

PRICING AND HEDGING OF VIX DERIVATIVES IN MODIFIED STOCHASTIC MODELS

JIEFEI YU 

(Received 4 May 2023; first published online 23 June 2023)

2020 *Mathematics subject classification*: primary 91G20; secondary 60H30.

Keywords and phrases: VIX option, 4/2 model, saddlepoint, GARCH, quadratic hedging.

After CBOE launched the VIX and its corresponding derivatives, investors have maintained a high level of interest in this unique index. Unlike indices that focus on asset prices, the VIX can reflect investors' subjective expectations of the market by incorporating varying volatility into the calculation. However, in the past literature, the prices of the VIX derivatives have often just been obtained from models with fixed parameters [1] or considered only the long-term volatility of the asset price [2]. Also, we find sparse discussions on the compatibility of the VIX valuation process with modifications of time series stochastic models, such as the regime switching and the subordinator method. Lastly, we note little literature specifically mentions hedging VIX derivatives and the relevant strategy-obtaining process. To address these issues, we explore the pricing and hedging of VIX derivatives in this thesis, using the VIX European call option as an example. In the pricing process covered in Chapters 3 and 4, we incorporate the regime switching factor into the continuous 4/2 model and discretise the model, based on Heston–Nandi's idea in combination with various modifications, to improve the model's capture of various volatility and changes in the market environment. After comparing the results obtained by the saddlepoint method, we find that those modifications significantly improved the quality of the model, increasing the accuracy of the pricing results and allowing the model to adapt to a more general market environment. In Chapter 5, we hedge the VIX options based on the GARCH framework using a local quadratic hedging approach. After taking advantage of the GARCH model, we optimise the method of obtaining option hedging strategies by reducing the weights of stochastic simulations and reducing the number of simulations required while enhancing the model accuracy.

Thesis submitted to the University of New South Wales in August 2022; degree approved on 23 January 2023; supervisors Leung Chan and Feng Chen.

© The Author(s), 2023. Published by Cambridge University Press on behalf of Australian Mathematical Publishing Association Inc.

References

- [1] T. Wang, Y. Shen, Y. Jiang and Z. Huang, 'Pricing the CBOE VIX futures with the Heston–Nandi GARCH model', *J. Futures Markets* **37**(7) (2017), 641–659.
- [2] W. Zheng and Y. K. Kwok, 'Saddlepoint approximation methods for pricing derivatives on discrete realized variance', *Appl. Math. Finance* **21**(1) (2014), 1–31.

JIEFEI YU, School of Mathematics and Statistics,
University of New South Wales, Sydney, New South Wales 2052, Australia
e-mail: jiefeigy1989@gmail.com