

The proof of the construction where  $(\theta + \phi)$  is less than  $180^\circ$  is illustrated in Fig. 3, where A, B and C are, as before, the observed marks and P is the required position. AP is produced to cut the circle through BCP in E and CP is produced to cut the circle through ABP in D.

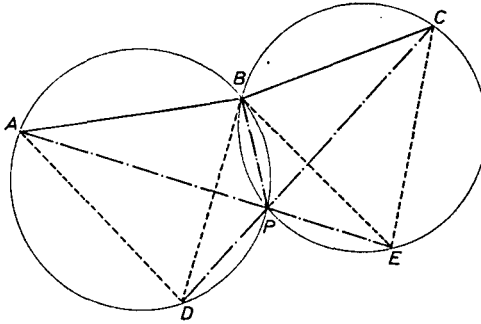


FIG. 3

Now  $\Omega = \angle ABD = \angle APD$  (since they are both subtended by the chord AD; Euclid III.21):

$$\angle APD = 180^\circ - (\angle APB + \angle BPC)$$

$$\therefore \Omega = 180^\circ - (\theta + \phi).$$

Similarly  $\Omega = \angle CBE = \angle CPE = 180^\circ - (\theta + \phi)$ .

In the cyclic quadrilaterals ABPD and CBPE:

$$\angle BAD = \angle BPC = \phi$$

$$\angle BCE = \angle BPA = \theta \text{ (Euclid III.22).}$$

A similar proof is applicable to the case where  $(\theta + \phi)$  exceeds  $180^\circ$ .

Although in practice this method is no quicker to plot than other recognized methods, it does have the advantage of not requiring compasses. The construction is easily performed with a parallel ruler from the compass rose on the chart, but a circular or Douglas protractor is much more convenient.

The author wishes to thank Inst. Lt. Rickards, R.N. for reading the original paper and suggesting a clearer presentation.

## The Calculation of Position Line Data with a Computer Calculator

Henry L. Podmore

As a result of the recent marketing of small pocket computers I have found much interest among navigators who would like to use them for working out

celestial sights. One computer calculator (the Hewlett-Packard HP-35) in particular appears to be of considerable interest due to its ability to compute trigonometric functions with algebraic logic as well as to store and recall derived intermediate values. Not having been able to find any published work on the use of this type of equipment for the rapid and precise working of sights I have given some thought to the matter and as a result believe the following notes may well be of interest to others.

The usual method of calculating zenith distance using the haversine formula overcomes the obvious difficulties of handling negative functions, but this computer calculator can handle negative values of sines and cosines without difficulty; moreover multiplication can be carried out without the need for logarithms. Hence the computer facilitates the use of the basic spherical cosine formula<sup>1</sup> which when applied to the PZX triangle may be expressed as:<sup>2</sup>

$$\cos P = \frac{\cos ZX - (\cos PZ) (\cos PX)}{(\sin PZ) (\sin PX)}$$

where P = Local Hour Angle

ZX = Zenith distance

PZ = Co-lat (= 90° - Lat)

PX = Polar distance = 90 - Dec, where Lat and Dec are of the same name  
or 90 + Dec (Lat and Dec Diff. Name)

Since  $\sin x = \cos (90 - x)$  we can substitute and transpose as follows:

$$\cos \text{LHA} = \frac{\cos ZX - (\sin \text{Lat}) (\sin \text{Dec})}{(\cos \text{Lat}) (\cos \text{Dec})} \quad (\text{Lat and Dec Same Name})$$

$$\text{or } (\cos \text{LHA}) (\cos \text{Lat}) (\cos \text{Dec}) = \cos ZX - (\sin \text{Lat}) (\sin \text{Dec})$$

from which  $\cos ZX = (\cos \text{LHA}) (\cos \text{Lat}) (\cos \text{Dec}) + (\sin \text{Lat}) (\sin \text{Dec})$ .

Where Lat and Dec have Different Names:

$$\begin{aligned} \cos ZX &= (\cos \text{LHA}) (\cos \text{Lat}) (\sin 90 + \text{Dec}) + (\sin \text{Lat}) (\cos 90 + \text{Dec}) \\ \text{or } \cos ZX &= (\cos \text{LHA}) (\cos \text{Lat}) (\cos \text{Dec}) + (\sin \text{Lat}) (-\sin \text{Dec}) \\ \cos ZX &= (\cos \text{LHA}) (\cos \text{Lat}) (\cos \text{Dec}) - (\sin \text{Lat}) (\sin \text{Dec}) \end{aligned}$$

It will be seen that the formulae are the same for both conditions, except that when Lat and Dec have Different Names the arithmetic sign in the formula is negative. The formula in this form is eminently suitable for use on the computer, providing one first converts (on the computer) the various values expressed as degrees and minutes into degrees and fractions of a degree, to (say) 3 decimal places.

Let us consider the calculation of the zenith distance where:

$$\begin{aligned} \text{LHA} &= 349.067^\circ \\ \text{D.R. Lat} &= 53.167^\circ \text{N} \\ \text{Dec} &= 15.005^\circ \text{N} \end{aligned}$$

Substituting in the above formula (Lat and Dec Same Name) we get:

$$\cos ZX = (\cos 349.067) (\cos 53.167) (\cos 15.005) + \sin (53.167) (\sin 15.005)$$

The necessary sequence of operations on the computer is shown below. The

digital display (column 2) facilitates the entry of correct figures and accurate operation.

Press	See	Comments
1) 349.067	349.067	
2) Cos	0.9818496387	
3) 53.167	53.167	
4) Cos	0.5994846873	
5) ×	0.5886038236	Gives product of values in first two brackets
6) 15.005	15.005	
7) Cos	0.9659032362	
8) ×	0.5685343381	Gives product of values in first three brackets
9) STO	0.5685343381	Storage of this result permits processing of last two brackets
10) 53.167	53.167	
11) Sin	0.8003862253	
12) 15.005	15.005	
13) Sin	0.258903337	
14) ×	0.2072226646	Gives product of values in last two brackets
15) RCL	0.5685343381	Recalls value in step 9
16) +	0.7757570027	Gives value of Cos ZX
17) arc	0.7757570027	
18) Cos	39.12628446	Gives angle in degrees (ZX) having Cos as in step 16
19) 39	39	
20) -	0.12628446	This leaves decimal degrees to be converted to mins
21) 60	60	
22) ×	7.5770676	Gives decimal degrees as minutes

Hence Zenith distance =  $39^{\circ} 7.6'$

The standard program can be summarized thus:

- 1) LHA
- 2) Cos
- 3) D.R. Lat
- 4) Cos
- 5) ×
- 6) Dec
- 7) Cos
- 8) ×
- 9) STO
- 10) D.R. Lat
- 11) Sin
- 12) Dec
- 13) Sin
- 14) ×
- 15) RCL
- 16)  $\left\{ \begin{array}{l} + \text{ (If Lat and Dec Same Name)} \\ \times \text{ followed by } - \text{ (If Lat and Dec Different Name)} \end{array} \right.$
- 17) arc
- 18) Cos
- 19) Enter 18 as whole number & note result (Z.D. in degrees)

- 20) —  
 21) 60  
 22) × Note minutes to first decimal place

Steps 19 and 22 combine to give the ZD in degrees and minutes.

Recently Hewlett-Packard have introduced a new instrument, the HP-65, which enables the above program to be recorded on magnetic tape and then subsequently used to calculate automatically the ZD for any given values of LHA, D.R. Lat and Dec. This particular computer handles degrees, minutes and seconds directly—which can be an advantage.

The calculation of LHA is, of course, carried out in the usual manner. The intercept is also found in the normal way by comparing the calculated zenith distance with the observed zenith distance. The azimuth can easily be found from ABC<sup>3</sup> or similar tables.

The time taken to plot a position line using this method is appreciably less than can be achieved by the use of sight reduction tables. Moreover, while it is agreed that reduction tables provide adequate accuracy, the possibility of making an error, particularly in interpolation, is much more likely than when using a computer calculator.

The main advantage, however, of the proposed method results from the fact that the position line can be plotted from the D.R. position—there is no need to work out a chosen position. Hence if a number of simultaneous sights are used they can all be plotted from the same point, i.e. the D.R. or Estimated Position of the ship. It follows that if the Estimated Position is reasonably accurate and the plot is made from this position, then the intercept will be short and the accuracy of the plot correspondingly high.<sup>4</sup>

#### REFERENCES

- <sup>1</sup> Brown, H. H. *Nicholl's Concise Guide*, volume II. Glasgow.
- <sup>2</sup> Cotter, C. H. *The Complete Nautical Astronomer*. London.
- <sup>3</sup> Norie, J. W. *Norie's Nautical Tables* by F. N. Hopkins, London.
- <sup>4</sup> *Admiralty Manual of Navigation*, volume II. London.

## Air Navigation with a Pocket Electronic Calculator

J. D. Proctor

SOME navigators may prefer other methods and other tools, but the Hewlett Packard HP-45 has some fascinating abilities. It might be called the poor man's INS computer, though it does not of course find position. Its only disadvantages are the need for a hood in sunlight and the need for concentration and freedom from distraction. It can be used on the knee for no table is required, and it replaces conversion tables and several other mathematical tables. To a large extent it can replace plotting charts, flight planning charts, Dalton computers, protractors and dividers.

It simplifies ground speed/distance/time and fuel calculations especially, with its ability simply to convert hours, minutes and seconds or degrees, minutes and seconds to and from decimals. It calculates in the decimal mode.