrating from $q = 0$ to $q = r$, we find

$$\alpha = \frac{\mu}{\lambda + \mu}[1 - e^{-r(1/\lambda + 1/\mu)}] , \tag{6}$$

and α is now the observed fraction of the sky covered by visible stars corrected for interstellar absorption. Equation (6) reduces to equation (4) in a transparent ($\mu = \infty$) universe. In an infinite universe, or a universe that extends a distance r much larger than $\lambda\mu/(\lambda+\mu)$, the observed covered-sky fraction is

$$\alpha = \mu/(\lambda+\mu) \ . \tag{7}$$

Interstellar absorption dominates when $\mu \ll \lambda$, and in this case $\alpha \ll 1$, and the sky is dark at night when $\mu \ll \lambda$. This is the solution proposed by Chéseaux and Olbers. In his paper, Olbers made no attempt to calculate the background distance λ, and he guessed that μ has a value 33 times the distance of Sirius, roughly 300 light-years. With Chéseaux's background distance λ of 3×10^{15} light-years and Olbers's absorption distance μ of 300 light-years, we find that the fraction of the sky covered by visible stars is roughly 10^{-13}.

Chéseaux and Olbers proposed that the interstellar medium absorbed most of the radiation emitted by stars. At first glance their proposal seems quite harmless and even plausible. Let us remember, however, that the starlight they sought to abate was 180,000 times more intense than sunlight at the Earth's surface. (This is why the modern tendency to substitute galaxies for stars fails to avert the starlight catastrophe implicit in Olbers's paradox.) John Herschel spotted the flaw in the absorption argument. He explained that the absorbing medium would soon heat up and the emission of radiation would equal absorption (Herschel 1848). Neither Chéseaux nor Olbers, nor apparently any astronomer until the second half of the nineteenth century, could fully appreciate the catastrophic physical consequences of the intense starlight implied by the riddle of darkness. Olbers even thought that life might adapt to the continuous glare of a sky covered by stars, and the main disadvantage in a bright-sky universe would be the plight of astronomers attempting to observe stars against a bright sky. William Herschel at one stage even supposed that living creatures existed on the surface of the Sun.

4. THE VISIBLE UNIVERSE

Ole Roemer's discovery of the finite speed of light in 1676, rejected by Cartesian natural philosophers until vindicated by James Bradley's discovery of the aberration of starlight in 1729, provided a natural solution to the riddle of cosmic darkness—a solution that now seems natural and even attractive to us in the twentieth century, but was not in the least natural and certainly not attractive to most astronomers before the rise of modern cosmology.

A line of sight is a null-geodesic; it extends out in space from the observer and, owing to the finite speed of light, also back in time (Figure 2). In an unbounded universe of finite age t, a line of sight extends a finite distance, and if unintercepted, it terminates at the beginning $t = 0$ of the universe. Starlight travels only a finite distance and the *visible universe*—the region visible to the observer bounded by the particle horizon—has a radius (in a static universe) equal to the age t of the universe multiplied by the speed of light c. Beyond the visible universe lie objects whose light has yet to reach the observer.

A possible solution of the riddle of cosmic darkness—of why stars fail to cover the sky—is that the light from numerous background stars has yet to reach us. If the background distance λ, given by equation (1), exceeds the size ct of the visible universe, then the observed stars are insufficient to cover the entire sky. This solution of the riddle applies to nonstatic as well as static universes, provided we adjust the size of the particle horizon or of the visible universe in the case of nonstatic universes. If Chéseaux had multiplied his estimate of 1.8×10^{20} AU for the background distance by the time that light takes to travel from the Sun to the Earth, a time known to be between 8 and 9 minutes, he would have found a background

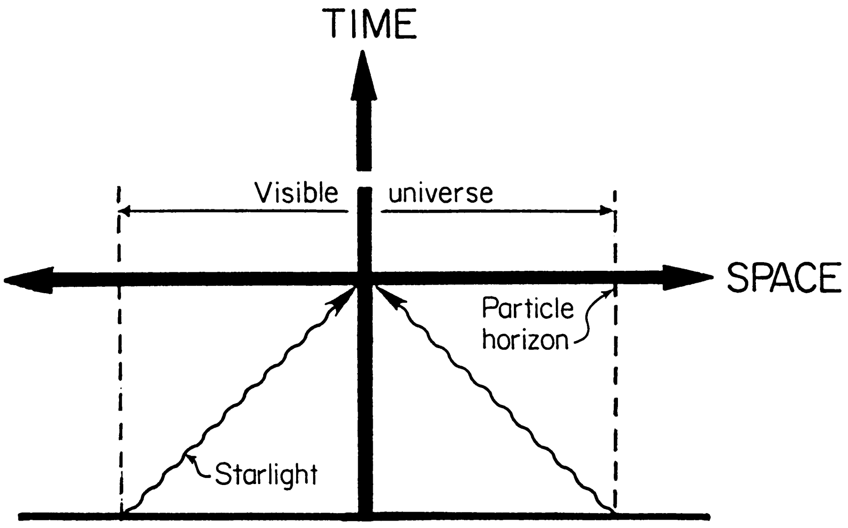

Figure 2. The visible universe of finite age with the observer at the center is bounded by the particle horizon and contains a finite number of stars. In an evolving universe, darkness at night is generally the result of finite age of the luminous sources.

light-travel time of 3×10^{15} yr. From this result he might have realized that the visible stars in a universe of age less than 3×10^{15} yr are too few to cover the whole sky.

When the background distance is less than the size of the visible universe ($\lambda<ct$ for a static universe), the first interpretation of the riddle is correct; stars cover the sky and the problem consists of determining what has happened to the missing starlight. Chéseaux and Olbers said interstellar absorption was the answer. When, however, the background distance exceeds the size of the visible universe ($\lambda>ct$), the second interpretation is correct; stars fail to cover the sky, and the dark gaps separating the visible stars are actually devoid of stars. Where are the missing stars? They lie outside the visible universe and their light has yet to reach us. If the luminous lifetime of stars is much less than the age of the universe, then the size of the visible universe must be modified (the age t of the universe is replaced by the luminous lifetime of the stars). Notice that in a static universe of finite age, such as the Cartesian and Newtonian systems, the visible universe expands at the speed of light, and stars not yet seen will be seen in the future.

A study of the history of the riddle of cosmic darkness discloses a curious inhibition in the thoughts of astronomers from the late seventeenth century to the early twentieth century. Nowadays, we realize that in looking out in space we look back in time and observe distant regions of the universe as they were in the distant past. Often, the realization requires conscious effort even in modern times; but we do not resist the idea, and we do not fear to discuss its cosmological significance. Astronomers in the eighteenth and nineteenth centuries recoiled from the monstrous idea that when we look out in space we look back in time to the creation of the universe (Harrison 1989). Many of these astronomers were ordained ministers active in the affairs of society and dependent on patronage. Few felt impelled to affront society with a quixotic attack on the Book of Genesis and the Mosaic chronology.

References to the travel time of light over astronomical distances are rare gems in the history of science. Francis Roberts wrote in 1694 that "Light takes up more time in Travelling from the Stars to us than we in making a *West-India* voyage (which ordinarily is performed in six Weeks)" (Roberts 1694). John Arbuthnot, Alexander Pope, Jonathan Swift, and other members of the Scriblerus Club, under the name Martinus Scriblerus (Kerby-Miller 1966), asked in 1741, in reference to the velocity of light, "How long a new star was lighted up before its appearance to the inhabitants of our earth?" [7] Thomas Young in *Night Thoughts* wrote in 1741 that the nocturnal stars are so distant that

> 'twere not absurd
> To doubt that beams set out at Nature's birth
> Had yet arrived at this so foreign world
> Though nothing half so rapid as their flight!

William Herschel in 1802 remarked "that a telescope with a power of penetrating into space, like my 40-foot one, has also, as it may be called, a power of penetrating into time past" (Herschel 1802). He considered the time light takes to travel from the very faint nebulae observed with his reflector (Figure 3), and said:

> Hence it follows, that when we see an object of the calculated distance at which one of these very remote nebulae may still be perceived, the rays of light which convey its image to the eye, must have been . . . almost two millions of years on their way; and that, consequently, so many years ago, this object must already have had an existence in the sidereal heavens, in order to send out those rays by which we now perceive it.

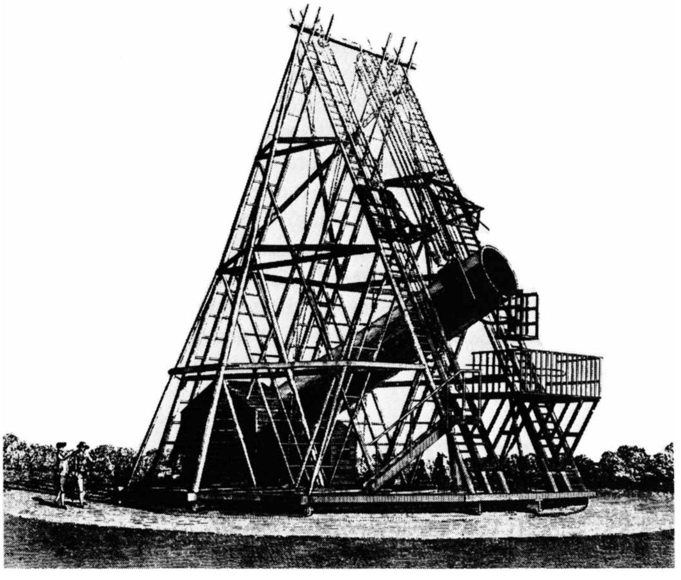

Figure 3. William Herschel's 40-foot telescope in 1795.

In the early nineteenth century, John Nicol and Thomas Dick marveled at the vastness of the visible world, lavished praise on the heavenly works of the supreme being, and referred cautiously to the immensity of light-travel time of distant stars. Alexander von Humboldt in *Kosmos* in 1845 wrote that "the light of remote heavenly bodies presents us with the most ancient perceptible evidence of the existence of matter" (Humboldt 1845).

More inspired and bolder statements came from the poet and essayist Edgar Allen Poe and the astronomer Johann von Mädler. Poe in 1845 in a short essay referred to the "golden walls of the universe" composed of "myriads of the shining bodies that mere number has appeared to blend into unity." In 1848, a year before he died at age forty, he published his famous essay *Eureka*. In this cosmological masterpiece he wrote:

> the only way we could comprehend the voids which our telescopes find in innumerable directions, would be by supposing the distance of the invisible background so immense that no ray from it has yet been able to reach us at all.

The finite speed of light and the finite age of the universe have for the first time come together explicitly for the purpose of solving the riddle of cosmic darkness.

Mädler, in the fifth edition of his *Populäre Astronomie*, published in 1861, wrote:

> The velocity of light is *finite*, a *finite* time has passed from the beginning of Creation until our day, and we, therefore, can only perceive the heavenly bodies out to the distance that light has traveled during that finite amount of time.

The sky at night is necessarily dark, he said, because light from the distant stars has not yet reached us, and the absorption of starlight proposed by Olbers is unnecessary [8].

Astronomy, the most socially respectable of the sciences, possessed evidence concerning the vast age of the universe, and it stayed respectable because astronomers did not publicize the sensational and dreadful facts. We can imagine what might have happened if an astronomer had announced in a Victorian drawing room, in the presence of the bishop, that the creation of the universe was exposed to telescopic observations!

In the ancient world, cosmological issues formed an inseparable part of religion. This remains true today in societies and institutions that still cling to the ancient creation myths. Cosmology and theology went hand in hand in Western Europe during the early, high, and late Middle Ages up to the time of the scientific revolution. In the sixteenth century they began to separate, and cosmology has since developed into a scientific study independent of theological doctrine. The separation occurred slowly and many, if not most, astronomers and scientists in the eighteenth and nineteenth centuries held to the belief that the Solar System and even the sidereal heavens had been created a few thousand years previously in accordance with biblical testimony. Those who disagreed prudently nursed their disbelief in silence. The roadblock to the finite-age solution of the riddle of cosmic darkness and to a theoretical study of the early universe was the biblical creation story of the Judaic-Christian-Islamic world.

5. KELVIN'S FINITE–AGE SOLUTION

Much of the credit for the finite-age solution of the riddle must go to Kelvin, who derived this solution quantitatively for the first time in a masterful manner (Thomson 1901). He first showed that the night sky remains dark when we consider only the stars of our own galactic system. His result was similar to our equation (2). For the distance r he used Simon Newcomb's 3.09×10^{16} km (roughly 3000 light-years) for the radius of the Galaxy, and he obtained a sky-cover fraction α of roughly 4×10^{-13}. Kelvin wrote:

> This exceedingly small ratio will help us to test an old and celebrated hypothesis that if we could see far enough into space the whole sky would be seen occupied with discs of stars all perhaps the same brightness as our sun, and that the reason why the whole night-sky is not as bright as the sun's disc is that light suffers absorption in traveling through space.

He then showed that the sky-cover fraction α would be as large as 4% if luminous stars stretched away to a distance r of 4×10^{14} light-years. In earlier work on stellar evolution and the release of gravitational energy he had demonstrated by "irrefragible dynamics" that the luminous lifetime of the Sun cannot exceed 10^8 yr. We may assume that this maximum lifetime applies to other stars, he said, and in that case stars beyond 10^8 light-years cannot be seen. Imagine a great sphere of stars of radius 4×10^{14} light-years, with the Sun at the center. The "time taken by light to travel from the outlying stars of our sphere to the centre would be about three and a quarter million times the life of a star." It would therefore be impossible to cover the night sky with luminous stars having a distribution density as in the neighborhood of the Sun. Kelvin pointed out that in a great sphere many times the size of the Galaxy the average distribution density of stars would be much less than in our Galaxy, thus reducing further the sky-cover fraction.

Kelvin played a leading role in the development of the science of thermodynamics. A sky everywhere as bright as the surface of the Sun, as envisaged in Olbers's paradox, implies thermodynamic equilibrium in which stars emit as much radiation as they receive. (That stars might not exist under such conditions is irrelevant.)

Let us take a first-law view of Olbers's paradox and consider a cavity of constant volume V that has perfectly reflecting walls and is sufficiently large to contain average conditions. If u denotes the average radiation energy density in space and u^* the radiation density at the surface of a star, then

$$d(Vu)/dt = (Vc/\lambda)(u^* - u) , \tag{8}$$

where the stars act as sources and sinks. In the traditional way it is assumed that all stars are alike and similar to the Sun. We note that the luminosity of a star (assumed constant and equal to that of the Sun) is $L_\odot = u^*cS$, the total incident flux on the surface of each star is ucS, and equilibrium is attained when $u^* = u$. By integration of equation (8) we obtain

$$u = u^*(l - e^{-ct/\lambda}) \tag{9}$$

for $u = 0$ at $t = 0$. This equation shows that the radiation density u increases with time and approaches the value u^* on a time scale λ/c. We see that λ/c is the fill-up time in a static universe (also cavity). The sky-cover fraction α, from equations (4) and (9), with r set equal to the radius ct of the visible universe, is

$$\alpha = u/u^* , \tag{10}$$

thus demonstrating the truth of Kelvin's statement that "α is the ratio of the apparent brightness of our star-lit sky to the brightness of our Sun's disk." Hence α becomes unity with the attainment of thermodynamic equilibrium when the radiation density in space equals the radiation density at the surface of stars. Extreme thermodynamic disequilibrium exists when $ct \ll \lambda$, and then

$$u/u^* = ct/\lambda .$$

With $ct = r$ and $\alpha = u/u^*$, equation (9) becomes equation (4).

The customary expression for the radiation density

$$u = \int_0^r (nL_\odot dq/c)\exp(-\int_0^q dq'/\lambda) , \quad (11)$$

with $nL_\odot/u^* = c/\lambda$, is the integral of the differential equation (8). This radiation equation, when written as a sky-cover fraction,

$$\alpha = \int_0^r nSdq \exp(-\int_0^q dq'/\lambda) , \quad (12)$$

expresses the traditional form of Olbers's paradox. Olbers's paradox, or the riddle of cosmic darkness, asks why the night sky is dark when plausible arguments show that the sky-cover fraction should be unity. Hence equation (12), not equation (11), is the historically correct mathematical formulation of the riddle. The twentieth-century habit of substituting radiation density (or intensity) for the sky-cover fraction often misleads the inquirer into mistaking the true nature of the riddle of why the sky is dark at night. The frequent omission of occultations, for example, leads to the conclusion that $u \rightarrow \infty$ as $r \rightarrow \infty$, and this scientific and historically fallacious result is often referred to as Olbers's paradox.

Much of the fascination of Olbers's paradox lies in its unfailing ability to puzzle and confuse. Enlightening calculations such as those performed by Kelvin are rare in the extensive literature of Olbers's paradox. The absence of Kelvin's remarkable paper from his collected works and the bibliographies of his publications probably accounts for its neglect by scientists and historians of science (Harrison 1986).

6. BONDI'S REDSHIFT SOLUTION

Apart from occasional comments in the scientific literature, the riddle of a dark night sky received little further attention in the first few decades of the twentieth century. With the rise of general relativity and the discovery of the expansion of the universe, cosmology became an abstract and complex subject in the minds of many astronomers. Hermann Bondi and Thomas Gold reawakened interest in the riddle in their paper proposing an expanding steady-state universe (Bondi and Gold 1948). These authors referred to Olbers's work, discussed the absence of cosmic thermodynamic equilibrium, and said, "this disequilibrium is the direct consequence of the motion of expansion." Bondi explored the question of why the sky is dark at night in his classical *Cosmology* text and, unaware of the long history of the riddle, referred to it as Olbers's paradox (Bondi 1951), a name that now enjoys unassailable popularity.

In several publications, Bondi discussed various aspects of Olbers's paradox and showed that the curvature of space in a homogeneous and isotropic universe cannot affect the background radiation density. He pointed out that finiteness in the age of the universe and also in the luminous lifetime of stars can account for the observed thermodynamic disequilibrium. But Bondi at that time was primarily interested in the steady-state universe that possesses an infinite past and future in which stars are perpetually luminous; the continuous creation of matter ensures that newborn stars replace dying stars. Within the framework of steady-state cosmology, he argued that the redshift of distant sources solves the riddle of cosmic darkness. This solution was widely adopted and uncritically applied to all models of an expanding universe, both steady-state and big-bang. Kelvin's finite-age solution lay forgotten, and many astronomers were convinced that expansion of the universe was a necessary and sufficient condition for cosmic darkness.

The easiest way to treat the background radiation in a steady-state universe (and in any homogeneous and isotropic universe) is to consider an imaginary comoving cavity of volume V that has perfectly reflecting walls and contains an average sample of sources and sinks. In the simple case where sources also act as sinks, as implied in Olbers's paradox, the first law gives

$$d(uV^{4/3})/dt = (cV^{4/3}/\lambda)(u - u^*) \tag{13}$$

in place of equation (8) for a static universe, and as before, u denotes the radiation density in interstellar space and u^* the radiation density at the sources. The terms u, u^*, and λ stay constant in a steady-state universe, and hence

$$\frac{u}{u^*} = \frac{D_H}{4\lambda + D_H}, \tag{14}$$

where $H = V^{-1/3} dV^{1/3}/dt$ is the Hubble term and $D_H = c/H$ is the Hubble distance. Using the values $\lambda = 10^{23}$ light-years and $D_H = 10^{10}$ light-years, we find that u/u^* is of order 10^{-13}.

Certain questions concerning radiation in a steady-state universe have never been fully answered, perhaps because this universe lost its appeal with the discovery of the 3 K cosmic background radiation. For example, how does the luminous lifetime of stars affect equation (14), and what fractions of the sky are covered by luminous and nonluminous stars? Some questions can be answered, provided we neglect the effect of a finite luminous stellar lifetime. Recent interest in an inflationary era of the early universe and the similarity of inflation and continuous creation is focusing attention once again on problems peculiar to the steady-state universe.

Equation (14) shows that expansion or inflation cannot automatically guarantee a state of cosmic darkness, contrary to what has been stated on numerous occasions. The brightness or darkness of the night sky in an expanding steady-state universe depends on the relative magnitudes of λ and D_H. Darkness of $u \ll u^*$ prevails when

$$D_H \ll 4\lambda \tag{15}$$

as a result of expansion. When, however, $D_H > 4\lambda$, then $u \approx u^*$, and the sky blazes bright, as envisaged in Olbers's paradox. Thus we can have dark-sky and bright-sky expanding steady-state universes, and expansion is a necessary but not a sufficient condition for darkness. (We must of course bear in mind the improbability of stars existing in a bright-sky universe.)

The observer's backward lightcone stretches out in space and back in time (Figure 4), and in a steady-state (or de Sitter) universe, its distance y from the observer's world line, as a function of redshift, is

$$y = D_H z/(1+z). \tag{16}$$

The lightcone approaches the surface of the Hubble sphere (at distance D_H) asymptotically in the infinite past at infinite redshift z. The surface of the Hubble sphere acts as an event horizon (no events can ever be observed beyond D_H), and because no particle horizon exists (the backward lightcone intersects all world lines), the finite-age solution fails to apply. A line of sight stretches out inside the Hubble sphere and intercepts the surface of a star somewhere in the infinite past. The first interpretation of a dark night sky (when $D_H \ll 4\lambda$) is correct; the sky is covered with invisible stars and most stars are invisible because their rays have been enfeebled by expansion. Edgar Allan Poe's golden walls are redshifted into invisibility.

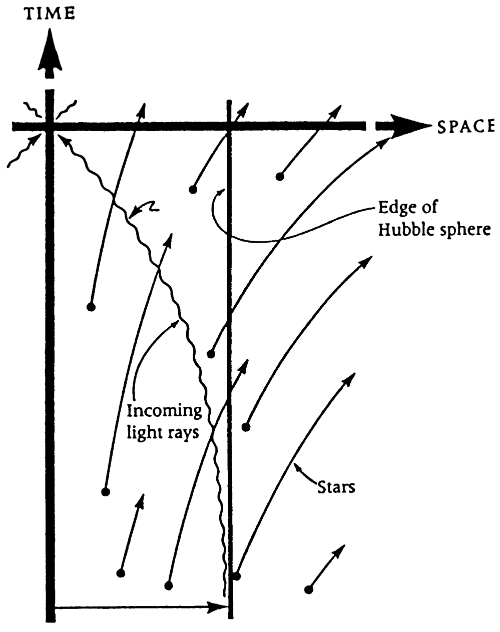

Figure 4. The visible region in a steady-state universe is the Hubble sphere. The observer's backward lightcone asymptotically approaches the event horizon, which in this case is the surface of the Hubble sphere. In this universe of finite age, darkness at night is the result of expansion.

7. THE BIG–BANG SOLUTION

Bondi's redshift solution of Olbers's paradox, valid for the steady-state universe, became the general solution for all expanding universes. But even before the discovery of the 3 K cosmic background radiation, it was evident that expansion could not be the solution of Olbers's paradox in an evolving universe (Harrison 1964).

The bright star-covered sky visualized in Olbers's paradox corresponds to a thermodynamic equilibrium state of $u = u^*$. Pre–twentieth century astronomers naturally did not realize that such radiation equilibrium requires energy far in excess of that available to stars. The radiation energy density in a bright-sky universe is $u^* = aT^{*4}$, where a is the radiation energy density constant and T^* (of order 6000 K) is the surface temperature of a sunlike star. This corresponds to a mass density u^*/c^2 of order $1{\times}10^{-20}$ g cm^{-3} that amounts to 10^{10} times the average density 10^{-30}g cm^{-3} of stellar matter. Alternatively, sunlike stars shining for 10^{23} yr (equal to λ/c) create a bright sky, and each star radiates a total energy that has a mass 10^{10} times the mass of the star. There is not enough energy in the universe, as constituted at present, to create the bright sky visualized in Olbers's paradox.

Leaving aside the steady-state universe, which requires special treatment because of its nonconservation of matter and energy, we find that the conditions for a low intensity of starlight in an expanding universe are remarkably similar to the low intensity of starlight in a static universe.

The intensity of starlight (if we ignore other sources and sinks of light in the night sky) is about 10^{-13} of the thermodynamic equilibrium intensity anticipated in Olbers's paradox. The solution to the problem of darkness at night must explain this vast difference between the observed and the anticipated intensities. The expansion of the universe dilutes and redshifts radiation, but as we can show, the magnitude of this effect is insufficient to account for a dark night sky [9]. The effect of expansion is in fact surprisingly small: it reduces the radiation energy density that would be observed in a static universe by only a few percent. This can be seen most easily with equation (13)

$$d(uV^{4/3})/dt = (cV^{4/3}/\lambda)(u-u^*)$$

and with the simple power-law models

$$V/V_0 = (t/t_0)^{3n} \ , \tag{17}$$

where V is a comoving volume, t the age of the universe measured from the initial epoch when $V = 0$, and n a constant index; a zero subscript denotes an arbitrary epoch. The Hubble and decelerations terms have the values

$$H = n/t \ ,$$

$$q = (1-n)/n \ ,$$

and n lies in the range $0<n<1$ for decelerating expansion, as in the Friedmann models. For the matter-dominated Einstein–de Sitter model, in which n has the value 2/3, we have $q = 1/2$, $D_H = 3ct/2 \approx 15\times10^9$ light-years. The particle horizon that bounds the visible universe lies at distance

$$D_H = ct/(1-n).$$

If we assume that stars commence radiating at time t_1, we find from equations (13) and (17) that the radiation density at a later time t is

$$u = u_1\left[\frac{t_1}{t}\right]^{4n} + \left[\frac{u^*ct}{\lambda}\right]\frac{1-(t_1/t)^{1+n}}{1+n} \ , \tag{18}$$

where u_1 is the initial radiation density at the time stars commence to shine. We must assume that u_1 is zero (ignore the cosmic background radiation!), consistent with the traditional terms of the riddle, and for $t \gg t_1$, which is not unreasonable, equation (18) becomes

$$u = u^*ct/\lambda(1+n) \ . \tag{19}$$

The condition for a dark night sky is $ct \ll \lambda(1+n)$, and when $n = 0$, this result simplifies to equation (12) for a static universe.

In static and expanding universes of similar constitution, of equal age, in which stars have shone for equal periods of time $t-t_1$, the radiation density in an expanding universe is $(1+n)^{-1}$ times that in a static universe. We see that expansion reduces the radiation density by a factor between 1 and 1/2 when $0<n<1$, and in the Einstein–de Sitter model this factor is 3/5. A similarly small reduction of radiation occurs in Friedmann models of positive ($q>1/2$) and negative ($q<1/2$) curvature.

The usual method of determining the background radiation created by a population of sources of density n, each of luminosity L, consists of constructing an integral equation of the form

$$u = \int_{t_1}^{t} \frac{n(t')L(t')V^{4/3}(t')dt'}{V^{4/3}(t)} \exp-\left[-\int_{t}^{t'} \frac{cdt''}{\lambda(t'')}\right] . \tag{20}$$

This equation, which is independent of curvature in a spatially isotropic and homogeneous universe, is nothing more than the integration of equation (13). The differential equation serves as a much more convenient starting point, is physically more lucid, and prompts us not to omit the initial radiation field u_1. We can also construct an integral for the sky-cover fraction and find:

$$\alpha = \int_{t_1}^{t} \frac{n(t')SV(t')cdt'}{V(t)} \exp\left[-\int_{t}^{t'} \frac{cdt''}{\lambda(t'')}\right], \tag{21}$$

and equations (20) and (21), unlike equations (11) and (12), do not give $\alpha = u/u^*$ because of redshift. Thus, assuming that $\alpha = 1$, we find $u<u^*$ because of redshift, and this is Bondi's resolution of the riddle.

We have known for twenty-five years that expansion and the redshift effect cannot solve Olbers's paradox in an evolving universe. In general, in an evolving universe, either static or expanding, the sky at night is dark when stars shine for a period considerably less than λ/c. The light-travel time λ/c of the background, or the radiation fill-up time, is of order 10^{23} yr for our universe as presently constituted. If we use a luminous lifetime of 10^{10} yr, typical of sunlike stars, we obtain a starlight intensity only 10^{-13} of that anticipated by Olbers's paradox. All other proposed solutions, such as the absorption of starlight, the hierarchical clustering of stars, and the expansion of the universe, merely serve to decrease a radiation intensity already sufficiently low to be consistent with the observed dark night sky.

8. THE CLIMAX OF OLBERS'S PARADOX

A dark night sky has two interpretations. According to the first, stars cover the sky, and this interpretation is true in the steady-state universe. The solution of the riddle is that the light from the distant background stars has been diluted and redshifted by expansion. According to the second, stars fail to cover the sky, and this interpretation is true in the big-bang universes. The solution of the riddle in this case is that the distant background stars are invisible because their light is in transit and has not reached us (and may never reach us).

The finite-age solution, first expressed in a quantitative manner by Kelvin, applies with only minor modification to most big-bang models of the universe. Through the starless gaps of the night sky we look far out in space and far back in time to the early universe. In every direction a line of sight reaches back to a time when the universe was 10^5 yr old. We see what our immediate forebears perhaps feared to see: the creation of the universe written across the heavens. The realization that the big bang covers the dark night sky, a big bang that was once incandescent and is now redshifted by expansion into an infrared gloom, is a fitting climax to an old and celebrated riddle.

NOTES

[1] For historical references see Jaki (1967) and Harrison (1987).
[2] Halley's papers are reproduced in Jaki (1967) and Harrison (1987).
[3] Chéseaux's discussion is translated and reproduced in Harrison (1987).
[4] The prevailing Continental view of the universe was Cartesian: a boundless universe of sunlike stars separated not by a vacuum, as in the Newtonian system, but by a continuous distribution of matter in a tenuous state. It was not unnatural for Chéseaux to assume that interstellar absorption existed and was sufficient to solve the riddle of lost starlight.
[5] Olbers's discussion is translated and reproduced in Harrison (1987).
[6] If each shell contributes an equal amount of light and occultations are ignored, then the amount of light received increases linearly with distance without limit. Therefore, starlight attains infinite intensity in a universe of infinite extent. This conclusion, based on the fallacy that stars are geometric points, is often mistakenly attributed to Chéseaux and Olbers, most recently by Tipler

(1988). I am unaware, however, of any scientist in the history of Olbers's paradox who ignored occultations and made this error; it seems to be modern in origin. It is interesting to note that the expectation that stellar disks cover the sky implies thermodynamic equilibrium and the temperature of radiation of interstellar space therefore equals the surface temperature of stars. Stars of finite luminosity that are no more than geometric points have infinite temperature, and therefore, under these conditions, the intensity of radiation in space must also be infinite.

[7] I am indebted to Professor A. J. Meadow for this reference and for identifying Thomas Young as the author of the lines "'twere not absurd. . ."

[8] Tipler (1987) argues against the priority of Poe's contribution and assigns precedence to Mädler. The available evidence indicates that Poe, despite his poetic imagery, had the right idea a decade or so before Mädler, and Tipler's argument fails to convince.

[9] The energy aspect of the riddle and the differential equation method are introduced in Harrison (1964). The equivalence of the integral and differential equation methods is discussed in Harrison (1974, 1977), with other theoretical aspects of the subject.

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