PRECISION ESTIMATES OF UNIVERSAL TIME FROM RADIO-INTERFEROMETRIC OBSERVATIONS

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ABSTRACT

Considerable improvement in the determination of the motion of the Earth is possible by the potentially high accuracy inherent in very-long-baseline interferometry. Precisions of UT1 are estimated from time delay and fringe frequency measurements of extragalactic radio sources with positional uncertainties at the $0\overset{''}{.}01$ level. Case studies resulted in standard deviations about one order of magnitude smaller than those obtained by classical astrometric methods. The dependence of estimates on baseline orientations and source declinations is discussed.

1. INTRODUCTION

Standard deviations of UT1 as deduced by BIH from classical astrometric observations amount to $\sigma(\mathrm{UT1}) = 0\overset{s}{.}001$ for a 5 day mean average (Guinot, 1970; Feissel et al., 1972). Lunar laser ranging and very-long-baseline interferometry (VLBI) are promising techniques with the potential to further improve the accuracy. Currently, these techniques have reached a state which is competitive with classical methods: UT1 determination from lunar laser ranging, VLBI and BIH data indicates agreement at the level of 1 to 2 ms (King et al., 1977). Prospective improvements of VLBI techniques will enable measurements of UT1 with uncertainties equivalent to a few centimeters and with time-resolution finer than one day as compared with five day averages hitherto (Counselman, 1976).

An attempt is made to substantiate the predicted accuracy for time from VLBI measurements by a least squares estimate. In pursuing the approach realistically, baselines formed by the antennae of the Deep Space Network are mainly employed as they also figure in NASA's project for the determination of UT1, polar motion and clock synchronisation. Within the framework of this paper the dependence of parameter estimation on the declination of observed radio sources, the choice of observables, i.e. time delay and/or fringe frequency, and the geo-

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graphical location of baseline terminals is investigated on the assumption of known positions of radio sources which constitute an adopted radio reference frame.

2. SIMULATION AND PROCESSING OF OBSERVATIONS

In contrast to a rigorous treatment of real radio interferometric observations requiring corrections accounting for environmental and instrumental effects it is practical to restrict a parameter estimation study to a simplified model, which is solely defined by the interferometer geometry. For this purpose an inertial coordinate system with origin at the centre of the Earth is constructed. Vectors to the antennae are designated $\vec{r}_i(t)$, i = 1, 2,.... Let $\vec{r}_s$ be the unit vector to the radio source. Then, under these purely geometrical conditions, the zero-order baseline $\vec{b}$, time delay τ and fringe frequency f are defined by

$$\vec{b} = \vec{b}(t) = (\vec{r}_i - \vec{r}_k) = d\,\vec{r}_b \ , \qquad (1)$$

where d denotes the baseline length and $\vec{r}_b$ the unit vector along the line interconnecting two antennae;

$$\tau = \frac{d}{c}\,\vec{r}_s\,\vec{r}_b \qquad (2)$$

and

$$f = \omega\,\frac{d}{c}\,\vec{r}_s\,\dot{\vec{r}}_b \ , \qquad (3)$$

which holds for radio sources of negligible proper motion such as extragalactic sources. Here, ω denotes the signal frequency and is kept constant at the level of 2.3 GHz. For details on radio interferometry and astrometrical aspects reference is made to Cohen and Shaffer (1971) and Counselman (1976).

Observations were simulated for samples consisting of 8 radio sources and 2 or 3 baselines. In order to discover possible dependences of parameter estimations on source declinations the sources were subdivided in three groups, G1, G2, G3 ranging from $0^\circ < \delta < +25^\circ$, $+20^\circ < \delta < +45^\circ$ and $-20^\circ < \delta < +5^\circ$, respectively. G1 comprises the sources 3C 120, 3C 138, 0736+01, OJ 287, 3C 273, OQ 208, CTA 102, 3C 454.3; G2 the sources 3C 48, 3C 84, OJ 287, 4C 39.25, 3C 286, OQ 208, 3C 345, BL Lac; G3 the sources CTA 26, 3C 120, 0736+01, 3C 273, 3C 279B, 1741-038, OX 057, 2345-16.

Likewise, a variety of baseline configurations was chosen to examine the influence of large equatorial versus large polar baseline components on the variances and covariances. Essentially, the study rests on baselines constituted by the antennae of NASA's Deep Space Network (Goldstone, Calif.; Robledo, Spain; Tidbinbilla, Australia), tentatively augmented by the radio telescope located at Hartebeesthoek (South Africa).

Whenever mutual visibility for a source from the baseline terminals was secured observation times in steps of about 30 minutes were generated throughout the visibility periods during 24 hours. The observations were assumed normally distributed with mean zero and uncorrelated with standard deviations of $\sigma(\tau) = 10^{-9}$ s and $\sigma(f) = 10^{-4}$ Hz, which corresponds to a source position uncertainty of the order of 0.″01 for a baseline length of 5000 km.

The estimation of UT1 necessitates the setting up of the partial derivatives of the observables with respect to Greenwich sidereal time α_G. As the instantaneous baseline vector $\vec{b}$ is referred to the geographical position vectors $\vec{\rho}_1$ and $\vec{\rho}_2$ of the terminals through

$$\vec{b} = R_G\ R_P(\vec{\rho}_2-\vec{\rho}_1)_{CIO} \qquad (4)$$

the required partial derivatives are obtained from Eqs. (1) and (2) by differentiating $\vec{b}$ and $\dot{\vec{b}}$ accordingly. The matrix R_P accounts for polar motion, whereas the matrix R_G rotates the baseline vector to its instantaneous orientation by the angle $\alpha_G = \alpha_G(t)$, the right ascension of Greenwich.

3. DISCUSSION OF RESULTS

Variances averaged over one day are deduced from 100 observations each for different sets of radio sources and baselines from customary covariance matrix calculations (e.g. Liebelt, 1967).

For sources subjected to the criterion of simultaneous visibility from terminals the baseline Tidbinbilla-Robledo did not yield noteworthy source availability times and was, therefore, omitted. Thus, the parameter estimates concentrated on the following baseline configurations: Robledo-Goldstone, Goldstone-Tidbinbilla, (I), both of which are baselines with predominant equatorial projections; Robledo-Goldstone, Goldstone-Tidbinbilla, Hartebeesthoek-Robledo, (II), where the last baseline is characterized by a large polar component; Robledo-Goldstone, Hartebeesthoek-Robledo, (III), in which configuration the baselines of sizable equatorial and polar components have an equal share; Goldstone-Tidbinbilla, Hartebeesthoek-Robledo, (IV), with baselines of large polar-axis projection.

The standard deviations of UT1 for a three parameter model are depicted in Figure 1 arranged by source declinations and baseline configurations. While the standard deviations of UT1 for a combined solution of an equal number of time-delay and fringe-frequency observations are practically independent of source declinations they exhibit noticeable differences concerning the baseline configuration. Baselines with large polar-axis projection cause unfavourable standard deviations as demonstrated by configuration IV, whereas baselines with large equatorial projections provide the better estimates.

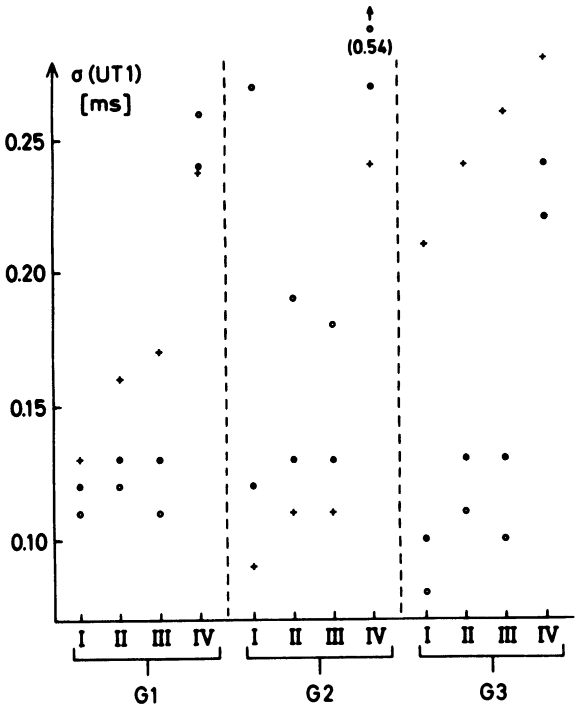

Fig. 1. Dependence of UT1 estimates on source declination (G1 to G4), baseline network (I to IV) and observables (τ : o, f : +, τ & f : ●)

This fact is even more pronounced when only one type of observable , τ or f, is analysed. Evidently, the determination of UT1 from time delay observations of sources in the equatorial zone (e.g. G3) is more precise than from any other distribution of source declinations. The results obtained for fringe frequency, however, suggest that UT1 is more sensitive when sources of higher declinations such as G2 participate in the observation campaign.

Since a covariance analysis is closely related to the derivatives of the observables with respect to the parameters to be estimated a partial explanation of the behaviour of the standard deviations, at least, can be expected from the analysis of the derivatives. Williams (1970) has performed such analysis for fringe frequency observations. Accordingly, UT1 is most accurately determined for sources at zero-degree declinations and improves with increasing equatorial projection of the baseline.

These results are confirmed by the current covariance analyses with the exception that the superiority of UT1 estimates, although minimal, from fringe frequency observations of sources in the zone $+20^\circ < \delta < 45^\circ$ contradicts the conclusions from partial derivatives which predict detection of the best estimates from sources at zero-degree declinations (e.g. G1 or G3). Presumably the discrepancy can be attributed to the choice of source declinations from a narrow zone around zero-degree, thus giving rise to near - linearity of observational partial derivatives and leading to ill-conditioned covariance matrices.

4. CONCLUSION

Table 1 summarizes the present state of accuracy for UT1 and lists the prospective estimates deduced from observations obtained through VLBI. In the course of the estimation process no allowance was made for signal disturbance caused by the propagation-medium, the instrumentation and the clock offset. Further, it was assumed that the effects of Earth tides, continental drift and possible others were properly accounted for in the data reduction. For these reasons the estimates may have turned out too optimistic and are likely to deteriorate under real conditions. To the extent that the current results are representative, determination of UT1 by VLBI techniques appears to be most effective for time-delay and fringe-frequency observations of sources in the equatorial zone acquired from a net of baselines in which large equatorial and

Method of Observation	σ(UT1) [s]
Classical astrometry 5 day resolution (Guinot, 1970; Feissel et al., 1972)	0.001
Lunar Laser Ranging 5 day resolution (Harris and Williams, 1977)	0.0014
Lunar Laser Ranging (King et al., 1977)	∿0.001
VLBI - pilot study (Shapiro et al., 1974)	0.002
VLBI - parameter estimate 1 day resolution (Moran, 1973)	0.0008
VLBI - parameter estimate 1 day resolution (c.f. this paper)	0.0002

Table 1. Standard errors of UT1

polar-axis projections are kept in balance.

REFERENCES

Cohen, M.H., Shaffer, D.B.: 1971, Astron. J. 76, pp. 91-100.

Counselman III, C.C.: 1976, Annual Rev. of Astron. and Astrophys. 14, pp. 197-214.

Guinot, B.: 1970, in "Earthquake Displacement Fields and the Rotation of the Earth", ed. L. Mansinha, D.E. Smylie, A.E. Beck; D. Reidel Publ. Comp., Dordrecht, Holland, pp. 54-62.

Harris, A.W., Williams, J.G.: 1977, in "Scientific Applications of Lunar Laser Ranging", ed. J.D. Mulholland; D. Reidel Publ. Comp., Dordrecht, Holland, pp. 179-190.

King, R.W., Clark, T.A., Knight, C.A., Counselman III, C.C., Robertson, D.S., Shapiro, I.I.: 1977, in "Scientific Applications of Lunar Laser Ranging", ed. J.D. Mulholland; D. Reidel Publ. Comp. Dordrecht, Holland, pp. 219-220.

Liebelt, P.B.: 1967, An Introduction to Optimal Estimation, Addison-Wesley Publ. Comp., Reading, Massachusetts.

Moran, J.M.: 1973, in "Space Research XIII", Vol. 1, ed. M.J. Rycroft, S.K. Runcorn; Akademie-Verlag, Berlin, pp. 73-82.

Shapiro, I.I., Robertson, D.S., Knight, C.A., Counselman III, C.C., Rogers, A.E.E., Hinteregger, H.F., Lippincott, S., Whitney, A.R., Clark, T.A., Niell, A.E., Spitzmesser, D.J.: 1974, Science 186, pp. 920-922.

Williams, J.G.: 1970, Space Programs Summary 37-62, Vol. II, Jet Propulsion Laboratory, Pasadena, Calif., pp. 37-50.

DISCUSSION

K. Johnston: Did your calculations include limitations due to source structure?

Dr Carter presented data in which the accuracy of the baseline was about 5 cm; this is better than the value you have assumed in your calculations.

H.G. Walter: No limitations due to source structure were included, but the assumed precision of the source positions was set at 0.01 seconds of arc.

A time delay precision of 1 ns has been tentatively assumed for this computer simulation; it could be improved to the level quoted by Dr Carter.