INTEGRALS OF MOTION IN AN ELLIPTICAL GALAXY MODEL

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The structure of a galaxy model is described completely by its phase–space distribution function f. By Jeans' Theorem f can be written as a function of the integrals of motion admitted by the potential of the model. Various independent combinations of the integrals may be used as arguments of f; in many cases the action integrals are to be preferred. For a general N–body model, these can be obtained by numerical integration and subsequent spectral decomposition of each orbit (Binney and Spergel 1984).

The orbital structure in triaxial elliptical galaxies resembles closely that in the general Stäckel potentials, for which the Hamilton–Jacobi equation separates in ellipsoidal coordinates, and all orbits have three exact integrals of motion H, I_2 and I_3, say (de Zeeuw 1985). Thus, for N–body models of elliptical galaxies, approximate integrals of motion can be found by the fitting of Stäckel potentials.

FITTING

In confocal ellipsoidal coordinates (λ, μ, ν) a Stäckel potential V_S is of the form

$$V_S = -\frac{F(\lambda)}{(\lambda-\mu)(\lambda-\nu)} - \frac{F(\mu)}{(\mu-\nu)(\mu-\lambda)} - \frac{F(\nu)}{(\nu-\lambda)(\nu-\mu)},$$

where $F(\tau)$ is an arbitrary function $(\tau = \lambda, \mu, \nu)$. Given a potential $V(x^2, y^2, z^2)$, we want to find the Stäckel potential V_S that approximates it most closely. Thus, we have to specify an ellipsoidal coordinate system and the function $F(\tau)$. This can be done by means of the following procedure, described in detail by de Zeeuw and Lynden–Bell (1985):

- Pick coordinates (λ, μ, ν), by specification of two pairs of their foci.
- Transform $V(x^2, y^2, z^2)$ to $V(\lambda, \mu, \nu)$ on a rectangular grid in (λ, μ, ν)–space.
- Construct an auxiliary function $\chi(\lambda, \mu, \nu) = -(\lambda-\mu)(\mu-\nu)(\nu-\lambda)V(\lambda, \mu, \nu)$
- Determine $F(\lambda)$, $F(\mu)$ and $F(\nu)$ by calculation of various weighted averages over $\chi(\lambda, \mu, \nu)$. The weighting functions may be chosen to ensure convergence; most weight can be put where most of the density is.
- Repeat this process for a different choice of the positions of the two pairs of foci, until the best fit is obtained.

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INTEGRALS

The integrals H, I_2 and I_3 in a Stäckel potential are known explicitly. Their values determine the family (boxes, short–axis tubes, inner long–axis tubes or outer long–axis tubes) to which an orbit belongs. We can obtain approximate integrals for all the particles in the N–body model by substitution of the instantaneous position and velocity coordinates in the expressions for H, I_2 and I_3 in the best–fitting Stäckel potential. Similarly, the action integrals J_λ, J_μ and J_ν can be calculated. This requires a simple quadrature. It follows that all orbits can be classified, and the approximate distribution function $f(H, I_2, I_3)$ or $f(J_\lambda, J_\mu, J_\nu)$ can be calculated by binning.

APPLICATION

We have applied the above procedure to the stationary triaxial galaxy simulated by Wilkinson and James (1982). We have fitted the 20000–body model at two timesteps, the first halfway through its evolution, and the second at the end of the run, i.e., at times equivalent to 0.5 and 0.9 of a Hubble time t_H. In both cases the potential can be fitted accurately with a Stäckel potential out to large radii. Based on $\sim$ 2500 orbits we obtain the orbit classification given in the Table. The uncertainty in the fractions are of the order of at most a few percent. For comparison, the estimates based on the visual classification of Wilkinson and James (1982) for the final timestep are given also.

We conclude that the orbital structure can be found at individual timesteps, and with good accuracy. The distribution function of the model is smooth in action space, and does not change very much during the second half of the run. For N-body models of elliptical galaxies this way of classifying orbits is much faster than spectral stellar dynamics.

Table: Orbit Classification

Orbit Family	$0.5t_H$	$0.9t_H$	By Eye
Boxes	83.8%	91.5 %	75.1 %
Short Axis Tubes	10.8	4.8	7.5
Inner Long Axis Tubes	2.8	1.3	
Outer Long Axis Tubes	1.2	0.4	6.5
Unbound	1.4	2.0	4.3
Unclassified			6.6

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