HIPPARCOS DATA REDUCTIONS

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ABSTRACT

The raw data produced by the European Space Agency's astrometry satellite HIPPARCOS -- mainly photon counts from the primary detector (IDT) and star mappers -- will be analysed in full by two independent reduction consortia, FAST and NDAC. The aim is however to generate, by 1993, a single catalogue of positions, proper motions, and parallaxes for the 10^5 programme stars, with additional information on photometry, multiple stars, and minor planets. This review outlines some practical and theoretical features of the data reductions.

1. INTRODUCTION

The relative complexity of the HIPPARCOS data reductions is not caused, as we might think at first, by the extreme accuracy aimed at, nor by the sheer size of the problem. Indeed, conditions in space are in several respects easier to modelise accurately than those pertaining to ground-based observations, and the data quantities involved in this project are not much greater than in many other space projects. I think we must rather blame the circumstance that HIPPARCOS observations are completely relative to each other and therefore themselves defining the system in which they are made. To be more specific: the basic kind of measurement we can make with the optical telescope with its modulating grid and detector, is simply the instantaneous location of a star image with respect to the slit pattern deposited on the grid. This can be derived from the raw photon counts by some rather elementary and unprejudiced calculations. Such a measurement depends however not only on the celestial position of the object, but equally on the pointing of the instrument (attitude) and the relation between optical field angles and the slit pattern (effective distortion). Thus we can write the observation equation very schematically as follows

$$\begin{matrix}\text{celestial}\\ \text{position}\end{matrix} - \begin{matrix}\text{instrument}\\ \text{pointing}\end{matrix} + \begin{matrix}\text{instrument}\\ \text{distortion}\end{matrix} + \text{noise} = \begin{matrix}\text{observed}\\ \text{location}\end{matrix} \qquad (1)$$

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The first three terms in (1) can be expressed as functions of several unknowns: the astrometric parameters of the object, attitude parameters, and distortion parameters. Depending on the modelisation of the attitude and distortion (e.g. as polynomials of time and field angles, respectively), their parameters will be the same for all observations in a time interval of minutes up to several hours. As a consequence, the astrometric parameters of any given star is indirectly related, via the attitude and distortion parameters, to those of hundreds or thousands of other stars observed within the same limited time interval. In the course of the full mission, the star is effectively connected to every other object in the observing programme. It is then impossible to isolate a given object from the remaining programme; the whole body of data must be treated as an entity.

In response to an Announcement of Opportunity issued by the European Space Agency (ESA, 1981), two independent groups of scientists were set up in order to carry out the reductions: the FAST consortium (Fundamental Astronomy by Space Techniques) with 17 participating institutes under leadership of J. Kovalevsky, and NDAC (Northern Data Analysis Consortium) lead by E. Høg and with participants from six institutes (Perryman, 1982). In the following I shall attempt to outline the data reductions in fairly general terms, which hopefully apply with small modifications to both consortia; a good deal of bias towards the methods and terminology of NDAC is still detectable. Alternative aspects are taken up e.g. by Kovalevsky (1980) and in the proceedings of the Asiago FAST Thinkshop (ed. Bernacca). Some familiarity with the mission concepts will be assumed below; see e.g. Høg (1980), Kovalevsky (1982, 1984) and Bouffard and Zeis (1983).

2. THE OBSERVATION MODEL

One way to regard the data reductions is as a problem of statistical estimation: the raw data are the unique realization of a many--dimensional stochastic function the parameters of which are to be estimated. Using e.g. the principle of Maximum Likelihood (ML) the solution is in theory quite simple, once we have a complete statistical description of the function (the outcome of a hypothetical mission) in terms of all the parameters. The purpose of an observation model is to approximate such a description.

The raw data which are to be 'explained' by the observation model, are mainly

(a) the complete record of photon counts from the primary detector (IDT) $\{N_\ell\}$, $\ell = 1$ to $\sim 10^{11}$ (sample frequency 1200 Hz);

(b) stretches of star mapper counts $\{N^*_\ell\}$ at the expected transits of programme stars across the star mapper slits (sample frequency 600 Hz);

(c) the complete record of gyro readings $\{\underline{g}_r\}$, $r = 1$ to $\sim 10^9$, where $\underline{g}$ is a 3-vector with one component for each active gyroscope.

The model parameters which characterize the idealized physical system (stars + instrument) at any instant are basically the following:

(A) object parameters, for a single star the five astrometric parameters α_0, δ_0, μ_α, μ_δ, Π and photometric parameters such as an HIPPARCOS magnitude, H, and colour, B-V.

(B) attitude parameters describing the temporal evolution of three pointing angles $\alpha_z(t)$, $\delta_z(t)$, $\omega(t)$;

(C) some 10 - 20 time dependent large-scale distortion parameters $\{g_{mn}(t),\ h_{mn}(t)\}$.

There are of course many other data than (a) - (c) which are needed for the reductions; these we shall regard as 'given', either by external sources (the Input Catalogue, ephemerides of the satellite and bodies of the Solar System) or by special calibrations (small-scale distortion, maps of the photometric sensitivity and OTF over the field).

From the object parameters and the barycentric motion of the satellite we have the proper direction to the object at any given instant, $\underline{u}$. This takes into account its space motion, parallax, gravitational deflection by the Sun and Earth (at least), and aberration. The object direction will now have to be expressed in a coordinate system defined by the instantaneous pointing of the telescope.

A fictitious point at the centre of the grid would define two viewing directions $\underline{p}$ and $\underline{f}$ (Fig. 1). The telescope system T = $[\underline{x}\ \underline{y}\ \underline{z}]$ is then defined by

$$\underline{z} = \langle\, \underline{f} \times \underline{p}\, \rangle, \qquad \underline{x} = \langle\, \underline{f} + \underline{p}\, \rangle, \qquad \underline{y} = \underline{z} \times \underline{x} \tag{2}$$

where <> signifies vector normalization. The celestial orientation of T can be expressed e.g. by means of the R.A. and Dec of $\underline{z}$ (α_z, δ_z) and the angle in the xy-plane from the Equator to $\underline{x}$ (ω). These angles in turn can be parametrized, as functions of time, by using piecewise polynomials or other suitable basis functions. FAST simulations (van der Marel, 1983) indicate that a representation accuracy better than 1 mas (0.001 arcsec) is possible with about one free parameter per axis and minute of time.

The field angles (η, ζ) of the object are established by writing its proper direction in T,

$$\underline{u} = \underline{x} \cos\zeta \cos(\eta \pm \tfrac{1}{2}\gamma) + \underline{y} \cos\zeta \sin(\eta \pm \tfrac{1}{2}\gamma) + \underline{z} \sin\zeta \tag{3}$$

where $\gamma = \arccos(\underline{f}'\underline{p})$ is the basic angle and upper (lower) sign is chosen for the preceding (following) field. The origin of field angles is $\underline{p}$ (or $\underline{f}$) with the η axis approximately in the scanning direction normal to the slits on the primary grid (Fig. 2).

The 'effective distortion' of the instrument describes the slit pattern in terms of field angles. On the primary grid we assign an integer 'grid coordinate' G to the centre of each slit, with G increasing by +1 for each new slit in the direction of +η, and write

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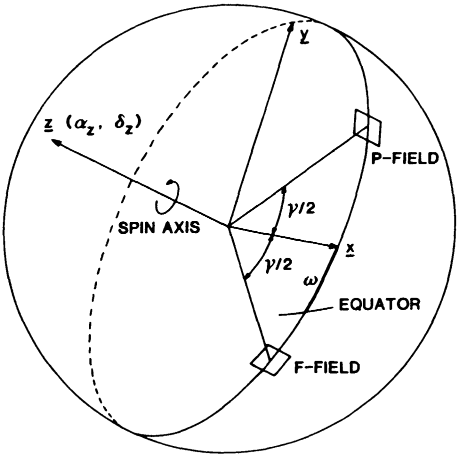

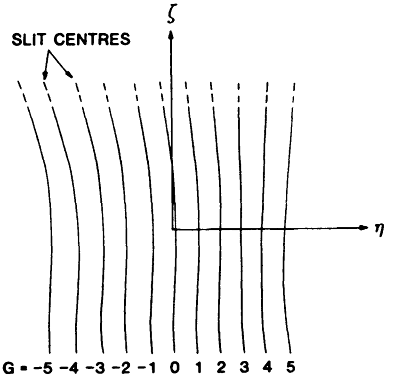

Figure 1 (left). Definition of telescope frame [x y z] and pointing angles α_z, δ_z, ω

Figure 2 (right). Illustrating the relation between field angles (η, ζ) and grid coordinate (G), i.e. the effective distortion

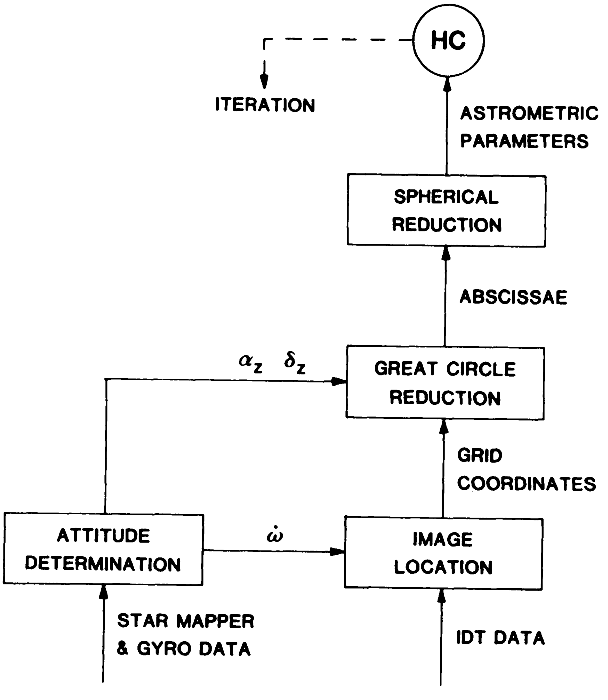

Figure 3. The four main stages of reduction leading up to the HIPPARCOS Catalogue of astrometric results

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$$G = \Sigma_m \Sigma_n (g_{mn} \pm h_{mn}) \eta^m \zeta^n + d(\eta, \zeta), \quad 0 \leq m+n \leq 3 \tag{4}$$

The large-scale distortion represented by the polynomial in field angles includes optical effects which may be different in the two fields; this corresponds to $h_{mn} \neq 0$. The small-scale irregularities is represented by the function d, and is here assumed to be known.

If G_ℓ is the grid coordinate of a point object at the time of the ℓth IDT sample, we can now assume that the observed number of photon counts in that sample, N_ℓ, is a Poisson process with mean value

$$E(N_\ell) = B + A[1 + M_1\cos(2\pi G_\ell) + M_2\cos(4\pi G_\ell + v_2)] \tag{5}$$

in which B and A stand for the instantaneous brightness of the background and object, respectively, and M_1, M_2, and v_2 are functions of the field angles known from the OTF calibrations. Note the absence of a phase term in the fundamental harmonic ($v_1 = 0$), which can be taken as definition of the centre of the slit (G = integer). In reality we shall have $M_1 \cong 0.6$, $M_2 \cong 0.25$, and $|v_2| \ll 1$ rad.

The star mapper slits are arranged in two groups: a 'vertical' with slits nearly parallel to $\underline{z}$ or the ζ axis, and an 'inclined' chevron group with slits at 45^o to the vertical. Each group consists of four parallel slits, but we need for the moment only consider the central line of each group, described by some equation $\eta = \eta_c(\zeta)$ different for the two groups and perhaps also for the two fields. The counts from the star mapper PMTs can now be modelled as a Poisson process with mean value

$$E(N^*_\ell) = B^* + A^* S[(t^*_\ell - \tau)\Omega] \tag{6}$$

B* and A* are the background and stellar brightness, S the known star mapper response function (consisting of four humps corresponding to the slits in a group), τ is the time of transit across the central line, and $\Omega = (d/dt)[\eta - \eta_c(\zeta)]$ is the effective scanning speed across the group.

The gyroscope readings are taken at a frequency (~10 Hz) much higher than the bandwidth of perturbing torques (~0.01 Hz) and can therefore be regarded as measurements of the instantaneous inertial rotation about the input axis of each gyro (which is of course very nearly fixed in T). If ω_x, ω_y, ω_z are the angular velocities about $\underline{x}$, $\underline{y}$, $\underline{z}$ (readily expressed in terms of the time derivatives of the pointing angles), we write

$$\underline{g}_r = \underline{K}\ (\omega_x\ \omega_y\ \omega_z)' + \underline{b} + \underline{n}_r \tag{7}$$

The rows of the (3,3)-matrix $\underline{K}$ are determined by the sensitivity and orientation (in T) of the gyro input axes and are known from laboratory calibrations. $\underline{b}$ is a vector of slowly varying biases (drift rates) and $\underline{n}_r$ is a noise vector; these could for instance be represented by polynomials and white gaussian noise, respectively.

Equations (3) - (7) plus an astrometric model of the object constitute a complete, albeit not very sophisticated, observation model.

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3. MAIN STAGES OF THE REDUCTIONS

3.1. Attitude Determination

The natural starting point for the data reductions is the estimation of the pointing angles α_z, δ_z, ω, or rather the polynominal coefficients (or similar) by which they are represented. A least-squares fit to the real-time attitude may provide a first approximation, accurate to about 1", adequate for linearizing the equations. This is then improved by processing observation equations for the star mapper transits and gyro readings. Basically, each stellar transit across one of the slit groups of the star mapper is an observation of the absolute pointing about an axis parallel to the slits, and gives an observation equation

$$\underline{C}\ (\Delta\alpha_z \quad \Delta\delta_z \quad \Delta\omega)' + \text{noise} = \tau_{obs} - \tau_{calc} \tag{8}$$

for the corrections to the attitude angles, $\underline{C}$ being a known row vector and τ the transit time. The observed τ is obtained via the model (6) by feeding the photon counts N^*_ℓ through a numerical filter matched to the response function S; this is closely related to a ML estimation of τ.

The gyro readings measure the angular velocities about certain axes fixed in T and give observation equations similar to (7) with velocity components written in terms of the derivatives of the attitude angles.

In the final iteration, the attitude must be known to about 0.1" in all three axes. This cannot be achieved immediately, since the precision in (8) is limited to 1-2" as set by the precision of the Input Catalogue. A first pass of updating the positions is thus required before the final attitude determination.

3.2. Image Location

The next stage shall determine the grid coordinates (G) of observed objects at given instants, e.g. the fram mid-times. (A frame is the period of 2.133.. s = 2560 IDT samples in which the sensitive detector spot is cyclically switched in a fixed pattern between the stars in the fields of view). From the attitude determination we know the angular velocity of the instrument perpendicular to the slits (essentially $\dot{\omega}$) and hence, to a first approximation, $\dot{G}$. The grid coordinate in (5) is consequently known up to an integration constant, say the grid coordinate at mid-frame $G(t_o)$. The photon counts can now be Fourier analysed with respect to the reference phase

$$H_\ell = 2\pi[G_\ell - G(t_o)] \tag{9}$$

yielding ML estimates of the trigonometric coefficients in

$$N_\ell \sim a_0 + a_1\cos H_\ell + a_2\cos 2H_\ell + b_1\sin H_\ell + b_2\sin 2H_\ell \tag{10}$$

By means of the model (5) it is now easy to find the mid-frame coordinate as well as the stellar and background intensities A and B. It is also possible to make a ML fit of (5) directly to the counts.

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3.3. Great-Circle Reduction

The observed locations -- in the form of grid coordinates at frame mid-times -- shall now be combined as in (1) to improve the position of the object, the pointing angles, and the instrument distortion. It would be easy to set up such observation equations directly with the astrometric parameters as unknowns. But in order to solve these, one would then have to combine such observations from the entire length of the mission, and the number of unknowns would become totally prohibitive.

By restricting to observations in a time interval of 6-12 hours, in which the satellite scans some 3-6 complete circles on the sky, there are only about 2000 stars involved at a time and we get normal equations of reasonable size. Since the observations are confined to a rather narrow strip on the sky (a few degrees wide), they can however only give measurements in one coordinate, i.e. along the scans. For such a set of observations we introduce a provisional coordinate system on the sky, with abscissae measured along a fixed reference great circle (RGC) and ordinates perpendicular to it. The origin of abscissae can be chosen quite arbitrarily (in fact we have at this stage no means of relating the abscissa exactly to R.A. and Dec; this is the purpose of the stage called Spherical Reduction).

With $\Delta\upsilon_i$ denoting the correction to the abscissa of star number i, we have

$$D\, \Delta\omega_i - E\, \Delta\omega_k + \Sigma_m \Sigma_n\, \eta^m \zeta^n\, (\Delta g_{mn} \pm \Delta h_{mn}) + \text{noise} = G_{obs} - G_{calc} \quad (11)$$

where D and E are known coefficients and $\Delta\omega_k$ is the correction to the third pointing angle in the kth frame.

When solving the system of equations (11) for a set, various assumptions can be made concerning the smoothness of the attitude motion. In one extreme case, the geometrical solution, the corrections $\Delta\omega_k$ are solved independently for each frame. This is far simpler but also less accurate than using the so-called numerical (or dynamical) smoothing, in which case a polynomial or other smooth representation as function of time is inserted instead of $\Delta\omega_k$. In either case the attitude unknowns can be eliminated successively while forming the normal equations, leaving a system with about 2000 unknowns, viz. the abscissa corrections and 10-20 distortion corrections.

For successful determination of the distortion parameters it is necessary to process at least a few, partly overlapping great-circle scans, so that the same stars cross the fields at different ζ-coordinates. The finite width of such a set requires, on the other hand, that the distances of stars from the RGC are known to about 0.1", in order that projection errors onto the RGC from the slightly skew scans shall be negligible; for a similar reason the pointing angles α_z and δ_z are needed to similar accuray. The great-circle reductions will therefore have to be iterated after a first improvement of the astrometric results.

The method of least-squares solution chosen by NDAC is via the reordered normal equations and Cholesky decomposition. Due to the unspecified abscissa origin the system has a rank defect of 1, which is handled with a simple modification producing the generalized solution. -- FAST has considered several alternative solution methods, including iterative ones (van Daalen, 1983; Tommasini-Montanari, 1983).

3.4. Spherical Reduction

The great-circle reductions produce star coordinates (abscissae) in many independent systems: each set of 6-12 hours' observation has its own RGC and a quite arbitrary abscissa origin. The purpose of the spherical reduction is to bring together the results in a common, global coordinate system. There are several possible ways of doing this; the method now adopted by NDAC was proposed by this author in 1976 and depends on solving first for corrections to the abscissa origins. Let υ_{ij} be the abscissa derived for star number i in the great-circle reduction of set number j, and let c_j be the correction required to bring the abscissae in the set on a global system. The equations input to the least-squares solution are

$$\underline{F}\ (\Delta\alpha_{oi}\ \Delta\delta_{oi}\ \Delta\mu_{\alpha i}\ \Delta\mu_{\delta i}\ \Delta\Pi_{i})' - c_j + \text{noise} = \upsilon_{ij}^{obs} - \upsilon_{ij}^{calc} \tag{12}$$

with $\underline{F}$ a known row vector. If the abscissae have been sorted by star number, we can eliminate the corrections to the astrometric parameters while forming the normal equations, so that in the end we have a system of linear equations with the set zero points c_j as only unknowns; there are about 2-3000 of these. In this process it is important to avoid using suspected multiple stars and other 'bad' observations which might introduce systemtatic errors into the global coordinates.

This system has a rank defect of 6, due to the unspecified orientation and rotation of celestial coordinates. A generalized solution method can be used as in the great-circle reductions. Having solved the zero points c_j, it is a simple matter to insert them in the eliminated equations and obtain the solution for the astrometric parameters.

A different approach was recently proposed by Betti et al. (1983) in the FAST consortium: the results of the great-circle reductions are used to get the angles between stars separated by approximately the basic angle (58^o); these are then expressed directly in terms of the astrometric parameters. The latter are thus determined for groups of, say, 6000 stars, yielding systems of equations with ~ 30,000 unknowns.

3.5. Iteration

The four main stages of the data reductions are schematically shown in Fig. 3. As already indicated, the full precision of the astrometric parameters (a few mas) will not be reached in a first pass through these stages, the positional errors in the Input Catalogue being too large to permit sufficiently accurate attitude determination and projection of measurements onto the RGCs. But it will probably not be necessary to iterate the full solution if the star catalogue is regularly updated as the mission progresses. Initially, it will be advantageous to use exclusively star mapper observations for updating those stars which are bright enough to give good transits; this eliminates the slit ambiguity problem for these stars and gives a quicker improvement in two coordinates.

4. SPECIAL PROBLEMS

4.1. Double Stars

It is estimated that some 10-15 % of the programme stars are double or multiple systems of a category that may require special treatment in the reductions (magnitude difference < 3, separation 0.1 - 20"). Only part of them will be known prior to the mission, and automatic procedures for recognizing potential double stars must be established. These will be based on several statistical tests on the intensity modulation and residuals of various solutions. After recognition, increasingly complex object models will be tried until a satisfactory fit is obtained. The associated statistical and numerical problems are very considerable and to this date only superficially studied.

4.2. Slit Ambiguities

The expression for the modulated intensity (5) or (10) is invariant to an integer shift in grid coordinate. The image location taken by itself is therefore ambiguous as to which slit the coordinate refers to; this will have to be decided from the current catalogue position. Since the grid period is $\partial\eta/\partial G \cong 1.2''$, of the same order as the positional errors of the Input Catalogue, it will very often happen that initially the wrong slit is chosen. For stars brighter than 10th magnitude the star mapper observations will help to update the catalogue positions to a precision where this problem is avoided. For fainter stars it is possible to find the correct solution in the spherical reduction by trying several positions in a small area around the catalogue position. This is basically a non-linear minimization problem with periodic instead of quadratic loss function.

4.3. Adjustment to an External Reference System

HIPPARCOS observations are by nature purely differential (albeit on a global scale) and external information must be injected to fix the orientation and rotation of the system of positions and proper motions. It is intended to link the HIPPARCOS system to FK5 but also to the extragalactic VLBI frame by means of radio stars, which are observed directly with the satellite, and optical quasars via the Space Telescope (Froeschlé and Kovalevsky, 1982; Duncombe and Hemenway, 1986). It is imperative that the internal inconsistencies of other systems such as FK5 are not allowed to propagate into the HIPPARCOS system. This is avoided if the reductions are made entirely without regard to prior system information and the results only afterwards transformed into agreement with external data by rigid rotation of the system.

5. ORGANIZATION AND STATUS OF PLANNING

The scientific organization of HIPPARCOS has been outlined by Perryman (1982), who also lists the participating institutes. The reduction consortia will receive data on high-density tapes 1-2 times per week throughout the scientific operation of the satellite (ca 1988.5-91.0). The European Space Operations Centre (ESOC) in Darmstadt (Germany) is responsible for operating the satellite, receiving, decommutating, monitoring and distributing the data.

For the FAST consortium the processing will be done entirely at the CNES computing centre in Toulouse (France) using mainframe computers (type CDC CYBER 170). NDAC has taken a different approach and will distribute the work load between England (attitude determination and image location) and Denmark (great-circle and spherical reductions), using moreover super-minicomputers type VAX11/780 (RGO) and HP9000 (DSRI). Both consortia use the FORTRAN 77 programming language.

Considering the voluminal difference between the two consortia it is not surprising that they have also tried rather different ways to arrive at the final design of the software. FAST has explored a wide range of possibilities -- as demonstrated most impressively by the Asiago Thinkshop -- and emphasized the detailed simulation of payload and mission. NDAC has by necessity restricted analysis to a single and largely preconceived plan for the reductions and will not make very extensive simulations. Both consortia now seem to have reached the point where the functional architecture is relatively frozen and some of the actual coding started.

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Discussion:

de VEGT: What is the scale value in the grid plane (in units of "/mm)?
LINDEGREN: We don't have a millimeter scale - we have arcseconds per slit. The focal length is 1.4m.
de VEGT: The system of original condition equations in nonlinear. Is it necessary to iterate the original system for better linearization?
LINDEGREN: This is one of the reasons for redoing the equations.
EICHHORN: Are you making a one-shot global solution or are you using an iteration scheme?
LINDEGREN: It is an iteration scheme but the system is effectively linear.
GUINOT: Do you assume that the relativistic deviation of light is known a priori or do you intend to solve for its amplitude?
LINDEGREN: General relativity is assumed in the reductions, but we have the possibility to solve for corrections to it in terms of a more general metric. This will certainly be tried although we do not expect any deviations at the level of a fraction of a milliarcsecond.
GUINOT: If this amplitude is solved for, would it provide a check of the general theory of relativity?
LINDEGREN: Since we are observing about 19° from the Sun, the average deviation is 0".004. We probably can detect this to a fraction of one percent. I don't know whether we can establish any deviations from relativity at that level.
THORNBURG: How much data processing do you do on board the satellite?
LINDEGREN: The raw photon counts are transmitted and used for the reduction, no processing is made on board except as needed for operating the satellite.
THORNBURG: So if you get unexpected distortion, etc., after launch, you can (try to) correct it on ground?
LINDEGREN: Yes.
HARRINGTON: What guarantees, numerical or analytic, are built into this system to assure convergence of this colossal iterative scheme?
LINDEGREN: We know that it converges for fewer than 100,000 stars. Adding more stars improves the rigidity of the system.
EICHHORN: I believe that Jefferys proved some time ago that systems of this type converge. Are you smoothing the attitude of the satellite numerically or dynamically? If the former, how do you take care of the discontinuities between different sets of parameters?
LINDEGREN It is possible to use either dynamical or empirical modeling. Gas jet actuations and shadowing effects will give discontinuous derivatives which would make spline functions very suitable for empirical modeling.
de VEGT: Can you describe the structure of the original normal equations (Full number of unknowns)?

LINDEGREN: The matrix of the full normal equations including the attitude unknowns is very sparse and of simple structure. After elimination of the attitude parameters the normal equation for the great-circle solution are much more complex and perhaps 20% full. The normal equation for the global solution are nearly 100% full.

HUGHES: I understand that there are two groups who are doing the data reduction. What is the difference between the two groups? Are they carrying out the data reduction parallel or by different methods?

LINDEGREN: There is a lot of difference in modeling, the way we do the computations etc. The two consortia essentially work independently of each other.

HØG: Let me add that the two consortia will work independently in parallel but we will make comparisons on simulated and real data at different levels of the data reduction. This is an important part of our strategy to enable us to arrive at one unique HIPPARCOS catalogue in the end.

KOVALEVSKY: Both consortia cooperate, but the more our methods will be different, the more confidence one can have in the final results.