THE SPONGE-LIKE TOPOLOGY OF LARGE SCALE STRUCTURE IN THE UNIVERSE

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ABSTRACT. We describe and apply a quantitative measure of the topology of large scale structure: the genus of density contours in a smoothed density distribution. For random phase (gaussian) density fields, the mean genus per unit volume exhibits a universal dependence on threshold density, with a normalizing factor that can be calculated from the power spectrum. The topology of the observational sample is consistent with the random phase, cold dark matter model.

Recent deep redshift surveys by de Lapparent, Geller and Huchra (1986) and Haynes and Giovanelli (1986) have renewed interest in the topology of large scale structure in the universe.

If we look at a distribution of galaxies we see a series of points distributed in 3-dimensional space. We would like to know the smooth underlying distribution from which the points could have been obtained by a sampling process. To begin, therefore, we smooth the data with a gaussian smoothing window $W = e^{-r^2/\lambda^2}$ where r is the distance and λ is a smoothing length picked to be larger than the mean galaxy-galaxy separation. Information about the topology is then carried in the density contour surfaces of this smoothed density distribution.

The topology of an object is mathematically specified by its genus. We may define the genus of a contour surface as

$$g_s = (\text{no. of holes}) - (\text{no. of isolated regions}) \tag{1}$$

where 'hole' means 'hole' like a donut has (Gott, Melott, Dickinson 1986). Suppose we have a density contour that shows 50 isolated spherical clusters, then $g_s = -50$. A contour may also have a multiply connected, sponge-like topology, in which case its genus is positive. In what follows we study g_s as a function of threshold density (Gott, Weinberg, Melott 1986; Weinberg, Gott, Melott 1986).

An important advantage of this method of looking at topology is that in the standard big bang - inflationary model we can relate the topology seen today to that present in the initial conditions. Imagine looking at the small-amplitude density fluctuations present at a red-

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shift of $z = 25$. These fluctuations arise from random quantum fluctuations that are gaussian with random phases with a power spectrum $P(k)$ which can be calculated directly from the primordial Zel'dovich inflationary spectrum. We now smooth the data with a smoothing length λ, and construct density contours. Hamilton, Gott, and Weinberg (1986) and Bardeen et al. (1986) have shown that the mean genus per unit volume of these surfaces will be given by

$$g_s = N(1 - \nu^2)e^{-\nu^2/2}. \tag{2}$$

Here $\nu = \delta_c/\xi(0)^{1/2}$ is the number of standard deviations by which the contour threshold density δ_c departs from the mean density. And $N = [\langle k^2\rangle/3]^{3/2}/4\pi^2$ is an appropriate moment of the smoothed power spectrum. The amplitude of the $g_s(\nu)$ curve depends on N and therefore on $P(k)$, but the form $g_s(\nu) \propto (1 - \nu^2)e^{-\nu^2/2}$ is completely independent of the initial power spectrum. N is positive definite so that g_s is positive (sponge-like topology) for $\nu=0$, $f=0.5$ (the median density contour) regardless of $P(k)$. If we make a cut where the fraction of the volume in the high density region is less than $f = 16\%$ ($\nu > 1$) then $g_s < 0$ and we encounter isolated clusters. If we choose a contour such that the fraction of volume in the high density region is greater than $f = 85\%$ ($\nu < -1$) then $g_s < 0$ and we find isolated voids.

Now we evolve the model to the present epoch, and generate biased subsets of the mass distribution in the usual way (Melott and Fry 1986).

Figure 1 plots the $g_s(\nu)$ curves for the initial, final, and biased conditions (averaged results of four simulations) together with the theoretical curve $g_s(\nu) = N(1 - \nu^2)e^{-\nu^2/2}$ expected for the CDM initial conditions. Since N is determined from $P(k)$, which is known for the CMD model, there are no free fitting parameters for this at all. The agreement is remarkable.

Why is this so? As long as the fluctuations stay in the linear regime they just grow in place increasing in amplitude, and the topology changes not at all. In the cold dark matter model the scale at which the mass covariance function goes to unity, $r_m = 3.6$ Mpc, is considerbly smaller than our smoothing length $\lambda = 10$ Mpc, so we are mainly looking at fluctuations that are just now beginning to come out of the linear regime and whose topology remains unchanged from the initial conditions. Biasing increases the 'contrast' of the picture, but it basically leaves the luminosity density a monotonic function of the underlying mass density. Thus contours of constant luminosity density are contours of constant mass density -- just the values are shifted. If we draw contours as a function of the volume enclosed we find that the biased and unbiased data sets are essentially identical.

Figure 2 shows results from a volume limited sample of the northern CfA redshift survey. It is a cube with a side length of 58 Mpc and includes galaxies brighter than $0.72\ L_*$ out to a maximum redshift of 5000 km/s (see GMD for details). As in the analysis of the N-body simulations, we use a periodic boundary condition for smoothing. The smoothing length $\lambda = 10.8$ Mpc. Given the small volume of the region surveyed, we would like to see results from larger observational samples before drawing any firm conclusions. Nonetheless, the

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data as they stand are remarkably consistent with the random phase model and with the amplitude expected for a CDM power spectrum.

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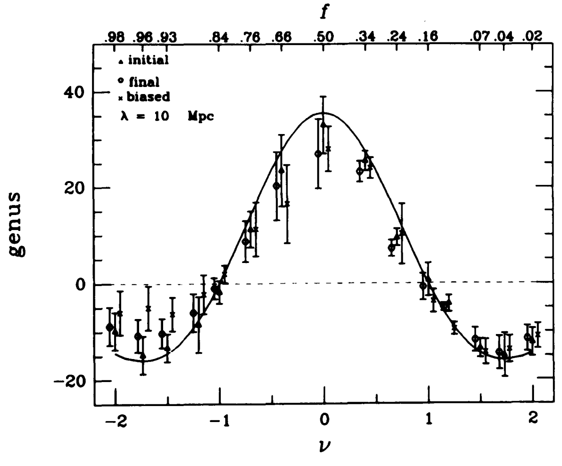

Figure 1. Cold Dark Matter Simulation Results

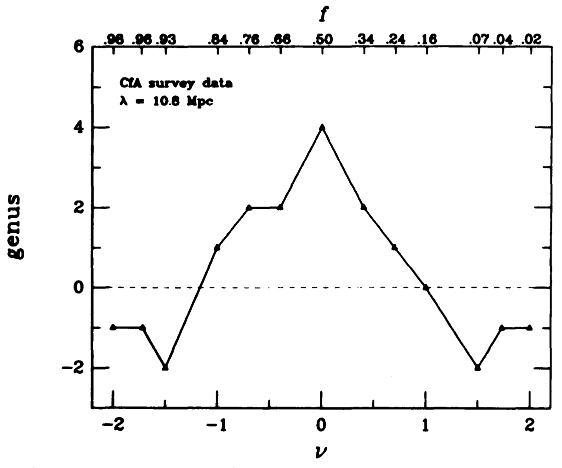

Figure 2. CfA Survey Results

DISCUSSION

PECKER: In what sense does your technique differ from that of Mandelbrot in building either cheese and bubbles, - or sponges, using various 3-dimensional fractal distributions?

GOTT: Mandelbrot has modeled galaxy distributions by starting with a uniform high density region and excising random spherical voids from it. With just a few voids excised this gives a swiss cheese topology, but if enough voids are excised so that they percolate and the voids form one connected low density network then this will make a sponge. Mandelbrot has also done models where galaxies are laid down on a random walk. In the sponge-like topology produced by random phase perturbations clusters are linked to each other by a network of filaments and voids are linked to each other by a network of tunnels. It is important to note that the topology is a three dimensional issue. A thin slice of swiss cheese and a thin slice of a sponge can look identical, both showing holes. The question is how the low density regions are connected up in three dimensions. In this regard it is interesting to note that the $\Omega = 1$, cold dark matter, biased models (by our group and also by White et al.) which are known to have a sponge-like topology in three dimensions, do reproduce remarkably well in thin slices the cellular appearance seen in the thin slices of the de Lapparent, Geller and Huchra survey.

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