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RESEARCH ARTICLE

San Martín's accurate longitude measurements on Magellan's circumnavigation: luck or mastery?

Romeu Gaspar*

X&Y Partners, Lisbon, Portugal.
*Corresponding author. E-mail: romeu.gaspar@thisisxy.com

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Abstract
During the 1519–1522 Magellan expedition, the astronomer Andrés de San Martín made two remarkably accurate longitude measurements, an order of magnitude better than what was typical for the 16th century. How he managed to do so remained shrouded in mystery for the past 500 years. Using modern ephemerides, we have retraced San Martín's observations and calculated their error signatures, clarifying the method he used (a simplified version of lunar distances) and why two out of his six measurements were accurate (a rather fortuitous cancellation of errors). It would be rash to dismiss San Martín's work as sheer luck though, as he was an exceedingly rare combination of a capable astronomer and a knowledgeable mariner.

1. Introduction

On 19 September 1519, Magellan and a fleet of five ships left Spain with a hefty set of goals: find a yet-unknown passage from the Atlantic to the Pacific, sail across the vast expanse of the latter, locate the miniscule Spice Islands (today part of the Moluccas, Indonesia), prove that they are part of Spain's realms, and retrace the entire trip back with a full haul of cloves. By March 1521, one and a half years later, the first two objectives had been achieved, albeit at no small cost. In Patagonia, Magellan quelched a rebellion of disgruntled mariners, executing or marooning the mutiny's instigators. Next, he lost one ship while looking for a passage to the Pacific, and another one deserted while exploring the newly discovered Strait of Magellan. On the brutal three-month Pacific crossing that ensued, further lives were lost to scurvy. The fleet was now at Suluan Island – part of modern-day Philippines, then unfamiliar to Europeans – restoring their strength before resuming the search for the Spice Islands, which they suspected were close by.

The interregnum was a good opportunity to try to prove that Suluan, plus the nearby Spice Islands, were part of Spain's claimed territories. In 1494, Spain and Portugal signed the Treaty of Tordesillas, splitting the globe in half: newly discovered territories more than 370 leagues west of the Cape Verde islands belonged to Spain,[1] while Portugal kept those to the east. With this treaty, the two neighbouring countries hoped to avoid any further clashes between Spain's western explorations – triggered by Columbus's recent landfall in the Americas – and Portugal's eastern maritime trading route to India. The demarcation line extended only from pole to pole, splitting the Atlantic but not reaching the other side of the globe. Little was known in Europe about the geography of Southeast Asia at the time, so neither signatory was too worried about the lapse. Over the years, however, as Portugal expanded across Asia, Spain increasingly argued that the treaty implicitly assumed an anti-meridian demarcation line that

[1] Approximately 47° west of Greenwich.

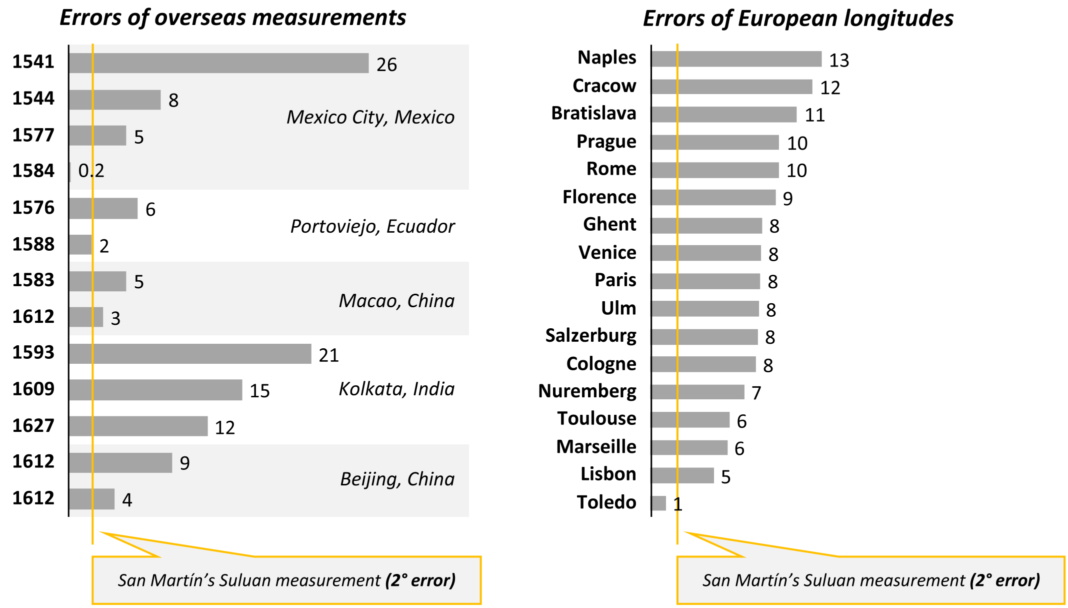

Figure 1. *Examples of errors of 16th- and 17th-century European longitude readings (all errors in degrees of longitude); Sources: Randles (1985) for overseas measurements; Zacut (1502) for European longitudes.*

limited Portugal's eastward expansion. The issue became unavoidable in 1512 when Portugal claimed the rich Spice Islands. If geopolitics had somehow led Portugal and Spain to split the world horizontally instead of vertically, Magellan's expedition would perhaps have never happened, as the true latitude of the Spice Islands was not up for debate. However, longitude was far more difficult to measure accurately in the 16th century.

Magellan's pilots were not able to determine longitude, as the ability to chart longitude for open-sea navigation would not be possible until two centuries later. There was one exception: Andrés de San Martín,[2] who served triple-duty as a pilot, astronomer, and astrologer. Combining astronomy and astrology was routinary in the 16th century,[3] but seaworthy astronomers were far scarcer, if not unheard of. Magellan, aware of the importance of determining longitude, had insisted on including the uncommon combination amidst the fleet's roster. Now, more than halfway across the globe, it was time for San Martín to justify his keep.

And that he did. The Spanish astronomer took out his instruments and placed Suluan Island 9° west of the Tordesillas antimeridian (130° east of Seville). Magellan surely did not like the result, as it meant they were already in the Portuguese hemisphere. However, from a scientific standpoint, it was an extraordinarily accurate measurement, just 2° off Suluan's true position. This had not been San Martín's only longitude measurement. While the fleet was still in South America, the astronomer made at least five other readings,[4] although the results of only two of them have survived: on 17 December 1519, San Martín placed Rio de Janeiro 269° west of Seville (an error of 232°) and, on 17 April 1519, he placed San Julían, in Patagonia, 61° west of Seville, a mere error of 0.6°.

It is worthwhile to put these results into context (Figure 1). Subsequent 16th- and 17th-century European attempts to measure longitude overseas routinely had errors one order of magnitude above San Martín's best results, often requiring multiple observations over the course of several years to

[2]See García (2020) for a biography of San Martín.
[3]In fact, the terms astronomer (or the more commonly used cosmographer) and astrologer were often used interchangeably.
[4]Barros (1628) refers to a possible additional longitude measurement of 42 min (10.5° from Seville, see Appendix A). As its date and location cannot be determined, its error signature is not assessed in this paper.

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narrow it down to reasonable results. Even within Europe, with access to a larger pool of experienced astronomers and better equipment, errors of more than 10° were not uncommon.

How did San Martín manage such extraordinarily accurate measurements at San Julían and Suluan? What went wrong at Rio de Janeiro? And could the three lost measurements have been accurate?

To our knowledge, these questions have remained unanswered for the past 500 years. Laguarda Trías (1975), a Uruguayan historian, made the most strides by compiling the known information sources on San Martín's measurements and theorising that they had been made using the lunar distances method. He urged others to verify this hypothesis using current ephemerides, a plea that has lingered unheeded until now.

2. Using error signatures to reconstruct San Martín's longitude measurements

As summarised in Table 1 and further detailed in Section 3, much of the mystery surrounding San Martín's measurements stems from the fact that his original notes have been lost, and the existing reliable second-hand reports provide only an incomplete picture of them.

In this paper, *error signatures* are used to fill in these gaps. As far as we know, these have not been used as a quantitative research tool for the 'longitude problem' in the 16th century, but they are routinely used in many other areas, both within and outside academia. Error signatures provide an accuracy range for any given measurement, reflecting the compounded effect of its individual sources of error, both random and systematic. Readers of this journal may be familiar with the error signature of GPS devices, or that of longitude readings made with the lunar distances popularised in the 18th century. Everyday examples of error signatures include those of weather forecasts and election polls.

The first requirement to compute an error signature is to establish an accurate baseline, something against which all errors can be measured. With modern ephemerides (Folkner et al., 2014) and software planetariums (Zotti et al., 2021), one can rewind the clock 500 years and see the exact same sky San Martín witnessed during his observations. The *exact* is not a hyperbole, as NASA's Jet Propulsion Laboratory (JPL) DE431 ephemerides can pinpoint the moon's present-day orbit with submeter accuracy and are suitable for computing celestial body positions from approximately 13,000 years into the past to approximately 17,000 years into the future.[5] Appendix C lists the DE431 ephemerides data used on this paper.

The second requirement is a hypothesis for how San Martín measured longitude. This was provided by the Rio de Janeiro observation, the only surviving description of his method (a simplified version of lunar distances, see Section 5), which also provided clues on the two almanacs the Spanish astronomer had at his disposal (Zacut, 1498, 1502; see Appendices A and B).

The third requirement is knowing what instruments San Martín used to make astronomical observations, and how accurate they were. From the *Casa de la Contratación* inventories of what was acquired for Magellan's fleet (Navarrete, 1837), we know that San Martín had at his disposal marine astrolabes, quadrants and compasses, which he would have used to measure the altitude and azimuth of celestial bodies. Malhão Pereira (1994) has conducted experiments aboard the *NRP Sagres* with period-faithful replicas of these instruments and measured an average standard deviation accuracy of 15.9 arcminutes for altitude readings.

San Martín's astronomical observations also required the determination of local time. For its associated errors, we have used the work of Steele and Stephenson (1998), who have compiled some 30 local time readings taken by Regiomontanus and Walther during eclipse observations. Despite using a variety of methods,[6] both these astronomers measured local time with a standard deviation accuracy of 9 min. Medieval Muslim astronomers, such as Ibn Yunus, al-Battani and al-Blrunl, achieved similar

[5]In March 2021, NASA JPL introduced the DE441 ephemerides, which are expected to further improve accuracy but have so far been less-thoroughly tested. For the analysis of San Martín's observations, DE431 and DE441 produce the same results, as the positional differences between the two models (<1 arcsecond) are well beyond the 1 arcminute precision of the ephemerides that San Martín had at his disposal.

[6]Regiomontanus favoured measuring the altitude of the sun, the moon or a bright star, and converting it to local time with spherical trigonometry, whereas Walther used both astronomical instruments – e.g. a planispheric astrolabe, a triquetrum, an armillary sphere – and a mechanical clock.

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Table 1. *Extant reliable information of San Martín's longitude measurements.*

Location	Date	Observation	Method	Result (Seville meridian)	Sources
✓Rio de Janeiro	✓17/12/1519	✓Moon and Jupiter (conjunction)	✓	✓−268.8°	• **Location:** Barros, Herrera, Pigafetta, pilot logbooks • **Date, observation:** Barros, Herrera • **Method, result:** Herrera
✓La Plata River	✓01/02/1520	✓Moon and Venus (opposition)	χ	χ	• **Location:** Pigafetta, pilot logbooks • **Date, observation:** Barros
✓Gulf of San Matias	✓24/02/1520	✓Moon and Sun (opposition)	χ	χ	• **Location:** Pigafetta, pilot logbooks • **Date, observation:** Barros
✓San Julian	✓17/04/1520	✓Moon and Sun (eclipse)	χ	✓−61.0°	• **Location:** Barros, Castanheda, Pigafetta, pilot logbooks • **Date, observation:** Barros, Castanheda • **Results:** Castanheda
✓Pacific Ocean	✓23/12/1520	✓Moon and Sun (opposition)	χ	χ	• **Location:** Pigafetta, pilot logbooks • **Date, observation:** Barros
✓Suluan Island	✓16/03/1521	χ	χ	✓+130.0°	• **Date, location:** Pigafetta, Albo and other pilot logbooks • **Result:** Albo

✓Reliable information available.
χReliable information not available.

results (Said and Stephenson, 1997) with planispheric astrolabes or tables to convert the altitude of a celestial body to local time. San Martín did not seem to be as well versed in spherical trigonometry as Regiomontanus (see Section 5) and certainly did not take Walther's bulkier instruments aboard. A planispheric astrolabe was portable enough to take aboard, but to be precise it would need to include a set of latitude-specific templates. The *Casa* inventories do not include an expense for the many templates needed to cover at least 70° of latitude,[7] so it seems more likely that San Martín determined local time with the aid of tables. In any case, as noted previously, these various methods for determining local time had a similar accuracy of 9 min, which was our assumption for San Martín's measurements.[8]

We have used these elements (i.e. the hypothesis that San Martín used the simplified lunar distances method described in the Rio de Janeiro notes for all his observations, the errors of the almanacs and the instruments he used, plus modern ephemerides as a baseline) to calculate the error signature of each one of the Spanish astronomer's longitude measurements. Table 2 describes the errors considered, and Appendix D lists their values for each observation.

The propagation of these individual sources of error is complex, as there are several dependencies to account for (e.g. the parallax error is affected by the instrument, local time and refraction errors, and will, in turn, influence the geometric error). To validate that these dependencies were reliably accounted for, we have used two independent numerical models, one calculating the individual contribution of each error and another one directly computing the total error of the longitude measurement. The maximum deviation between the two models is well below the 1 arcminute precision of 16th-century almanacs (see Appendix D)

As the next sections detail, the resulting error signatures provide robust evidence to answer the three questions that we have set to address: (i) the accurate measurements of San Julían (Section 6) and Suluan (Section 7) are entirely achievable with San Martín's simplified lunar distances method (as suggested by Laguarda Trías), but only due to a fortuitous cancellation of large errors; (ii) at Rio de Janeiro (Section 5), San Martín made a calculation error but, even with that misstep, he would have still exaggerated longitude by over 40°; and (iii) it is unlikely that any of the three lost measurements (Section 8) had errors below 80°.

3. Assessing the reliability of the extant information sources

San Martín's original notes were lamentably lost to time, so everything that is known about the astronomer's observations comes from second-hand reports, either written by other members of Magellan's crew or by 16th-century chroniclers who had access to San Martín's original notes. This section assesses which of these information sources can be trusted.

The description of the Rio de Janeiro calculations – the most interesting piece of this puzzle, as it lays out San Martín's method – is penned by Herrera (1601), a Spanish chronicler. The historian's claim to have faithfully transcribed the astronomer's words is credible, as he did not have the means to do otherwise even if for some inscrutable reason he wanted to. Indeed, falsifying an astronomical observation (i.e. correctly positioning celestial bodies in the Rio de Janeiro sky at a given moment in time) would require both knowing the correct coordinates of the place and having accurate ephemerides, which was far from being the case in the 16th century.

Information about the other four observations made in South America comes from Barros (1628) and Castanheda (1554), two Portuguese chroniclers. Rather than transcribing San Martín's full notes, as Herrera did, they only mention the dates of the measurements, the observed celestial bodies and a single longitude result (for San Julían). While the selective reporting is certainly odd, Barros and Castanheda did not have any discernible motive to falsify records and could not have invented the aspects that were

[7]From Seville to the mouth of the La Plata River, the last location plotted in the charts taken aboard (Gaspar, 2019).

[8]One may wonder if San Martín was as skilled in measuring local time as some of the greatest astronomers of this period. The only surviving record of his local time determinations (at Rio de Janeiro, see Section 5) is within the 9 minute accuracy range, but that might have been a fluke. For computing the error signatures, we have inclined to the side of caution and given San Martín the benefit of the doubt, as this narrows the range of the error signature (in other words, one could justify any longitude measurement by making a wide enough error range, whereas making it narrower decreases the likelihood of false positives).

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Table 2. *Errors included in the calculation of the error signatures.*

Type	Source	Error	Description	Calculation method
Systematic	Almanac	Equations	Errors in the tabulated ecliptic longitude values for the Moon and other celestial bodies (computed based on Ptolemy's equations of motion)	Comparison of each relevant table entry with the DE431 ephemerides (see Appendix C)
		Meridian	Error in the true longitude of the reference meridian (the location of most European cities in the 16th century was not accurately known)	Comparison of the almanac's longitude for the reference meridian with its true longitude
		Interpolation	Error of assuming a linear motion of the celestial bodies between two table daily entries	Comparison of the interpolated linear motion with the non-linear motion derived from the DE431 ephemerides
	Observation	Parallax	Error of not discounting the effect of parallax from the measured altitude of the Moon	Comparison of topocentric and geocentric altitude values from the DE431 ephemerides
		Refraction	Error of not discounting the effect of atmospheric refraction in the altitude of the celestial bodies	Comparison of with and without atmosphere values from the DE431 ephemerides
		Geometric	Error of not reducing altitude and azimuth measurements to ecliptic longitude	Comparison of altitude/azimuth values with ecliptic longitude values, as provided by the DE431 ephemerides
Random		Instrument	Measurement error of the instruments likely to have been used to measure altitude (marine astrolabe and quadrant) and azimuth (compass)	Based on standard deviation of measurements made with 16th-century instrument replicas (Malhão Pereira, 1994)
		Local time	Measurement error of determining local time based on the altitude of a celestial body	Based on standard deviation of measurements made by medieval and 15th/16th century astronomers (Steele and Stephenson, 1998)

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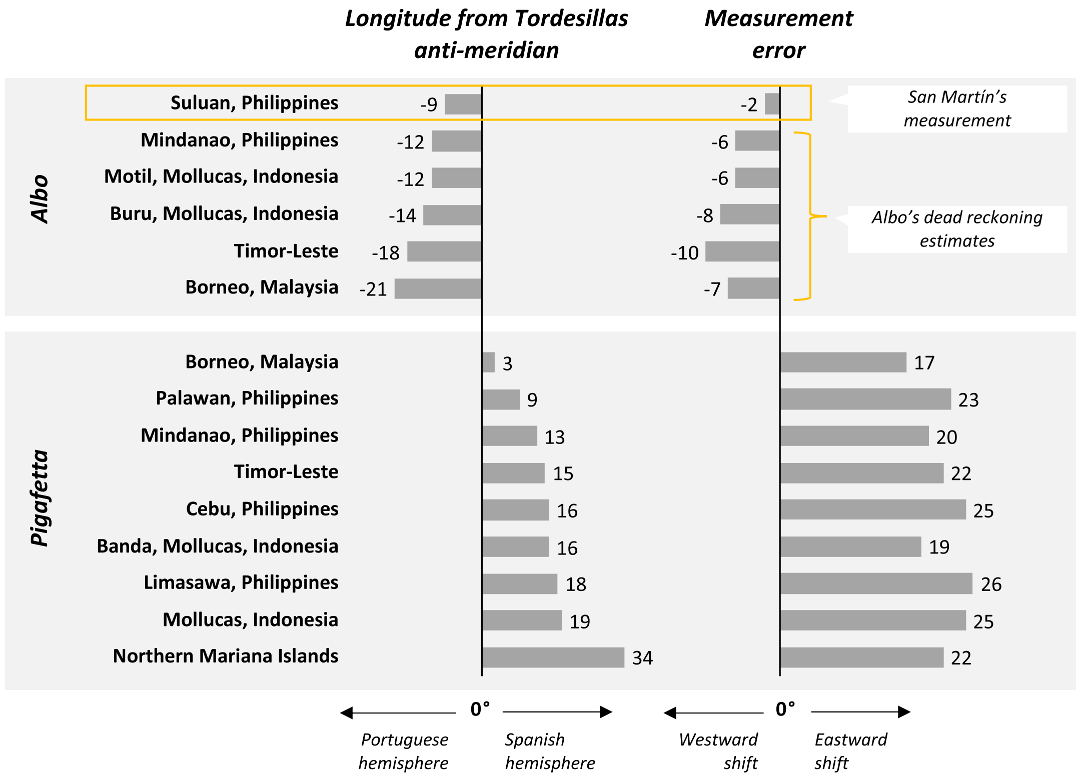

Figure 2. *Albo (1519)[9] and Pigafetta (1922) recorded longitudes in Southeast Asia; values compiled by Laguarda Trías (1975).*

visible to San Martín (as this would also require knowing the correct coordinates where the observations were made and having accurate ephemerides).[10]

None of the chroniclers discusses the last longitude measurement made at Suluan, and there is no known explicit reference to San Martín doing an observation after crossing the Pacific. However, both notes from Francisco Albo (a fellow pilot) and Antonio Pigafetta (an Italian nobleman on the expedition) include the longitude of the Philippines and of several other Southeast Asian locations (Pigafetta, 1922; Albo, 1519), shown in Figure 2. There are two possible explanations for these longitudes: they could be based on astronomical observations, or they could be dead-reckoning estimates. In South America, Albo had approximated the longitude of the Strait of Magellan by estimating how much the fleet had travelled west since San Martín had measured the longitude of San Julían. Dead reckoning is reasonably accurate across short distances but would have been utterly useless after a three-month Pacific crossing.[11] Instead, Albo likely based his dead-reckoning longitude estimates for the fleet's Southeast Asian ports of call on an initial astronomical observation at Suluan, which could only have been made by San Martín.

There is one hurdle left, though: Albo's and Pigafetta's reported longitudes do not match, with the Greek pilot placing all the Southeast Asian islands on the Portuguese hemisphere, and the Italian nobleman putting all of them on Spain's side. There are at least four good reasons to think that Albo's

[9] At Suluan, Albo does not specify which meridian he is using as a reference for the reported longitude. On the next readings, however, he clarifies that he is using the demarcation line as the prime meridian. Pigafetta also references the demarcation line, placing it 30° west of the Canary Islands (47° west of Greenwich). The chart (Reinel and Reinel, c1519) that Magellan used to present his case to the Spanish king, copies of which were certainly taken aboard, also placed the demarcation line 30° west of the Canary Islands.

[10] As detailed in Sections 5–8, the aspects reported by Herrera, Barros and Castanheda were indeed visible, as confirmed by the DE431 modern ephemerides.

[11] Before longitude measurements could be reliably made aboard, pilots would use compass bearings, latitude readings and estimated distance travelled to triangulate East–West progress. Because each new estimate was relative to the previous one, errors quickly compounded over long journeys, particularly if the course and the winds had changed often.

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numbers are the original ones, while Pigafetta's have been doctored. First, the Greek pilot's longitude errors increase as they get further away from Suluan (as one would expect from compounded dead-reckoning estimates), while the Italian's errors are seemingly random. Second, it is easier to believe Albo, known for steering clear of politics, than it is to trust Pigafetta, who gifted the Spanish king with a copy of his diary. Third, Albo's notes only resurfaced in the 19th century, long after Portugal and Spain's claims to the Spice Islands had been forgotten. And fourth, Barros reports that one of the fleet's sailors confessed to doctoring longitudes that did not support Spain's claims:

> And because they saw that the calculations of the astrologer and the estimates based on the route they had taken were more favourable to us [Portugal] than to them [Spain], they placed the lands along their route favourably to them and not according to Andrés de San Martín. And because this, and other things, were done maliciously, one of them, named Bustamante, confessed on his deathbed. He was sailing in one of our ships from Malacca to India, but died enroute in the Maldives, as he was very sick. And in his testimony, to ease his conscience, he declared that regarding some of the measurements made by the Castilian in the Moluccas, he gave false values to be favourable to them.[12]

4. Sixteenth-century methods for determining longitude

The gold standard for measuring longitude in the 16th century (and long before that, at least since the Ancient Greeks) resorted to lunar eclipses. Such an eclipse happens when the Earth passes between the sun and the moon, casting a shadow onto the latter. Two observers standing in different locations would take note of the local time at which they had witnessed the lunar eclipse, and then convene to convert the time difference into a longitude difference. To improve accuracy, the two observers could time the start of the eclipse (i.e. the moment when the Earth's shadow appears on the moon) and its end (i.e. when the Earth's shadow disappears) and average the results. The main source of error for this method was the determination of local time, usually derived from the altitude of a celestial body (see Section 2). Alternatively, a solar eclipse (i.e. when the moon passes in front of the sun) could be used, although these were more cumbersome than lunar eclipses, as the astronomers needed to account for parallax (due to its proximity to the Earth, the moon's altitude in the sky changes from location to location, so measurements need to be reduced to geocentric coordinates to be comparable).

In theory, measuring longitude from a lunar eclipse or a parallax-corrected solar eclipse was reasonably accurate, with an error of about 3 degrees.[13] In practice however, and as Figure 1 illustrates, longitude readings seldomly achieved such accuracy.

During the expedition, San Martín had to deal with two additional complications: one, eclipses are rare (typically only a handful per year)[14] and thus a poor method for positioning a moving fleet; two, 16th-century European ephemerides only predicted the timing of eclipses visible in Europe, so he could not arrange for simultaneous observations with a colleague back home.

Rui Faleiro, a Portuguese astronomer with whom Magellan had initially partnered to present his plan to the Spanish king,[15] had prepared a treatise that described three alternative methods of measuring longitude that circumvented these limitations.

Faleiro's preferred method, which took up a sizeable portion of the treatise, was devoted to the chimera of using magnetic declination to measure longitude, a method first described by the Portuguese pilot

[12] *E porque viam por estas operações do astrólogo, e assim por suas singraduras e estimativa ao modo da sua arte, ser mais em nosso favor que no seu, situavam as terras da derrota a seu propósito, e não segundo o que achava ele, Andrés de San Martin. E de estas, e outras cousas serem feitas com malicia, testemunhou à hora de sua morte um deles, per nome Bustamante, o qual, indo em ura navio nosso de Malaca pera a índia, foi ter as Ilhas de Maldiva, onde faleceu por ir muito enfermo. E no seu testamento disse que, por descargo de sua consciência, declarava que tal cousa e tal, em alguns instrumentos que os castelhanos tiraram era Maluco sobre aquele negócio, ele testemunhara o contrario da verdade, porque o fazia em seu favor.* (Barros, 1628, Folio 147)

[13] Assuming each observer determined local time with an error of 9 min (see Section 2), the error of their relative longitude measurement would be 13 min ($\sqrt{9^2+9^2}$), or ~3°.

[14] Although he was lucky enough to observe one in San Julian, see Section 6.

[15] He was later side-lined from the enterprise, and San Martín assumed his duties as the fleet's astronomer.

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João de Lisboa in 1514.[16] Knowing that magnetic declination was zero in the Canary Islands,[17] Faleiro postulated that it varied linearly along the East–West axis and could thus be used to measure longitude. While sometimes there is indeed a local correlation between magnetic declination and longitude (for instance, Portuguese pilots leveraged it to estimate how far they were from the Cape of Good Hope), it breaks down across long distances, as magnetic declination is caused by the random arrangement of iron ores in Earth's depths.

Faleiro did not put much effort into describing the other two methods of measuring longitude, devoting little more than a couple of sentences to each. One of them aimed to take advantage of the fact that the moon's orbital plane is tilted 5.1° relative to the ecliptic. On every revolution of the moon around the Earth, its ecliptic latitude will thus vary between +5.1° and −5.1°, changing 0.75° per day,[18] or 0.03° per hour. In theory, San Martín could compare the local ecliptic latitude with the reference ecliptic latitude given by an almanac, but the method would require a precision (both from the measurement and from the reference almanac) far beyond what was possible in the 16th century.

Faleiro described his third method as follows:

> The conjunction of the moon with one of the fixed stars at a certain time in Seville (or the moon's oppositions to the sun) can be known from almanacs. I will teach you that the number of hours earlier said conjunction is seen in the west corresponds to the longitude difference to Seville [. . .].[19]

The passage hints at the *lunar distances* method but seems to limit it to specific aspects. A few years earlier, a German mathematician named Johannes Werner had provided a broader description of this method, noting that anytime the moon is visible in the sky, its ecliptic longitude difference to a celestial body can be measured and compared with an almanac's computed values for the reference meridian (Werner, 1514). This greatly increases the opportunities of measuring longitude since the astronomer does not need to wait for specific aspects to be visible. This technique, which would come to be known as the lunar distances method, is a generalisation of the several procedures that use the motion of the moon to calculate longitude, e.g. a lunar eclipse (the moon and the sun with the same ecliptic latitude[20] and opposing ecliptic longitude), a solar eclipse (the moon and the sun with the same ecliptic longitude and, for the observer's position, the same altitude),[21] an occultation (the moon and another celestial body with the same ecliptic longitude and, for the observer's position, the same altitude), a conjunction (the moon and another celestial body with the same ecliptic longitude) and an opposition (the moon and another celestial body with opposing ecliptic longitude).

While Werner's text is the first known description of the lunar distances method, it is entirely possible that others before had already arrived at the same conclusion,[22] and that by the 16th century the method was relatively well known among astronomers, at least as a theoretical approach.

San Martín's own words, as reported by Castanheda, reinforce this hypothesis. When Magellan questioned San Martin and the other pilots about Faleiro's treatise, he dismissed all but the lunar distances method, which he stated to be well known among astronomers:

> And the pilots replied by writing that one could not use that treatise nor navigate by it. And such they signed; and the astrologer [San Martín] said the same thing about all the chapters except for the fourth one [a description of Faleiro's description of the lunar distances method, transcribed above, followed]. And about this chapter he said there if there was another way to measure the difference from north to south and east to west he did not know it [this mix-up between latitude differences ('north to south') and longitude differences ('east to west') comes from Castanheda's misquote of

[16]Later included in his *Livro de Marinharia* (Lisboa, 1560).

[17]This seems to have been indeed the case for the Canary Islands in the 16th century (Korte and Constable, 2005).

[18]5.1° per quarter revolution (or 20.4° per full revolution), divided by the 27.3 days it takes the moon to complete a revolution.

[19]*Otrosi, por la conjuncion que yo sé que la luna ha de tener com alguna de las Estrellas fixas á cierta ora en Sevilla, como por quales quier ciertos almanaques se puede saber ó por las oposiciones de la luna que se hazen en grado opósito del sol te puedo enseñar que las oras en que primero es en hocidente la dicha conjuncion, que aquella es la diferencia que estás mas al hocidente que Sevilla* [. . .] (Faleiro, c1519).

[20]Precisely the same for a total eclipse, approximately the same for a partial eclipse.

[21]Idem.

[22]For instance, Fernández-Armesto (2007) argues that it was known since antiquity.

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> San Martín, as noted by Barros (1628)]. And he added many other conjunctions and oppositions, and for more clarity he made a treatise out of it, where he alleged much astrology, and said that such rule was well known among astrologers and cosmographers.[23]

As we will see next in Section 5, San Martín's measurements at Rio de Janeiro may be the first recorded unequivocal use of the lunar distances method in the context of navigation. Molander (1992, 1996, 1997) has hypothesised that Christopher Columbus used this method to track his progress across the Atlantic, a theory that has, in our view, been convincingly dismissed by Pickering (1996, 1997, 1999). Amerigo Vespucci, in one of his letters, claimed to have used the same technique to pinpoint his location in 1499, close to the Venezuelan coast. While some historians defend that Vespucci did indeed apply the lunar distances method (Stein, 1950), the authenticity of this observation has been muddled by the controversy surrounding Vespucci's forged letters,[24] leading some to claim that that he merely copied Columbus's previous longitude estimates (Fernández-Armesto, 2007).

5. Rio de Janeiro: the blunder

The observation at Rio de Janeiro is of particular interest, not because it was accurate (it was not) but because it is the only surviving step-by-step description of how the Spanish astronomer applied this method in practice:

> At Rio de Janeiro on the 17th of December, at 4:30 in the morning, which was 7:30 before noon, we saw the Moon over the eastern horizon, at an altitude of 28°30′, and Jupiter above it, at an altitude of 33°15′. Subtracting the altitude of the Moon from that of Jupiter, we arrived at a difference of 4°45′, so, considering the backward movement of the Moon, the conjunction occurred 9:15 ago, in which the Moon moved the said 4°45′. Deducing it from 16:30 [the hour of the observation relative to the previous day's noon i.e. 4:30 + 12:00], it seems that the conjunction was on Friday the 16th, 7:15 after noon. According to Zacut's tables, it happens 1:20 after noon, in the Salamanca meridian, this Saturday; and on the Seville meridian, 1:12 after noon. And by Regiomontanus's almanac, they believe that it occurs this same Saturday, December 17th, in the Seville meridian, 1:10 after noon. In this meridian, it seems that the conjunction was on December 16th, 7:15 after noon, and hence there seems to be a difference of 17:55 to the Seville meridian [12:00–7:15 + 12:00 + 1:10], from which we infer to be an error in the table's equations of motion, because it is impossible for the longitude to be so large.[25]

The description is arguably confusing, as the typical 16th century's ample use of prose and aversion to negative numbers are further complicated by the several conversions between civil time (in reference to midnight) and astronomical time (in reference to noon). Figure 3 provides a contemporary breakdown of San Martín's calculations, a comparison with the true figures computed using the DE431 ephemerides, and the resulting error signature of the observation.

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[23] *E os pilotos responderam por escrito que não se podia usar aquele regimento, nem aproveitava para se navegar por ele. E assim o assinaram: e o astrólogo respondeu o mesmo a todos os capítulos do regimento que eram trinta salvo ao quarto* [a description of Faleiro's description of the lunar distances method, transcribed earlier, followed]. *E disse a este capítulo que não havia outro caminho para alcançar a diferença de altura de norte a sul a de leste a oeste se não aquele nem ele o sabia. E acrescentou ainda muitas outras conjunções e oposições, e por maior clareza fez sobre isso um tratado em que alegou muita astrologia, e disse que aquela regra era muito sabida por todos os astrólogos e cosmógrafos* (Castanheda, 1554).

[24] See Fernández-Armesto (2007, p. 6) for a summary of the doubts still surrounding the authenticity of Vespucci's letters.

[25] *Estando en este Rio de Genero a 17 de Diciembre, a las quatro horas i treinta minutos de la manana, que eran siete horas, i treinta minutos antes de Mediodia, se vio la Luna sobre el Oriçonte Oriental, en altura de 28 grados, i treinta minutos, i Júpiter elevado sobre ella, en altura de 33 grados, i 15 minutos: deduciendo el altura de la Luna de la de Júpiter, se halló de diferencia 4 grados, i 45 minutos, que bolviendo atrás con el movimiento de la Luna, a ponerse en la conjuncion de Júpiter, 9 horas, i 15 minutos: en cuio espado movió la Luna los dichos 4 grados, i 45 minutos: deduciendolos de las 16 horas, i 30 minutos de la Nota, parece que fue el Viernes 16 de Diciembre, a las 7 horas, i 15 minutos después de Mediodia. Viene por las tablas dei Caçuto a la una hora, y 20 minutos des- pues de Mediodia, en el Meridiano de Salamanca, este dia Sabado; i en el meridiano de Sevilla, a la una hora, i 12 minutos despues de Mediodia. Y por el Almanac de Juan de Monte-Regio hallaron que vino a ser dicho dia Sabado 17 de Diciembre, en el Meridiano de Sevilla a la una hora, i 10 minutos despues de Mediodia; i segun esta conjuncion, que parece que fue en este Meridiano, a los 16 de Diciembre, siete horas i quince minutos despues de Mediodia, pareció haver de diferencia de este Meridiano al de Sevilla, 17 horas, i 55 minutos; de lo qual infirieron haver error en la equadon de los movimientos en las tablas, porque es imposible ser tanta la longitud* (Herrera, 1601, p. 132).

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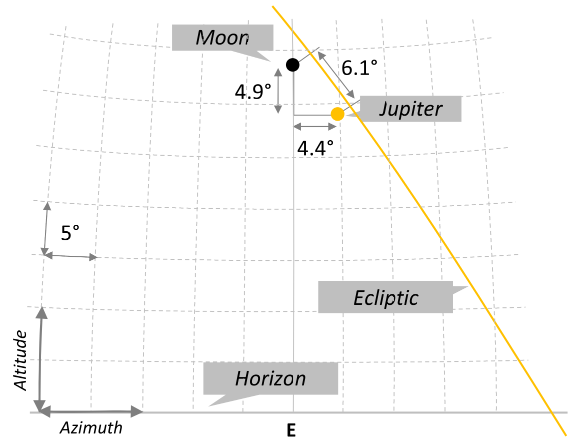

	San Martin	True	Compounded longitude error	
Local time *(mean time)*	4:30 am	4:26 am	Local time error	+0.9
Altitude difference *(apparent)*	4.8°	4.2°	Instrument error	-16.9
Altitude difference *(refraction corrected)*		4.2°	Refraction error	0.0
Altitude difference *(parallax corrected)*		4.9°	Parallax error	-16.9
Ecliptic longitude difference		6.1°	Geometric error	-5.8
Relative motion	0.5° per hour	0.5° per hour	Interpolation error	-0.9
Time to conjunction *(Rio, relative to noon)*	- 16:45	+ 4:05	Process error	-272.1
Time to conjunction *(Salamanca, relative to noon)*	+1:18	+6:45	Equations error	+78.6
Salamanca-Seville time difference	0:08	0:01	Meridian error	+1.7
Longitude difference	**268.8°**	**37.2°**	**Compounded error**	**-231.5**

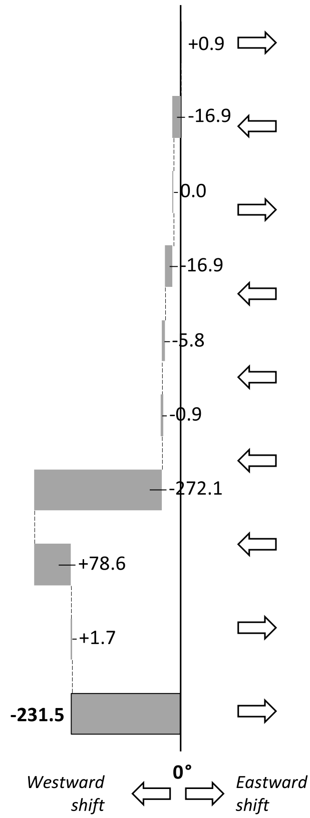

Figure 3. *Observed sky (DE431)*[26] *and error signature of San Martín's longitude measurement at Rio de Janeiro, 17 December 1519.*

The Spanish astronomer first mentions a local time of 4:30 in the morning. He does not specify how he computed it but, as detailed in Section 2, he likely measured the altitude of the moon or of a

[26]The provided true figures are for 4:26 a.m., when the moon was exactly over the East horizon, consistent with what San Martín refers in his text. Instead of assuming that the astronomer was able to measure the moon's azimuth with no error, one could also compute the true figures for 4:30 a.m. (i.e. assuming no local time determination error) or for 4:14 a.m. (i.e. assuming no altitude measurement error). All three scenarios produce the same compounded total error, with only minor differences in the distribution of individual errors.

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bright star and converted it to local time using a table. Since he would be comparing local time with the reference meridian time provided by an almanac, he also had to convert solar time into mean time.[27] San Martín's local time determination was off by less than 4 min, well within Regiomontanus's and Walther's standard deviation of 9 min.

He then measured the altitude of the moon and Jupiter, noting that they differed by 4.8° (an overestimation of 0.6°, somewhat higher than the 0.4° that could be expected from a 16th-century quadrant or marine astrolabe).[28] San Martín does not mention a refraction correction, but the resulting error is negligible.[29]

However, he also does not mention a correction for lunar parallax, the effect of which was far from insubstantial: an error of 0.7° in altitude, resulting in a ~17° westward shift in longitude. There is no evidence that San Martín had sufficient knowledge of spherical trigonometry to calculate parallax from the ground up, and the Zacut almanacs only provided correction tables for the latitude of the reference meridian (and are known to contain several copying errors and omissions, see Chabás and Goldstein, 2000). This was certainly not an exception in the context of 16th-century navigation, where the first known method to correct for lunar parallax is from 1634, by the pen of Jean-Baptiste Morin, a French mathematician. Even then, Morin's proposal was deemed impractical and never saw much use aboard ships (Grijs, 2020).

San Martín's next step further illustrates that his knowledge of astronomy was perhaps not quite up to the level of land astronomers. Instead of converting the altitude difference of the moon and Jupiter into an ecliptic longitude difference, San Martín assumed that the ecliptic longitude difference was the same as the altitude difference. This would have been approximately the case if the two bodies shared the same azimuth (which happened only several hours later, when the sun was already up and Jupiter was no longer visible),[30] but at the time of the observation there was still an azimuthal gap (4.4°) that led to a material geometric error, as the ecliptic longitude difference (6.1°) and the altitude difference (4.9°) differed substantially (Figure 3).

It seems unlikely that San Martín was unaware of the difference between altitude and ecliptic longitude. His timing of the Rio de Janeiro observation (and that of subsequent observations, see next sections) suggests instead he looked for cases when such an approximation can be reasonably made (in his own words, '*we saw the Moon over the eastern horizon* [. . .] *and Jupiter* ***above it***'), perhaps not realising that even small azimuthal differences will introduce large errors. If that was indeed the case, we can infer three more characteristics about the Spanish astronomer's method: (i) He could only measure vertical and horizontal angles, not oblique ones (indeed, the ship's inventories included marine astrolabes and quadrants – capable of measuring vertical angles – and compasses – capable of measuring horizontal angles – but not cross-staffs); (ii) he did not have with him a planispheric astrolabe loaded with a template for the latitude of Rio de Janeiro, which he could have used to convert altitude into ecliptic longitude; and (iii) he was not sufficiently versed in spherical trigonometry to make that conversion himself (not at all surprising in the 16th century, particularly for someone who was both a pilot and an astronomer, rather than a dedicated land astronomer).

San Martín then converted his approximation of the moon and Jupiter's ecliptic longitude difference into the time to conjunction (when the two bodies would have the same ecliptic longitude) by applying a relative motion of 0.5° per hour (an accurate estimate, likely computed by interpolating daily ecliptic longitude entries from the almanacs).

While the astronomer's estimate of the moon and Jupiter's relative motion was accurate, he applied it in reverse, i.e. instead of concluding that the conjunction had not yet happened (since the moon travels

[27] Sixteenth-century equations of time differed from those used today, most notably in the fact that they used only positive values. This has been considered in our comparisons with modern ephemerides.

[28] Compounded error of two measurements (altitude of the moon and altitude of Jupiter), each one with an error of 0.26° (Section 2).

[29] Refraction makes celestial bodies appear higher than they are, since the Earth's atmosphere bends light. The phenomenon was known since at least Ancient Greece, but Tycho Brahe (1546–1601) was the first to measure it with some accuracy. In any case, for San Martín's observation, the altitude difference between the moon and Jupiter was small, so the respective effects of refraction mostly cancelled each other.

[30] It would be precisely the case if the two celestial bodies were on a plane parallel to that of the ecliptic. For this observation, the moon and Jupiter had a 0.5° ecliptic latitude difference.

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east in the sky relative to other planets), he noted instead that it had '*occurred 9:15 ago*' (16:45 ago relative to noon). The source for this misstep seems to be a lapse in noting the position of the celestial bodies (San Martín states that the moon was below Jupiter, but it was the other way around, see Figure 3). Perhaps, in the hustle and bustle of taking altitude measurements, the astronomer swapped the readings for the moon and Jupiter. One hour later, the sun had already risen, hiding Jupiter and preventing him from double-checking the observation before continuing with his calculations.

The impact of this lapse is enormous (a ~270° westward shift in longitude), but it is one that is disarmingly easy to make, particularly for a Northern Hemisphere astronomer making his first observations in the Southern Hemisphere. To our knowledge, this mistake has remained undiscovered until know.

San Martín's final step was to find, from the almanacs, the time to conjunction in Seville, and compare it with his calculated time to conjunction at Rio de Janeiro (a 17 h:55 m difference, or about 269° of longitude). This final step introduced two more sources of error, none of which were the astronomer's fault: a small one (+1.7° longitude) because the longitude difference between Salamanca (the reference meridian of the 1502 Zacut almanac) and Seville (San Martín's chosen reference meridian) was not well known; and a large one (+78.6°, from the almanacs faulty equations of motion).

San Martín was acutely aware that the resulting longitude reading made no sense (after all, it placed Rio de Janeiro more than halfway around the world, in the middle of the Indian Ocean), but blamed the ephemerides for the sizeable error ('*we infer to be an error in the table's equations of motion*'). While his almanacs did indeed misplace the moon and Jupiter by a wide margin, San Martín's own blunder was the largest source of inaccuracy.

Even without the blunder, the Spanish astronomer would have overestimated Rio's longitude by >40°. However, could this simplified lunar distances method, even if San Martín made no subsequent improvements to it, explain the accurate longitude measurements at San Julían and Suluan? And could it have produced accurate results for the remaining three measurements, the results of which have been lost? The next sections assess these questions.

6. San Julían: the solar eclipse

On 17 April 1520, San Martín positioned Port San Julían, in Patagonia, 61° west of Seville, a remarkably accurate measurement (0.6° error). The astronomer's original notes for this observation have been lost, but its result was preserved by Castanheda (1554), and both he and Barros (1628) report that it was based on a solar eclipse. As detailed in Section 3, Castanheda and Barros had no motive to falsify this observation, nor the means to do so even if they had wanted to. Also, modern ephemerides confirm that a full solar eclipse was visible from San Julían on that date.

However, to accurately measure longitude with a solar eclipse (with an error margin of about 3° in the 16th century, see Section 4) requires two observers and correction for parallax. San Martín surely did not use two observers, as he calculated the longitude while still at San Julían and the eclipse was not visible in Europe. It is also unlikely that he corrected for parallax since he failed to do so in Rio de Janeiro (Section 5).

As this solar eclipse was not included in Zacut's almanacs (which included only eclipses visible in Europe), Laguarda Trías (1975) suggested that San Martín's accurate measurement was instead the result of multiple observations using the lunar distances method, made over the many months the fleet stayed in San Julían, weathering the winter. While that is certainly conceivable, there is no evidence that the Spanish astronomer made multiple observations. On the contrary, both Castanheda and Barros unequivocally state that the measurement was based on a solar eclipse.

There is another explanation, one that is both simpler and fits the available facts better: San Martín could have applied his simplified lunar distances method to the solar eclipse. After all, a solar eclipse is merely a specific type of conjunction (one where the sun and the moon have not only the same ecliptic longitude but, from certain locations, also the same topocentric altitude), and was tabulated as such in Zacut's almanacs (see Appendix B). In other words, while the Spanish astronomer had no way of

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knowing about the incoming eclipse, he knew that the sun and the moon would be in conjunction and planned for an observation, just as he had done at Rio de Janeiro.

Figure 4 shows the error signature of applying San Martín's simplified lunar distances method to the San Julían observation. Several differences vis-à-vis the error signature for Rio de Janeiro are worth highlighting. One, since we do not know the results of the astronomer's determination of local time, its associated error is presented as a range with a standard deviation of 2.2° of longitude, or 9 min (see Section 2). Two, there are no instrument or geometric errors, since the sun and the moon had the same ecliptic longitude (save for the impact of parallax, captured in the parallax error). Three, the two almanacs San Martín had with him produced different results.

As detailed in Appendix B, the discrepancy between the two almanacs was caused by a typographical error in the Toledo almanac. It is interesting to note that this error made this almanac more precise (+6° error vs. +19.8° for the Salamanca almanac), but San Martín could only have arrived at his accurate longitude measurement (+0.6° error) if he used the Salamanca tables (+0.1 ± 2.2°) and discarded the Toledo results (−13.8 ± 2.2°).

There are good reasons to assume he did this, either because he detected the typographical error (by noting that the daily entry was an outlier, see Appendix B), or by assessing the merits of the computed longitudes to determine which set of tables was 'correct'. Using the Toledo almanac, he would have placed San Julían 76° west of Seville and 37° west of the mouth of the La Plata River (Figure 5), the last plotted location on the charts taken aboard.[31] That would have been clearly an exaggeration, as the fleet had been mostly travelling down the American coastline and could not have progressed westward by almost as much as on the Atlantic crossing. With the Salamanca tables, the distance to the La Plata River reduced to 23°, a more reasonable figure that San Martín may have nevertheless suspected to still be inflated (the true difference is 12°, as 16th-century charts shifted the La Plata River eastward by 11°).

In summary, San Martín's uncannily accurate San Julían longitude measurement is consistent with the use of the Zacut (Salamanca) almanac and the application of the astronomer's simplified lunar distances method to the solar eclipse observation. These results also suggest that the nearly perfect measurement is merely the result of a fortuitous cancellation of large errors. In fact, if the astronomer had accounted for parallax, his measurement would be less accurate, as the remaining errors could not offset the almanac's flawed equations of motion.

Next, Section 7 assesses whether this cancelation of large errors can also explain the accurate Suluan longitude measurement.

7. Suluan Island: bittersweet results

As detailed in Section 3, San Martín also seems to be the author of the Suluan longitude reading included in Francisco Albo's logbook, which missed the true location of this small island in the Philippines by a mere 2°. The aspect that San Martín observed is not known but, in the <24 h the fleet spent at this island, there were two occasions where they sky was fitting to apply his simplified lunar distances method (i.e. with two celestial bodies at approximately the same azimuth): one at daytime (1:13 p.m.), with the moon and the sun; and another one at night-time (9:28 p.m.), with the moon and Mars (Figure 6).

The latter cannot explain San Martín's result, as its error signature is well off range (+105.9 ± 11.3°, 16th-century almanacs were notoriously inaccurate in predicting the position of Mars). However, the moon and sun observation had just the right characteristics to balance the errors of San Martín's simplified lunar distance method, suggesting that this was how he arrived at his remarkably accurate longitude measurement (−2° off Suluan's true position, well within the observation's −3.5 ± 11.3° error signature).

The Spanish astronomer had no way of knowing how accurate these readings were, but he would have had at least the satisfaction of knowing that they looked plausible, unlike the ones at Rio de Janeiro.

[31] Likely based on the Kunstmann IV chart (Reinel and Reinel, c1519) that Magellan had presented to the Spanish king to make his case (Gaspar, 2019).

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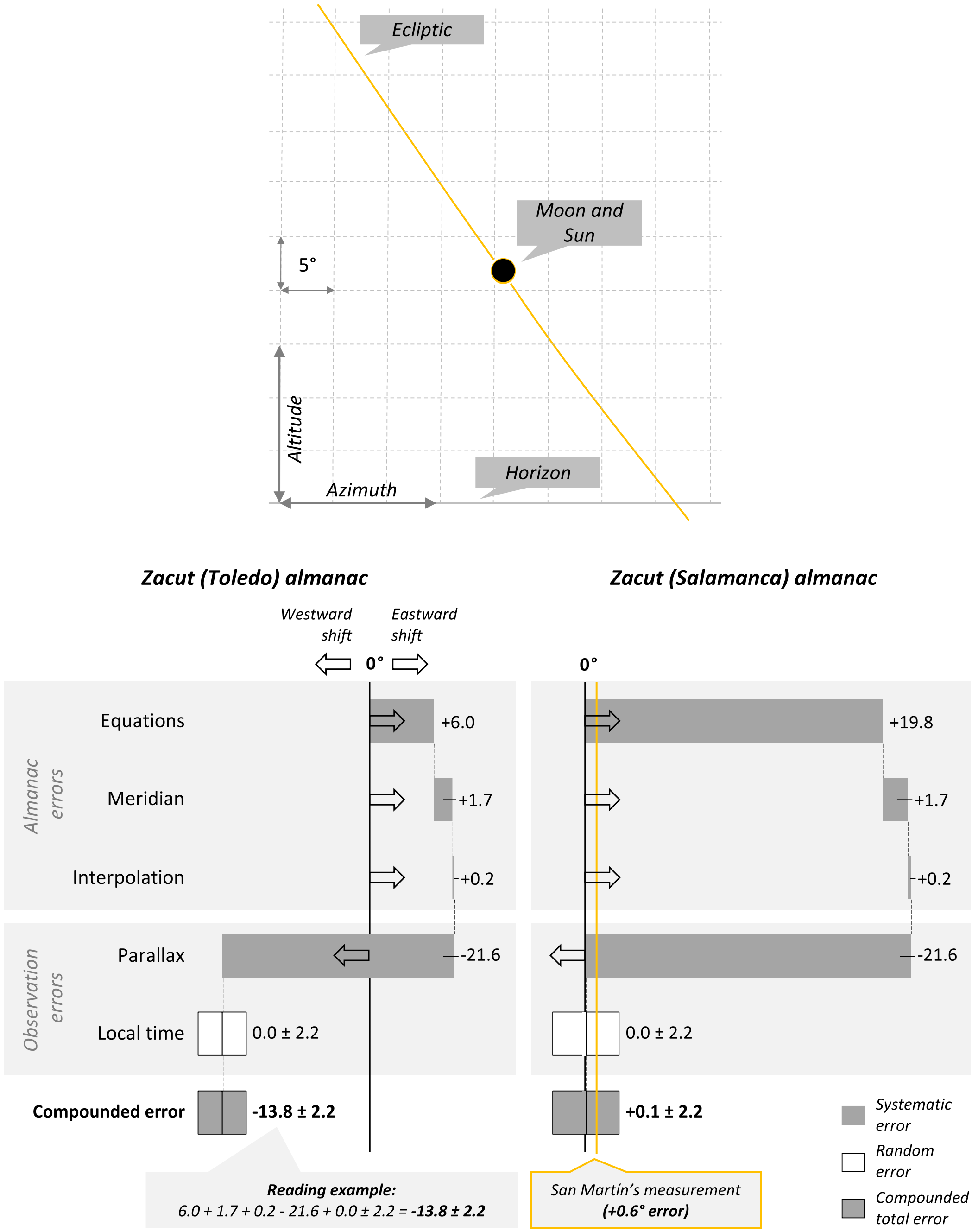

Figure 4. *Observed sky (DE431) and error signature of San Martín's longitude measurement at San Julián, 17 April 1520 (all errors in degrees of longitude); refraction error (<1 arcminute) not shown.*

It was a bittersweet result, however, as it placed Suluan (and likely the nearby Spice Islands) already on the Portuguese hemisphere. One of the key objectives of Magellan's expedition – to prove that these islands could be claimed by Spain – was crumbling to the ground.

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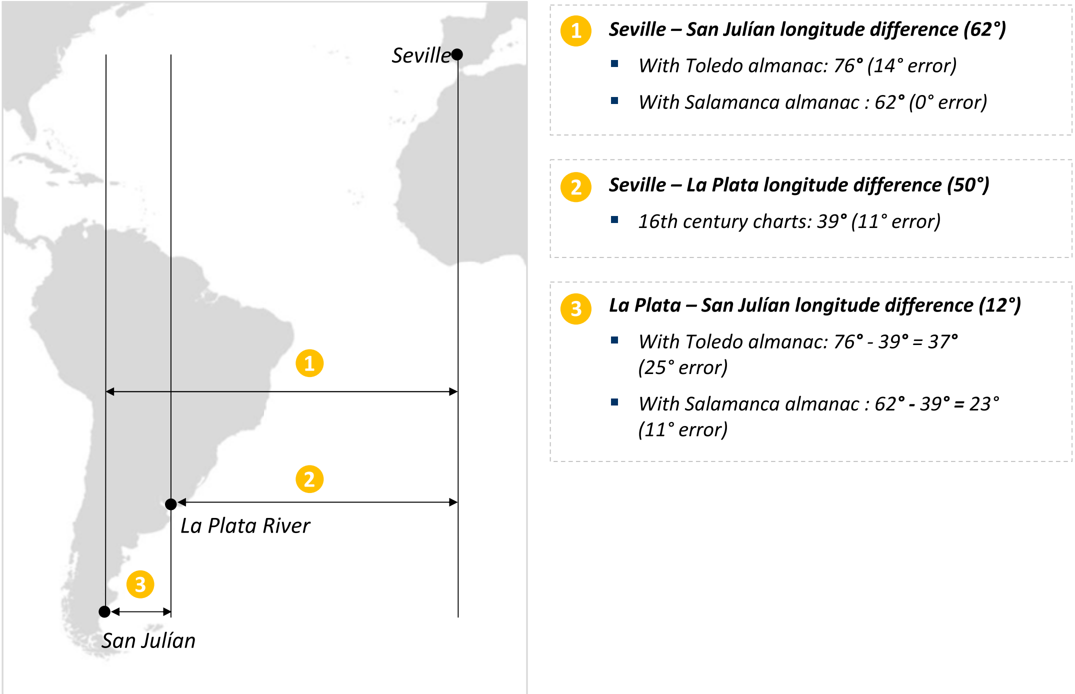

Figure 5. *Toledo ephemeris vs. Salamanca ephemeris longitude results for San Julián.*

8. The lost measurements

While still in South America, Barros's records show that San Martín made three more attempts to measure longitude. While the calculations and results have unfortunately been lost, the dates and observed celestial bodies have survived: at the mouth of the La Plata River, on 31 January 1520, an opposition of the moon and Venus; at the Gulf of San Matias, on 24 February 1520, an opposition with the sun;[32] and after entering the Pacific Ocean, on 23 December 1520, another opposition with the sun (with no land in sight, the latter had to be done aboard a swaying ship, adding yet another difficulty to the process).

These three configurations are interesting, because they suggest that San Martín attempted a variation of his simplified lunar distances method: instead of seeking celestial bodies at approximately the same azimuth, he looked for those at approximately the same altitude (Figure 7). And rather than using a marine astrolabe or a quadrant to measure altitude, he would have used a compass to measure azimuth. Using a compass to measure the horizontal angle between two celestial bodies was well within San Martín's competences both as a pilot and as an astronomer. Sixteenth-century compasses were often equipped with sighting devices that allowed for more accurate azimuth readings (1° markings instead of the 11.25° intervals of a regular compass), and Faleiro had also included in his treatise instructions on how to measure the azimuth of celestial bodies (as a step for calculating magnetic declination).

Unfortunately for San Martín, the moon and the reference body were far apart on all three observations, meaning that assuming their ecliptic longitude difference was approximately the same as their azimuthal difference introduced a large geometric error, rendering the measurements useless. As for the Rio de Janeiro observation, San Martín could have avoided this error by reducing the observations to ecliptic longitudes, either with spherical trigonometry or a planispheric astrolabe with latitude-specific templates. However, it would be contrived to assume he applied such a correction, since he failed to

[32] Strictly speaking, the aspect predicted by the Zacut almanacs was a quadrature (90° ecliptic longitude difference; see Figure 7) rather than an opposition (180° difference). San Martín seemed to use the 'conjunction' term less strictly to refer to any aspect that was not a conjunction, and Faleiro appeared to do the same thing (see Section 4).

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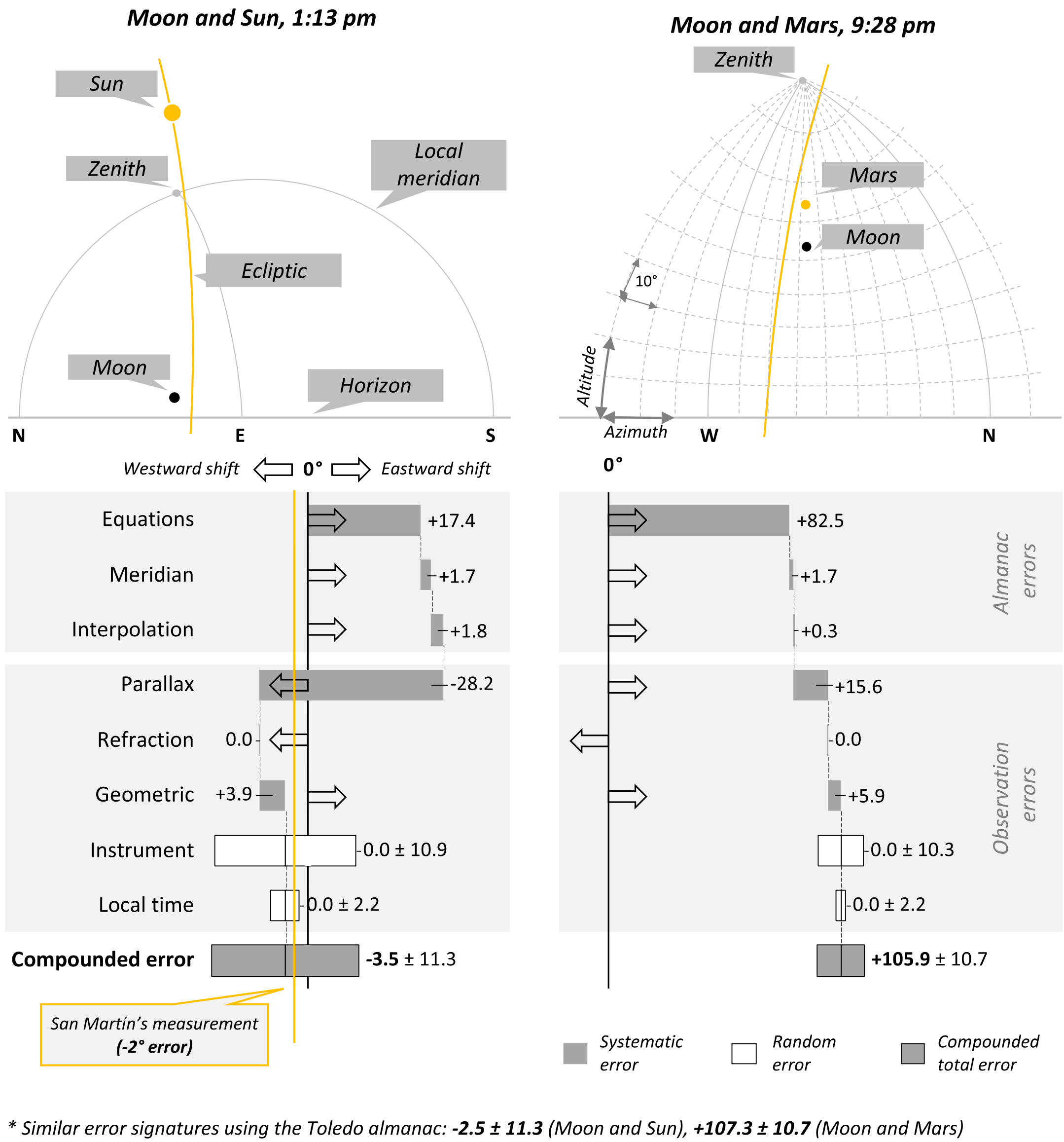

Figure 6. *Observed sky (DE431) and error signature in degrees for the two possible longitude measurements at Suluan, 16 March 1521, using the Zacut/Salamanca ephemeris (the error signatures of the Zacut/Toledo ephemeris are similar: -2.5 ± 11.3 for the moon and sun observation and $+107.3 \pm 10.7$ for the moon and Mars one).*

do so at Rio de Janeiro. Barros's derisive remarks about the Spanish astronomer's measurements (see Appendix A) further suggest that these lost measurements did not produce credible results. The pilot Albo, who often made use of San Martín's measurements as a baseline for subsequent dead reckoning estimates (see Section 3), also did not reference them.

9. Final remarks

To conclude, San Martín made at least six longitude measurements during the expedition, two of which had an accuracy that would take other navigators centuries to match. We have computed the error

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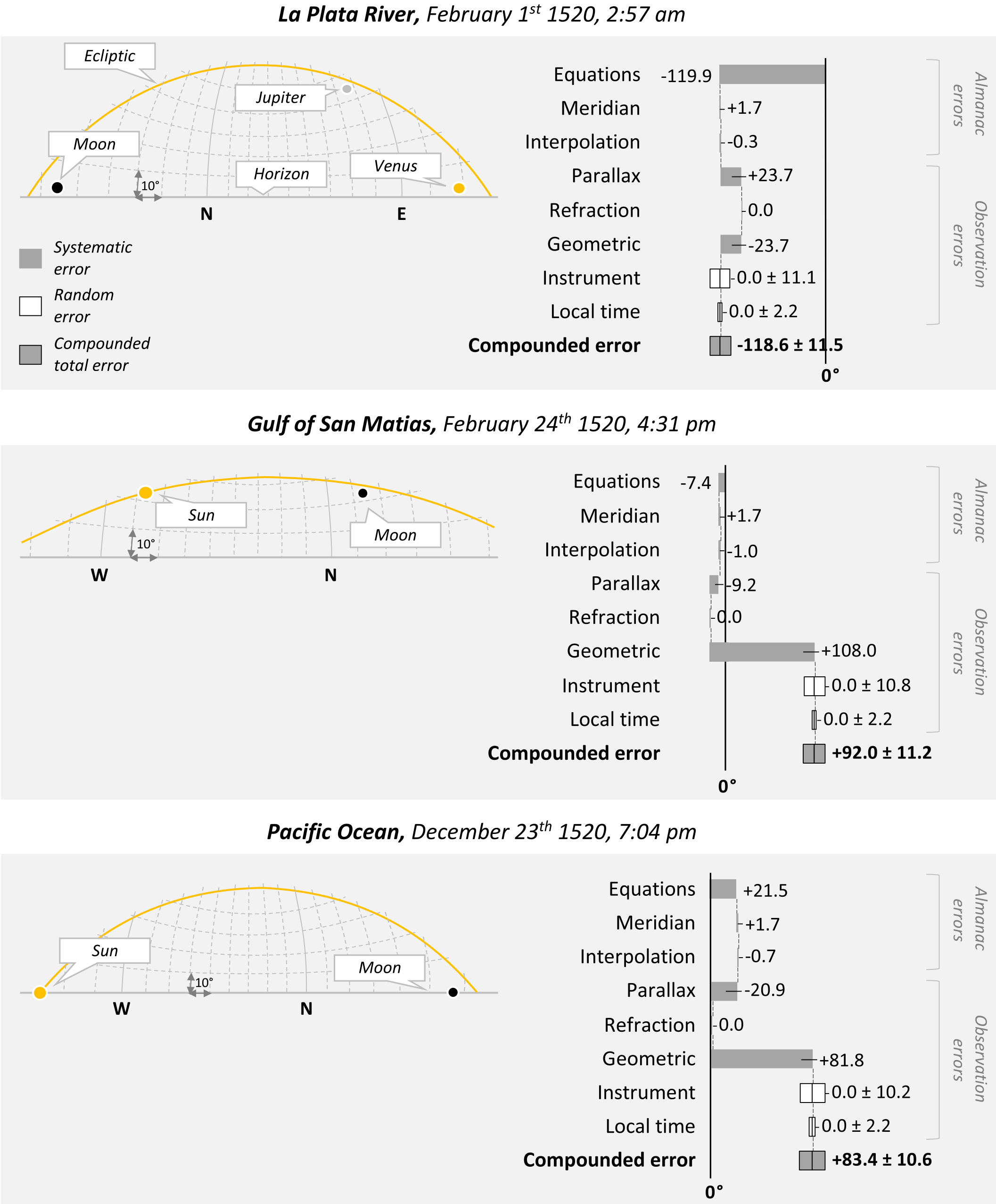

** Similar error signatures using the Toledo almanac: **-117.1 ± 11.5** (La Plata River), **+94.0 ± 11.2** (Gulf of San Matias), **+85.2 ± 10.6** (Pacific Ocean)*

Figure 7. *Observed sky (DE431) and error signature in degrees of San Martín's remaining longitude measurement in South America, using the Zacut (Salamanca) ephemeris. The error signature of the Zacut (Toledo) ephemeris is similar: −117.1 ± 11.5 (La Plata River), +94.0 ± 11.2 (Gulf of San Matias), +85.2 ± 10.6 (Pacific Ocean).*

Table 3. *Summary of error signatures.*

Date	Location	Error of San Martín's measurement (°)	Error signature of measurement (°)
17/12/1519	Rio de Janeiro	−231.5°	−231.5 ± 0.0°
01/02/1520	La Plata River	*not known*	−118.6 ± 11.5° (Zacut – Salamanca)
24/02/1520	Gulf of San Matias	*not known*	+92.0 ± 11.2° (Zacut – Salamanca)
17/04/1520	San Julian	+0.6°	+0.1 ± 2.2° (Zacut – Salamanca)
23/12/1520	Pacific Ocean	*not known*	+83.4.0 ± 10.6° (Zacut – Salamanca)
16/03/1521	Suluan Island	−2.0°	−3.5 ± 11.3° (Moon and Sun)

signatures of each one of these observations (Table 3), which support the following findings (which are, to our knowledge, original):

1. A comparison with the DE431 modern ephemerides confirms that the celestial configurations reported by Herrera, Barros and Castanheda as being used by San Martín were, in fact, visible at the dates and locations where the fleet was.
2. At Rio de Janeiro, San Martín applied a simplified lunar distances method. The two key simplifications were not reducing altitude differences to ecliptic longitude differences (by assuming that the ecliptic was approximately vertical) and not correcting for lunar parallax.
3. His erroneous longitude measurement (which shifted Rio de Janeiro westward by >230°) was not due to these simplifications, but mainly the result of a process error (applying the moon–Jupiter relative motion in reverse).
4. San Martín likely used this simplified lunar distances method, without making any improvements, to achieve the accurate measurements at San Julián and Suluan. This hypothesis is consistent with both the extant documentation and the computed error signatures (Table 3).
5. Such accuracy was merely the result of fortuitous cancelations of large errors, as the method is inherently unreliable.
6. For the three lost measurements, San Martín seems to have attempted a variation of his lunar distances method, equating azimuthal differences to ecliptic longitudinal differences (i.e. assuming that the ecliptic was approximately horizontal). This introduced a large geometric error that rendered the measurements useless, a conclusion that is consistent with what can be inferred from extant documentation.
7. All of San Martín's measurements appear to be based on two Zacut almanacs (1498, 1502) and are achievable using only pilot instruments available aboard (marine astrolabes, quadrants and sighted compasses). There is no evidence that he used specialised astronomer tools such as planispheric astrolabes, or that he was well versed in spherical trigonometry.

San Martín's simplifications of the lunar distances method indicate that his experience as an astronomer was not quite up to the level of well-known land-based astronomers. That is hardly a surprise, though, as San Martín was a pilot first and an astronomer second (García, 2020). This combination of skills was exceedingly rare in the 16th century, and better-known royal pilots from the *Casa de la Contratacíon*, such as Amerigo Vespucci (San Martín's friend and mentor), did not possess it.[33] Also, one can only image the hurdles of making astronomical observations during an expedition marred by death, extreme weather, wrecks and mutiny.[34]

[33] Even if assuming that the letter is not a forgery and Vespucci did not copy his longitude measurements from Columbus (Fernández-Armesto, 2007), the navigator's version of the lunar distances method is vastly oversimplified (Stein, 1950).

[34] San Martín was reportedly tortured for his role on the San Julían mutiny (García, 2020). It seems unlikely, though, that someone that endured *tratos de cuerda* would be able to make astronomical observations a couple of weeks after. This horrible form of punishment, quite popular in the 16th century, involved tying the subject's arms behind their back with a rope, which was then used to hoist them. Once dangling in the air, the victim was suddenly dropped, inevitably dislocating their shoulders and often leaving them disabled for life.

Paradoxically, a more capable astronomer would have produced *less* accurate results at San Julían and Suluan, since correcting for geometric and parallax errors would no longer offset the large almanac errors. Rather than few and far between, these large almanac errors were ubiquitous: Gaposchkin and Haramundanis (2007) found that 16th-century ephemerides had a 0.55° RMS error in the ecliptic longitude of the moon, which turns into a c15° error in longitude.

More than 200 years would pass before the lunar distances method became something more than a throw of the dice. Multiple European naval powers, including Spain, France and Britain, launched sizeable prizes in exchange for a practical way to measure longitude aboard ship, attracting contributions from some of the Renaissance's brightest minds. Eventually, the octant and the sextant replaced the quadrant and the astrolabe, substantially reducing the error of angle measurements. Ptolemy's geocentric representation made way for Copernicus's heliocentric model, putting the Earth where it belonged. Kepler determined that the planets moved in eclipses rather than in circles, and Newton discovered that it was gravity that made them move. The Royal Greenwich Observatory and other stations hosted rigorous astronomical observations that, coupled with the improved planetary equations, finally allowed for more accurate ephemerides.

Were San Martín and Magellan aware of the inescapable unreliability of 16th-century longitude measurements, or were they truly convinced that Suluan – and most probably the nearby Spice Islands – were already in the Portuguese hemisphere? After the tribulations of his multiple measurements, it is hard to imagine San Martín putting enough faith in his results to categorically state that they were a mere 9° away from the demarcation line. Even if he was unaware of the magnitude of parallax and geometric errors, he had complained several times about the inaccuracy of the ephemerides and may have noted at least one occasion where his almanacs contained typographical errors. As for Magellan, there is little doubt about his respect for science, patent in the case he put forward to the Spanish king and on his insistence in having an experienced astronomer aboard. However, there is also ample evidence of his stubbornness and willingness to pursue his goals at any cost. Would he throw in the towel because a number did not agree with him?

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Appendix A: The almanacs used by San Martín

By the time San Martín set sail in Magellan's expedition, two types of ephemerides enjoyed wide circulation in Europe: perpetual almanacs, such as those made by Zacut (1498, 1502); and yearly almanacs, typically in the format first popularised by Regiomontanus and later adopted by Stöffler and Pflaum (1499). While both types were based on the same Ptolemaic equations of motion (Gingerich, 2004) and had similar accuracy (Gaposchkin and Haramundanis, 2007), they offered different advantages and shortcomings. Yearly almanacs were easier to read (each page covered a specific year and month, providing side-to-side ecliptic longitudes for all visible celestial bodies), but had to be recomputed for every year. Perpetual almanacs were cumbersome to interpret but were – in theory, at least – perpetual.

Two passages are relevant to identify the almanacs San Martín took with him on the Magellan expedition: one is Herrera's (1601, p. 132) transcription of the Rio de Janeiro observation (reproduced in Section 5); and the other one is Barros's (who, like Herrera, had access to the astronomer's original notes; see Section 3) rather scornful commentary:

> And because the measurements did not suit their goals, he [San Martín] complained about Regiomontanus's tables, saying that its numbers must be wrong, and that he thought it was the printer's fault. And on one of his observations (we do not tell which one, as everything has its time), after he had calculated his equations, he said such words: 'The difference from this meridian to the

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> Seville meridian, assuming there is no error in the tables of said almanac, is 42 min [this could have been the result of an additional measurement, as it does not appear to be related to any of his known six observations]; however, because I think the difference should be much higher, I infer there is an error in the tables, but I am not sure who is at fault for it'. [End of citation, the following is Barros again] It is hard to believe that something so common and divulged as the almanacs of Regiomontanus, printed by João Lietersteim [sic] would have such [typographical] errors, given the credit of its printing. And stating that the error lies in Regiomontanus's equations of movement [as San Martín did in his Rio de Janeiro observation, see Section 5], also seems like a grave thing to say about such a well-regarded man and an authority in astronomy.[35]

From the Rio de Janeiro observation, we know that San Martín had with him two almanacs, and that one was made by Zacut and used Salamanca as the reference meridian. Five editions of Zacut's *Almanach Perpetuum* are known: the original edition, printed in Leiria in 1496; and four re-editions (1498, Venice, Johannes Santritter and Peter Liechtenstein; 1502, Venice, Peter Liechtenstein; 1525, Venice, Lucas Antonius Iunta; and 1528, Venice, Peter Liechtenstein). The 1525 and 1528 editions were printed after Magellan's expedition, and the 1498 edition uses Toledo as the reference meridian, so San Martín either had with him the original 1496 edition or the 1502 re-edition. The latter seems more likely, as it enjoyed wider circulation, but San Martín would have arrived at the same results with either one (the two editions provide the same ecliptic longitude for the six observations, see Appendix B).

San Martín himself states that his second almanac was made by Regiomontanus and printed by Peter Liechtenstein (although either he or Barros misspells his name to 'João Lietersteim'). However, Regiomontanus is not known for having computed yearly almanacs past 1506. Could San Martín be instead referring to Stöffler and Pflaum's *Almanach nova plurimis annis venturis inserentia* (1499), which continued Regiomontanus's work until 1531, and was printed by Liechtenstein? Perhaps, but it would be odd for the Spanish astronomer to misattribute this work to Regiomontanus, as Stöffler and Pflaum are clearly stated as the authors (although Regiomontanus is mentioned in the dedication).

Instead, it seems more likely that San Martín is referring to Liechtenstein's 1498 edition of Zacut's *Almanach Perpetuum*, which used Toledo as the reference meridian, and is often misattributed to Regiomontanus. As noted by Chabás and Goldstein (2000, p. 162), Zacut's name is never mentioned, but Regiomontanus is cited several times in the introductory text, and a couple of his tables (for geographical coordinates and the equation of time) are also included, explaining why San Martín (and many others after him) could assume that the almanac had been prepared by Regiomontanus. As shown in Appendix B, the tables of the 1498 edition not only have the same structure as those in the 1496/1502 editions, but also the exact same values,[36] except for typographical errors.

This hypothesis is further reinforced by San Martín's Rio de Janeiro notes, which show nearly identical times to conjunction at Seville for the two almanacs (1 h:12 min for one, 1 h:10 min for the other[37]). This would likely not be the case if almanacs computed by different astronomers had been used (while all of them relied on Ptolemy's equations of motion, they calibrated them with different observations), particularly if using distant reference meridians (Ulm for Stöffler and Pflaum).

In any case, would our conclusions be different if San Martín had onboard not two almanacs from Zacut, as assumed, but one from Zacut and one from Stöffler and Pflaum? No, since the accurate longitude measurements at San Julían (Section 6) and Suluan (Section 7) are explained by the Zacut

[35] *E de não lhe responderem a seu propósito sobre o negócio a quem iam, aqueixa-se de umas tábuas de Joanes de Monte Regio, dizendo: que não pode ser senão que os números estarão errados, & que lhe parecia que devia ser por culpa dos impressores. E numa destas observações (não dizem em que parte foi, porque tudo guardamos para seu tempo) depois de ter calculado suas equações, diz estas formais palavras. De maneira, que haveria diferença deste meridiano ao meridiano de Sevilha, não estando erradas as tábuas do dito Almanaque, quarenta e dous minutos de hora; porém, porque me consta ser muito mais a diferença, infiro haver erro nas Tábuas, que certo não sei a que o atribuía. Porque atribuído a vicio da impressão, não é de crer uma coisa tão comum e tão divulgada como os Almanaques de Joanes de Monte Regio, impressão de João Lietersteim abandonar de tantos vícios nela, por razão do crédito de sua impressão. Pois atribuí-lo a que Joanes de Monte Regio errasse a equação dos movimentos, também me parece grave coisa, dizer de um homem de tanta veneração & autoridade em Astronomia, ter errado sua obra* (Barros, 1628, Folio 147).

[36] Despite using different references meridians (Salamanca and Toledo), the editor may have not realised that including a different table of geographical coordinates changed the reference meridian, or may have assumed that the impact was small, because the two cities are relatively close.

[37] The true longitude difference between Salamanca and Toledo is 8 min, not 2 min, but the longitudes of European cities were not precisely known in the 16th century (see Section 1).

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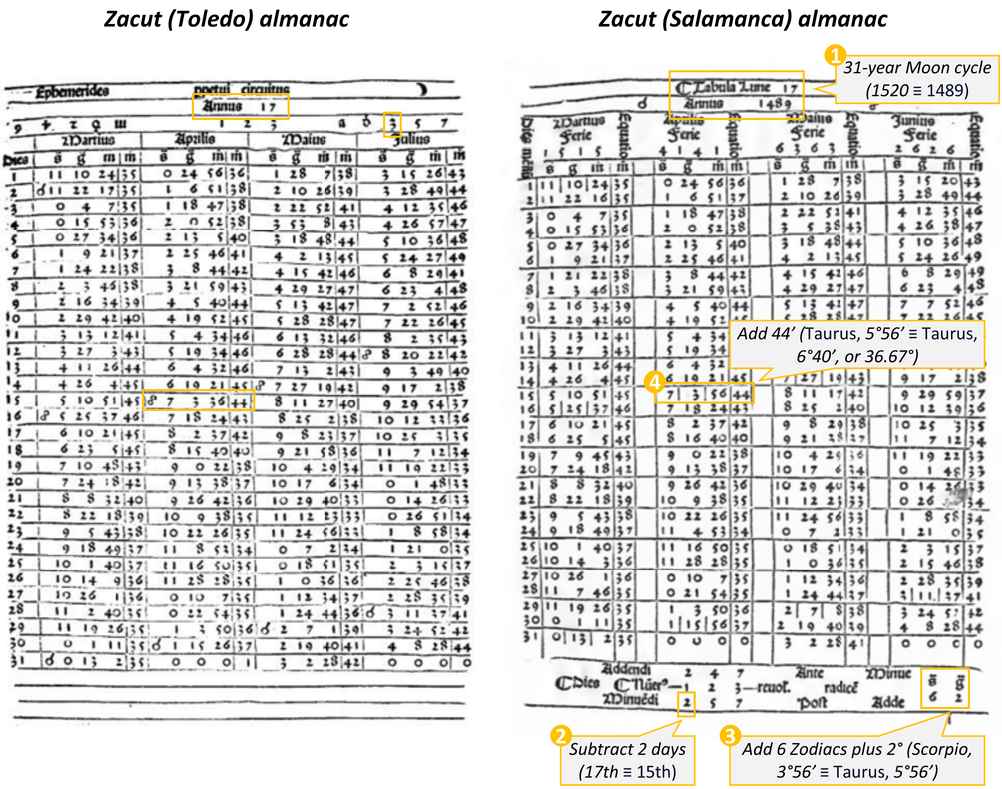

Figure 8. *Example (for the moon on 17 April 1520) of how to read Zacut's ecliptic longitude tables, also highlighting the different values provided by both tables for that specific date.*

(Salamanca) almanac, and the Stöffler/Pflaum almanac would also not have produced accurate results for the three lost measurements (Section 8).[38]

Appendix B: Source data from Zacut almanacs

Table B1 lists the source data extracted from Zacut (Salamanca) and Zacut (Toledo), while the next paragraphs provide an example of how to read these perpetual almanacs. Further details can be found on the comprehensive work done by Chabás and Goldstein (2000).

Figure 8 shows the almanacs' relevant pages for computing the moon's ecliptic longitude for the San Julían observation, made 17 April 1520: as moon data is provided on 31-year cycles, 1520 figures are to be found on the 1489 tables. Next, the matrix on the bottom-left corner of the Salamanca almanac (top-right for Toledo) shows by how many days one needs to go forwards ('*addendi*') or backwards ('*minuédi*') in the table to find the correct entry (2 days backwards for the 17 April 1520 observation, which is one 31-cycle ahead of the base year; e.g. if the observation had been made in 1427, one would need to add four days). In the Salamanca almanac, the resulting entry for 15 April 1489 shows an ecliptic longitude of 3°56′, in the 7th Zodiac (Scorpio). To convert this entry into the moon's ecliptic longitude for 17 April 1520, San Martín had to apply two additional corrections: the first one, highlighted on the page's bottom-right corner, is to add six Zodiacs and 2° (i.e. 182°); the second one, shown on the last column of the daily entry, is to add 44′. The final value is then 36°40′ (36.67°) for the Salamanca almanac, and 36°20′ (36.33°) for the Toledo almanac.

[38]For San Martín's observations, the Zacut and Stöffler/Pflaum errors are typically within 30 min of each other (7.5° of longitude). The exception is for the La Plata River, where Stöffler/ Pflaum were significantly more precise than Zacut, but still not enough to have supported an accurate longitude measurement.

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Table B1. *Source data from Zacut almanacs (the Salamanca values are extracted from the 1502 Venice re-edition, but the original 1496 Leiria edition produces the same results).*

Ephemerides	Location	Aspect	Date	Source values – Moon – Zodiac	Source values – Moon – Degrees	Source values – Moon – Minutes	Source values – Moon – Correction	Source values – Reference body	Source values – Reference body – Closest table entry	Source values – Reference body – Zodiac	Source values – Reference body – Degrees	Source values – Reference body – Minutes	Source values – Source pages
				#	°	′	′		d	#	′	′	
Zacuto (Toledo)	Rio de Janeiro	Conjunction	17/12/1519	1	10	11	36	Jupiter	16	7	13	19	77,209
			18/12/1519	1	22	37	38	Jupiter	24	7	14	40	77,209
	San Julían	Conjunction	17/04/1520	7	3	36	44	Sun	17	1	6	40	28,78
			18/04/1520	7	18	24	43	Sun	18	1	7	38	28,78
	La Plata River	Opposition	31/01/1520	9	12	47	39	Venus	31	9	18	42	77,170
			01/02/1520	9	25	6	38	Venus	1	9	19	58	77,170
	Gulf of San Matias	Quadrature	24/02/1520	8	0	8	42	Sun	24	11	15	14	23,77
			25/02/1520	8	13	45	41	Sun	25	11	16	14	23,77
	Pacific Ocean	Opposition	23/12/1520	8	29	42	43	Sun	23	9	11	43	28,80
			24/12/1520	9	13	27	41	Sun	24	9	12	45	28,80
	Suluan Island (Sun and Moon)	Quadrature	15/03/1521	9	2	10	44	Sun	15	0	4	16	22,81
			16/03/1521	9	15	39	44	Sun	16	0	5	15	22,81
	Suluan Island (Mars and Moon)	Quadrature	15/03/1521	9	2	10	44	Mars	11	3	24	53	81,190
			16/03/1521	9	15	39	44	Mars	16	3	25	49	81,190
Zacuto (Salamanca)	Rio de Janeiro	Conjunction	17/12/1519	1	10	11	36	Jupiter	16	7	13	19	134, 252
			18/12/1519	1	22	39	38	Jupiter	24	7	14	40	134, 252
	San Julían	Conjunction	17/04/1520	7	3	56	44	Sun	17	1	6	24	61,135
			18/04/1520	7	18	24	43	Sun	18	1	7	22	61,135
	La Plata River	Opposition	31/01/1520	9	12	47	39	Venus	31	9	18	43	134, 335
			01/02/1520	9	25	6	38	Venus	1	9	19	58	134, 335
	Gulf of San Matias	Quadrature	24/02/1520	8	0	8	42	Sun	24	11	15	14	62,134
			25/02/1520	8	13	45	41	Sun	25	11	16	13	62,134
	Pacific Ocean	Opposition	23/12/1520	8	29	42	43	Sun	23	9	11	44	68,137
			24/12/1520	9	13	27	41	Sun	24	9	12	45	68,137
	Suluan Island (Sun and Moon)	Quadrature	15/03/1521	9	2	10	44	Sun	15	0	4	16	61,138
			16/03/1521	9	15	39	44	Sun	16	0	5	15	61,138
	Suluan Island (Mars and Moon)	Quadrature	15/03/1521	9	2	10	44	Mars	11	3	24	53	138,307
			16/03/1521	9	15	39	44	Mars	16	3	25	49	138,307

Ephemerides	Location	Date	Derived values – Moon – Ecliptic longitude	Derived values – Moon – Motion	Derived values – Reference body – Ecliptic longitude	Derived values – Reference body – Interpolated ecliptic longitude	Derived values – Reference body – Apparent motion	Derived values – Lunar distance – Ecliptic longitude Δ	Derived values – Lunar distance – Relative motion	Derived values – Lunar distance – Time to aspect	True values (DE431) – Moon – Ecliptic longitude Δ	True values (DE431) – Moon – Motion	True values (DE431) – Reference body – Ecliptic longitude Δ	True values (DE431) – Reference body – Apparent motion	True values (DE431) – Lunar distance – Ecliptic longitude Δ	True values (DE431) – Lunar distance – Relative motion	True values (DE431) – Lunar distance – Time to aspect
			Decimal °	*Decimal °/h*	*Decimal °*	*Decimal °*	*Decimal °/h*	*Decimal °*	*Decimal °/h*	*Decimal h*	*Decimal °*	*Decimal °/h*	*Decimal °*	*Decimal °/h*	*Decimal °*	*Decimal °/h*	*Decimal h*
Zacuto (Toledo)	Rio de Janeiro	17/12/1519	222.78	0.52	223.32	223.49	0.01	0.70	0.51	**1.37**	221.35	0.53	224.93	0.01	3.58	0.52	**6.93**
		18/12/1519	235.25		224.67						233.95		225.12				
	San Julían	17/04/1520	36.33	0.62	36.67	36.67	0.04	0.33	0.58	**0.58**	36.02	0.62	36.72	0.04	0.70	0.58	**1.21**
		18/04/1520	51.12		37.63						50.85		37.68				
	La Plata River	31/01/1520	105.43	0.51	288.70	288.70	0.05	183.27	0.46	**7.11**	104.53	0.52	283.92	0.02	179.38	0.51	**-1.22**
		01/02/1520	117.73		289.97						117.07		284.30				
	Gulf of San Matias	24/02/1520	62.83	0.57	345.23	345.23	0.04	282.40	0.53	**23.62**	62.30	0.56	344.63	0.04	282.33	0.52	**23.78**
		25/02/1520	76.43		346.23						75.75		345.63				
	Pacific Ocean	23/12/1520	92.42	0.57	281.72	281.72	0.04	189.30	0.53	**17.60**	91.42	0.59	281.70	0.04	190.28	0.55	**18.70**
		24/12/1520	106.13		282.75						105.65		282.73				
	Suluan Island (Sun and Moon)	15/03/1521	94.90	0.56	4.27	5.25	0.04	-103.13	0.51	**-25.52**	93.97	0.56	4.20	0.04	-102.38	0.51	**-24.06**
		16/03/1521	108.38		5.25						107.60		5.22				
	Suluan Island (Mars and Moon)	15/03/1521	94.90	0.56	114.88	115.82	0.01	7.43	0.55	**13.60**	93.97	0.56	118.02	0.01	10.62	0.55	**19.43**
		16/03/1521	108.38		115.82						107.60		118.22				
Zacuto (Salamanca)	Rio de Janeiro	17/12/1519	222.78	0.52	223.32	223.49	0.01	0.70	0.51	**1.37**	221.40	0.53	224.93	0.01	3.53	0.52	**6.83**
		18/12/1519	235.28		224.67						234.00		225.12				
	San Julían	17/04/1520	36.67	0.60	36.40	36.40	0.04	-0.27	0.56	**-0.47**	36.08	0.62	36.72	0.04	0.63	0.58	**1.10**
		18/04/1520	51.12		37.37						50.92		37.68				
	La Plata River	31/01/1520	105.43	0.51	288.72	288.72	0.05	183.28	0.46	**7.13**	104.58	0.52	283.93	0.02	179.35	0.51	**-1.28**
		01/02/1520	117.73		289.97						117.12		284.30				
	Gulf of San Matias	24/02/1520	62.83	0.57	345.23	345.23	0.04	282.40	0.53	**23.59**	62.37	0.56	344.63	0.04	282.27	0.52	**23.65**
		25/02/1520	76.43		346.22						75.82		345.63				
	Pacific Ocean	23/12/1520	92.42	0.57	281.73	281.73	0.04	189.32	0.53	**17.61**	91.48	0.59	281.72	0.04	190.23	0.55	**18.61**
		24/12/1520	106.13		282.75						105.70		282.73				
	Suluan Island (Sun and Moon)	15/03/1521	94.90	0.56	4.27	5.25	0.04	-103.13	0.51	**-25.52**	94.03	0.56	4.25	0.04	-102.42	0.51	**-24.13**
		16/03/1521	108.38		5.25						107.65		5.23				
	Suluan Island (Mars and Moon)	15/03/1521	94.90	0.56	114.88	115.82	0.01	7.43	0.55	**13.60**	94.03	0.56	118.02	0.01	10.57	0.55	**19.33**
		16/03/1521	108.38		115.82						107.65		118.22				

Aspect: *Observation's target aspect.*

Date: *Date of the observation, plus the previous or next day (necessary to interpolate the motion of celestial bodies).*

Source values: *Zodiac (30° sector of the ecliptic plane, Aries = 0 to Pisces = 11); Degrees/minutes/correction (source table values); Closest table entry (for some celestial bodies, the almanacs do not provide daily entries, requiring an interpolation from the closest provided dates); Source pages (page numbers where the source data can be found).*

Derived values: *Moon and reference body (ecliptic longitudes and motions of the observed celestial bodies, derived from the source values applying the necessary corrections); Lunar distance (ecliptic longitude difference and motion of the Moon relative to the reference body time to the observation's target aspect).*

True values (DE431): *True values, from NASA JPL's DE431 ephemerides, used to calculate the almanac's errors.*

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Appendix C: Source data from DE431 ephemerides

Table C1. *Source data from the DE431 ephemerides.*

Location	Longitude	Longitude	Date	Hour	Celestial body	Topocentric, with atmosphere				Topocentric, without atmosphere				Geocentric, without atmosphere			
						Azimuth	Altitude	Ecliptic longitude	Ecliptic latitude	Azimuth	Altitude	Ecliptic longitude	Ecliptic latitude	Azimuth	Altitude	Ecliptic longitude	Ecliptic latitude
	°, to Greenwich	°, to Seville		LMST		Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °
Rio de Janeiro	-43.18	-37.20	17/12/1519	04:26:29		90.00	36.00	219.37	1.52	90.00	35.98	219.37	1.52	90.00	36.73	218.78	1.07
San Julian	-67.70	-61.72	17/04/1520	07:31:48		63.15	4.37	36.70	-0.02	63.15	4.20	36.70	-0.02	63.13	5.22	35.87	-0.57
La Plata River	-56.19	-50.21	01/02/1520	02:57:15		303.87	1.45	113.37	5.08	303.87	1.10	113.37	5.08	303.88	2.03	114.17	4.63
Gulf of San Matias	-64.53	-58.55	24/02/1520	16:31:09	Moon	13.98	22.72	67.45	2.83	13.98	22.68	67.45	2.83	13.98	23.57	67.13	2.00
Pacific Ocean	-76.14	-70.16	23/12/1520	19:04:30		56.95	0.07	99.25	5.28	56.95	-0.48	99.25	5.28	56.95	0.50	98.48	4.67
Suluan Island (Sun and Moon)	125.96	131.94	16/03/1521	13:13:00		63.22	9.48	104.35	5.15	63.22	9.40	104.35	5.17	63.22	10.35	103.38	5.17
Suluan Island (Mars and Moon)	125.96	131.94	16/03/1521	21:27:48		302.00	52.63	107.47	5.35	302.00	52.62	107.47	5.35	302.00	53.20	108.03	5.23
Rio de Janeiro	-43.18	-37.20	17/12/1519	04:26:29	Jupiter	94.40	31.83	224.90	1.02	94.40	31.80	224.90	1.02	94.40	31.80	224.90	1.02
San Julian	-67.70	-61.72	17/04/1520	07:31:48	Sun	63.13	4.37	36.70	0.00	63.13	4.20	36.70	0.00	63.13	4.20	36.70	0.00
La Plata River	-56.19	-50.21	01/02/1520	02:57:15	Venus	109.48	1.45	284.22	6.25	109.48	1.12	284.22	6.25	109.48	1.12	284.22	6.25
Gulf of San Matias	-64.53	-58.55	24/02/1520	16:31:09	Sun	282.80	22.72	344.98	0.00	282.80	22.67	344.98	0.00	282.80	22.67	344.98	0.00
Pacific Ocean	-76.14	-70.16	23/12/1520	19:04:30	Sun	242.92	0.00	282.22	0.00	242.92	-0.48	282.22	0.00	242.92	-0.48	282.22	0.00
Suluan Island (Sun and Moon)	125.96	131.94	16/03/1521	13:13:00	Sun	243.22	108.78	4.92	0.00	243.22	108.78	4.92	0.00	243.22	108.78	4.92	0.00
Suluan Island (Mars and Moon)	125.96	131.94	16/03/1521	21:27:48	Mars	301.98	63.62	118.23	2.98	301.98	63.62	118.23	2.98	301.98	63.62	118.23	2.98

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Appendix D: Computed error signatures

Table D1. *Computed individual errors for each observation.*

Almanac	Location	Almanac errors						Observation errors													Process errors		Compounded error				Validation				
		Equations		Meridian		Interpolation		Parallax			Refraction			Geometric		Instrument		Local time					Mean		+1 SD			Mean		+1 SD	
		Ecliptic longitude	Longitude	Ecliptic longitude	Longitude	Ecliptic longitude	Longitude	Altitude	Ecliptic longitude	Longitude	Altitude	Ecliptic longitude	Longitude	Ecliptic longitude	Longitude	Ecliptic longitude	Longitude	Local time	Ecliptic longitude	Longitude	Ecliptic longitude	Longitude	Ecliptic longitude	Longitude	Ecliptic longitude	Longitude	True longitude (Seville meridian)	Computed longitude (compounded errors)	Computed longitude (check)	Computed longitude (compounded errors)	Computed longitude (check)
		Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal h	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °	Decimal °
Zacuto (Salamanca)	Rio de Janeiro	2.71	78.56	0.06	1.67	-0.03	-0.95	0.75	-0.58	-16.91	0.02	0.00	0.00	-0.20	-5.80	-0.58	-16.91	0.06	0.03	0.88	-9.38	-272.09	-7.99	**-231.55**	-7.99	**-231.55**	-37.20	-268.75	-268.75	-268.75	-268.75
	San Julian	0.76	19.84	0.06	1.67	0.01	0.19	1.02	-0.83	-21.63	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.08	2.20	0.00	0.00	0.00	**0.07**	0.09	**2.27**	-61.72	-59.45	-59.45	-75.50	-75.50
	La Plata River	-4.05	-119.91	0.06	1.67	-0.01	-0.33	0.93	0.80	23.67	-0.02	0.00	0.00	23.53	-23.67	-0.37	-11.05	0.03	0.01	0.42	0.00	0.00	20.33	**-118.57**	19.97	**-129.20**	-50.21	202.70	202.70	192.71	192.71
	Gulf of San Matias	-0.26	-7.38	0.06	1.67	-0.04	-1.04	0.88	-0.32	-9.16	0.02	0.00	0.00	-8.72	107.95	0.37	10.80	0.03	0.01	0.42	0.00	0.00	-9.27	**92.04**	-8.88	**103.26**	-58.55	-315.29	-315.29	-324.58	-324.58
	Pacific Ocean	0.79	21.47	0.06	1.67	-0.02	-0.66	0.98	-0.77	-20.91	-0.07	0.00	0.00	3.00	81.82	0.37	10.19	0.03	0.02	0.42	0.00	0.00	3.06	**83.40**	3.45	**94.00**	-70.16	23.84	23.84	15.05	15.05
	Suluan Island (Sun and Moon)	0.60	17.37	0.06	1.67	0.06	1.78	0.95	-0.97	-28.18	-0.08	0.00	0.00	0.13	3.89	-0.37	-10.89	0.03	0.01	0.42	0.00	0.00	-0.12	**-3.47**	-0.48	**-13.95**	131.94	139.77	139.77	129.44	129.44
	Suluan Island (Mars and Moon)	3.00	82.47	0.06	1.67	0.01	0.28	0.58	0.57	15.55	-0.02	0.00	0.00	0.22	5.95	0.37	10.25	0.03	0.02	0.42	0.00	0.00	3.86	**105.92**	4.25	**116.59**	131.94	248.53	248.53	239.23	239.23
Zacuto (Toledo)	Rio de Janeiro	2.71	78.56	0.06	1.67	-0.03	-0.95	0.75	-0.58	-16.91	0.02	0.00	0.00	-0.20	-5.80	-0.58	-16.91	0.06	0.03	0.88	-9.38	-272.09	-7.99	**-231.55**	-7.99	**-231.55**	-37.20	-268.75	-268.75	-268.75	-268.75
	San Julian	0.23	5.99	0.06	1.67	0.01	0.19	1.02	-0.83	-21.63	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.08	2.20	0.00	0.00	-0.53	**-13.78**	-0.45	**-11.58**	-61.72	-75.50	-75.50	-73.30	-73.30
	La Plata River	-4.00	-118.43	0.06	1.67	-0.01	-0.33	0.93	0.80	23.67	-0.02	0.00	0.00	23.53	-23.67	-0.37	-11.05	0.03	0.01	0.42	0.00	0.00	20.38	**-117.09**	20.02	**-127.72**	-50.21	192.71	192.71	204.18	204.18
	Gulf of San Matias	-0.19	-5.45	0.06	1.67	-0.04	-1.04	0.88	-0.32	-9.16	0.02	0.00	0.00	-8.72	107.95	0.37	10.80	0.03	0.01	0.42	0.00	0.00	-9.20	**93.97**	-8.81	**105.19**	-58.55	-324.58	-324.58	-313.36	-313.36
	Pacific Ocean	0.85	23.29	0.06	1.67	-0.02	-0.66	0.98	-0.77	-20.91	-0.07	0.00	0.00	3.00	81.82	0.37	10.19	0.03	0.02	0.42	0.00	0.00	3.12	**85.21**	3.51	**95.82**	-70.16	15.05	15.05	25.66	25.66
	Suluan Island (Sun and Moon)	0.63	18.34	0.06	1.67	0.06	1.78	0.95	-0.97	-28.18	-0.08	0.00	0.00	0.13	3.89	-0.37	-10.89	0.03	0.01	0.42	0.00	0.00	-0.09	**-2.50**	-0.45	**-12.98**	131.94	129.44	129.44	140.75	140.75
	Suluan Island (Mars and Moon)	3.05	83.84	0.06	1.67	0.01	0.28	0.58	0.57	15.55	-0.02	0.00	0.00	0.22	5.95	0.37	10.25	0.03	0.02	0.42	0.00	0.00	3.91	**107.29**	4.30	**117.96**	131.94	239.23	239.23	249.90	249.90

Almanac and observation errors: *Individual errors included in the error signature, including compounded effects (see chapter 3 for descriptions), in terms of altitude (if applicable), and reduced to ecliptic longitude and longitude of the place; note that errors are shown net of dependencies so that they can be summed (e.g., for all observations except San Julían, where altitude was not taken, the local time error is net of the instrument error, as both these errors are assumed to follow a normal distribution).*

Process errors: *Only applicable to the Rio de Janeiro observation (see chapter 6).*

Compounded errors: *Total measurement errors, including compounded effects; mean values exclude random errors, +1 SD values include random errors at +1 standard deviation.*

Validation: *Error signature validation, by comparing the longitude of the place computed with the error signature and computed by applying the lunar distances method directly to the DE431 ephemerides data; both methods produce the same results within the precision of the model (1 arcminute).*

One could assume that the 20’ difference between the two almanacs was due to the use of different reference meridians (Salamanca and Toledo), but that does not seem to be the case. As both Figure 8 and Table B1 indicate, most of the figures from the two almanacs are the same, and the differences appear to be typographical errors (a common occurrence in the early days of printing). For the San Julían observation, San Martín may have noted the correct value was that of the Salamanca almanac, as the Toledo one resulted in an irregular lunar daily motion (14.8° from 15 April to 16 April, 14.3° from 16 April to 17 April, and then back up again to 14.8° from 17 April to 18 April). The astronomer may have also noted a second error on the same page: a ‘3’ instead of a ‘2’ on the ‘*minuédi*’ correction factor (by noting that the correction factor did not match those on the almanac’s next couple of pages).

Despite its typographical errors, the Toledo almanac did have one advantage over the Salamanca one: as highlighted in Figure 8, it provided a visual cue that the moon would be in conjunction with the sun (the symbol shown is for the 1489 opposition, which turns into a conjunction in 1520 after applying the six Zodiacs, or 180°, correction factor mentioned). This reminder was useful but not required, as the astronomer could reach the same conclusion by comparing the ecliptic longitude values.

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