CORRESPONDENCE.

ON THE VALUATION OF ENDOWMENT ASSURANCES IN GROUPS.

To the Editor of the Journal of the Institute of Actuaries.

DEAR SIR,—I have read Mr. King's recent note on the valuation of Endowment Assurances, and of Whole-life Assurances with limited premium payments, in groups, with considerable interest, but in connection with the actual authorship of the method in question, I may perhaps be allowed to communicate the following information.

At a recent Committee meeting in connection with the approaching Actuarial Congress, my Secretarial colleague, Dr. Julius Graf, Actuary of the "Generali", in Trieste, called my attention to the fact that Dr. August Zillmer, and not Dr. Johannes Karup, was

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the author, and referred me to the 1st edition of his (Zillmer's) Text-Book dated 1861.*

On p. 106, the following paragraph will be found—

Whole-Life Assurances with limited premium payments may also be treated as if the premiums were payable for the whole of life; i.e., they may be grouped according to valuation age. A further reserve will, however, be required in such cases, on account of the excess premiums.

The idea involved in the method is therefore here clearly stated. I may also mention that Regierungsrath Blaschke, Actuarial Adviser to the Ministry of the Interior, confirms Dr. Graf's statement.

I accordingly communicated with Mr. Altenburger, and he has replied as follows—

"Two or three years ago I have also read the 1st edition "of Zillmer's book, and found that the method in question is "already contained therein, so that it is Zillmer who first discovered "the method."

It would therefore appear that Dr. August Zillmer was the first to discover the method, although Dr. Johannes Karup may, perhaps, have been the first to apply it practically.

The method in question may also be applied with success to more complex tariffs.

(1) Here, on the Continent, a tariff very much sought after, is an Endowment Assurance, with decreasing premiums, merely popular on account of the total premiums paid, in the event of survival, being less than would be the case under an ordinary endowment assurance.

For example, we may assume that the 1st 5 premiums are constant at P, the 6th being ·97P, the 7th ·94P, the 8th ·91P, and so on, decreasing by ·03P each year until maturity.

Taking x, n, and π' as the age at entry, term of assurance, and net premium respectively, we have—

$$\pi'\{(1\cdot15 - \cdot03n)\mathbf{a}_{x\overline{n}|} + \cdot03\sum_{m=5}^{m=n-1}\mathbf{a}_{x:\overline{m}|}\} = \mathrm{A}_x + \frac{d\mathbb{N}_{x+n}}{\mathrm{D}_x}$$

so that

$$\pi' = \frac{\mathrm{M}_x + \mathrm{H}_{x+n}}{\mathbb{N}_x - (\mathrm{B}_{x+n} + \mathrm{C}_{x+5})}$$

where

$$\mathrm{H}_{x+n} = d\mathbb{N}_{x+n}$$

$$\mathrm{B}_{x+n} = \{(1\cdot15 - \cdot03n)\mathbb{N}_{x+n} - \cdot03\mathbb{S}_{x+n}\}$$

$$\mathrm{C}_{x+5} = \cdot03\mathbb{S}_{x+5}.$$

The constants, H and C, are at once obtained from the ordinary commutation columns, and B, which varies with the term of assurance, may be readily computed by a continued process.

For a grouped valuation, it will be necessary to separate the policies with a duration of less than five years from those having a

* This book is included in the Library of the Institute.—[Ed. *J.I.A.*]

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longer duration. Each category may then be grouped according to calendar year of birth, and valued by the following formulæ—

(α) $(t+\frac{1}{2})<5$

$$ {}_{t+\frac{1}{2}}V'_{\overline{x.n}|} = \left\{ A_{x+t+\frac{1}{2}} + \frac{H_{x+n}}{D_{n+t+\frac{1}{2}}} \right\} - \left\{ \pi'(\tfrac{1}{2} + a_{x+t+\frac{1}{2}}) - \frac{K_{x+n} + L_{x+5}}{D_{x+t+\frac{1}{2}}} \right\} $$

(β) $(t+\frac{1}{2})>5$

$$ {}_{t+\frac{1}{2}}V'_{\overline{x.n}|} = \left\{ A_{x+t+\frac{1}{2}} + \frac{H_{x+n}}{D_{x+t+\frac{1}{2}}} \right\} - \left\{ (1.12 - \cdot 03t)\pi'(\tfrac{1}{2} + a_{x+t+\frac{1}{2}}) - \frac{K_{x+n}}{D_{x+t+\frac{1}{2}}} - \cdot 015\pi'[(Ia)_{x+t} + (Ia)_{x+t+1}] \right\} $$

where $K_{x+n} = \pi'.B_{x+n}$; $L_{x+5} = \pi'.C_{x+5}$; and $\pi'(1.12 - \cdot 03t)$ is the current net premium.

(2) Dr. Zillmer's method may be applied to the formula of Dr. Johannes Karup for valuing Endowment assurances in groups according to exact duration (vide *J.I.A.*, xxxviii, 431).

For a policy effected at age x, with a term of m years, and which has been in force $(n+t)$ years, n being integral and t fractional, we have—

$$ \text{Value of sum assured} = A_{x+n} + t.\Delta A_{x+n} + \frac{H_{x+m}}{D_{x+n}} + t.H_{x+m}.\Delta D^{-1}_{x+n} $$

$$ \text{Value of net premium} = \pi.a_{x+n} + t\pi(\mathbf{a}_{x+n+1} - a_{x+n}) - \frac{K_{x+m}}{D_{x+n}} - t.K_{x+m}.\Delta.D^{-1}_{x+n} $$

where $H_{x+m} = d\,\mathbb{N}_{x+m}$ and $K_{x+m} = \pi.\mathbb{N}_{x+m}$.

In addition to the constants (SH) and (SK), we shall therefore require the values of (tS), (tSH), (tSπ), and (tSK) to be entered on the valuation cards. The policies will then be grouped according to calendar years of birth, and no further adjustment is required on account of distribution of premium income over the financial year.

In conclusion, I might add that Dr. Zillmer's method is employed by the "Generali" of Trieste, one of the largest Continental companies, having a premium-income (Life branch only) of Kr. 36,870,312 (=£1,500,000, about), with a sum assured under Endowment Assurances alone of Kr. 595,000,000 (=£24,800,000, about).

This company makes an annual valuation and distribution, computes its profit or loss on the death-strain annually (compulsory in Austria), and publishes its returns within the first three months of the year.

I am, Dear Sir,
Yours faithfully,
ROBT. S. B. SAVERY.

1, *Giselastrasse I, Vienna.*
2 *March* 1908.

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