FORUM

Distance Off by Vertical Sextant Angle

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The note by J. W. Crosbie[1] prompts the writer to the following comments. Lecky's Tables[2] do not cover vertical angles below the geometrical horizon (zenith distance = 90°), or, indeed, angles of less than 10′ above it. Crosbie's method, also, does not cater for angles below the geometrical horizon, though it could be adapted to do so. His extraordinary evolutions with the traverse tables, undertaken to avoid solving a quadratic equation by orthodox methods, are fascinating. Surely, a slide rule, with which the equation could be solved in a few seconds, is available in today's chartroom.

The following method of solving the vertical angle problem has been used by the writer. Its principles are obvious and there are no special things such as refraction corrections to remember; consequently it is convenient for occasional use. It works with angles right down to the visible horizon, but requires the use of a table of the distance of the visible horizon, or the formula $d = 1{\cdot}15\sqrt{h}$.

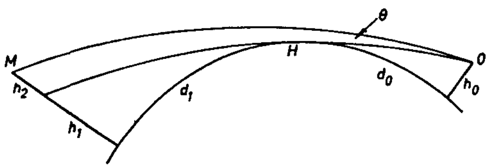

FIG. 1

The following notation is used (Fig. 1).

h_0 is the height of the observer O
d_0 is the distance of his visible horizon H
d_1 is the distance from H to the mountain M
h_1 is the height of the obscured part of M
h_2 is the height of the visible part of M
$d = d_0 + d_1$ is the total distance from O to M
$h = h_1 + h_2$ is the total height of M
θ is the observed vertical angle.

Proceed as follows:

1. Knowing h_0, use the tables to find d_0
2. Assume a value for d
3. Calculate d_1 $(= d - d_0)$
4. Use the tables to find h_1
5. Calculate h_2 $(= h - h_1)$
6. Calculate $\theta = 0{\cdot}565\, h_2/d$.

If this is not equal to the observed value start again at stage 2 with a new value of d, larger if the observed value of θ is less than the calculated value, and conversely.

When the correct value of θ has been bracketed, the value of d can be interpolated. The rate of change of θ with d gives an estimate of the potential accuracy of the observation, bearing in mind that abnormal refraction will reduce the accuracy.

The calculation of J. W. Crosbie's example ($\theta = 28.0'$, $h_0 = 56$ ft., $h = 5230$ ft.) goes as follows:

$h_0 = 56$, $d_0 = 8{\cdot}6$ M
Try $d = 60$ M, $d_1 = 60 - 8{\cdot}6 = 51{\cdot}4$ M
$h_1 = 2000$ ft, $h_2 = 5230 - 2000 = 3230$ ft.
$\theta = 0{\cdot}565 \times 3230/60 = 30{\cdot}4'$
Try $d = 65$ M, $d_1 = 65 - 8{\cdot}6 = 56{\cdot}4$ M
$h_1 = 2400$ ft., $h_2 = 5230 - 2400 = 2830$ ft.
$\theta = 0{\cdot}565 \times 2830/65 = 24{\cdot}6'$.

Interpolating for $\theta = 28{\cdot}0'$ gives $d = 62$ M. The rate of change of angle with distance in this case is about $1{\cdot}2'$ per mile.

Observations of small vertical angles on objects *between* the observer and his visible horizon are obviously incapable of giving accurate distances off. But when the object is beyond the horizon, moving away from it reduces the angle not only because the same object appears smaller, but also because more of the object disappears below the horizon. Consequently even quite small observed angles can give a useful position line. Provided that the height of the object is not less than twice that of the observer the rate of change of vertical angle with distance will not be less than $\frac{1}{2}'$ per mile.

REFERENCES

1 Crosbie, J. W. (1970). Distance off by vertical sextant angle. This *Journal*, **23**, 253.
2 Lecky, S. T. S. *The Danger Angle, and Off-shore Distance Tables*, 21st Edition, 1918.

Rhumb-line Sailing

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If Turner[1] is right that 'the underlying theory of the traditional approach (to rhumb-line sailing) is obscure' and that there is a 'lack of ready availability of a table of distances of parallels of latitude from the equator, it is certainly not the fault of this *Journal* which precisely 20 years earlier[2] published a paper which gave:

1. The correct mathematical theory of rhumb-line sailing on an oblate spheroid.
2. The name 'meridional distance' to what Turner now calls the $L(\phi)$ function.
3. A table to reduce latitude to meridional distance.

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