axiom of power objects is formulated for the setting of NF. The most interesting direction of the equiconsistency result is established by interpreting the set theory in the category theory, through the machinery of categorical semantics, thus making essential use of the flexibility inherent in category theory. Moreover, $\mathrm{ML}_{\mathsf{Cat}}$ is connected to topos theory as follows:

THEOREM. *For any category* $\mathbf{C} \models \mathrm{ML}_{\mathsf{Cat}}$*, with endofunctor* $\mathbf{T}$*, the full subcategory on the fixed-points*[1] *of* $\mathbf{T}$ *is a topos.*

The relevance of this categorical work lies in that it provides a basis for studying the dynamics of NF within the realm of category theory. In particular, it opens up for constructions of categorical models of intuitionistic versions of NF, and for stratified approaches to type-theory. It may also be relevant for attempts to prove or simplify proofs of the consistency of classical NF.

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SAEIDEH BAHRAMI, ***Self-embeddings of Models of Peano Arithmetic***, Tarbiat Modares University, Iran, 2018. Supervised by Ali Enayat. MSC: 03C62, 03F30, 03H15. Keywords: Peano arithmetic, nonstandard model, self-embedding, fixed point, strong cut.

Abstract

This thesis investigates the behavior of *fixed points* of embeddings between countable nonstandard models of fragments $\mathrm{I}\Sigma_{n+1}$ of PA (Peano arithmetic), for $n \in \omega$. Our concentration is on studying the fixed points of embeddings of a countable nonstandard model $\mathcal{M}$ into a countable nonstandard model $\mathcal{N}$ whose images are proper initial segments of $\mathcal{N}$, namely *initial embeddings*. This area of research was initiated by Friedman [4] (who proved that every countable nonstandard model of PA is isomorphic to a proper initial segment of itself), and continued by a number of researchers, including Dimitracopoulos and Paris [3], Ressayre [6], and Tanaka [7]. Most of the results in the thesis appear in [1] and [2]. The following is a summary of the highlights of the thesis:

(1) For a given pair of countable nonstandard models $\mathcal{M}$ and $\mathcal{N}$ of $\mathrm{I}\Sigma_{n+1}$ which share a proper cut I that is closed under exponentiation, a sufficient condition for the existence of a Σ_n-elementary initial embedding j from $\mathcal{M}$ into $\mathcal{N}$ which fixes each element of I is that both $\mathcal{M}$ and $\mathcal{N}$ have the same I-standard system and $\mathrm{Th}_{\Sigma_{n+1}}(\mathcal{M}, i)_{i\in \mathrm{I}} \subseteq \mathrm{Th}_{\Sigma_{n+1}}(\mathcal{N}, i)_{i\in \mathrm{I}}$ (this is a generalization of a theorem by Hájek and Pudlák [5]). Moreover, if there exists such a Σ_n-elementary initial embedding, then there are continuum many of them with distinct images (this generalizes a theorem of Wilkie [8] that asserts that every countable nonstandard model of PA is isomorphic to continuum many initial segments of itself).

(2) For every countable nonstandard model $\mathcal{M}$ of $\mathrm{I}\Sigma_{n+1}$, a proper cut I of $\mathcal{M}$ is closed under exponentiation iff there exists a proper Σ_n-elementary (initial) self-embedding j of $\mathcal{M}$ such that I is the longest initial segment of fixed points of j. The $n = 0$ case of this result is also extended to models of the subsystem WKL_0 of second-order arithmetic, thereby leading to a generalization of Tanaka's self-embedding theorem [7].

[1]An object A of $\mathbf{C}$ is a *fixed-point* of the (axiomatized) endofunctor $\mathbf{T}$ of $\mathbf{C}$ if $\mathbf{T}A \cong A$ and this is witnessed by a certain natural isomorphism characterized by axioms of $\mathrm{ML}_{\mathsf{Cat}}$.

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(3) The strong cuts I of a countable nonstandard model $\mathcal{M}$ of $I\Sigma_{n+1}$ which are Σ_{n+1}-elementary submodel of $\mathcal{M}$ are precisely those cuts of $\mathcal{M}$ for which there is a proper Σ_n-elementary (initial) self-embedding j of $\mathcal{M}$ such that I is the set of all fixed points of j.

(4) The standard cut $\mathbb{N}$ is strong in a countable nonstandard model $\mathcal{M}$ of $I\Sigma_{n+1}$ iff there is a proper Σ_n-elementary (initial) self-embedding of $\mathcal{M}$ which moves all Σ_{n+1}-undefinable elements of $\mathcal{M}$.

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Lukas Daniel Klausner, ***Creatures and Cardinals***, Technische Universität Wien, Austria, 2018. Supervised by Martin Goldstern. MSC: Primary 03E17, Secondary 03E35, 03E40. Keywords: cardinal characteristics of the continuum, continuum hypothesis, localisation cardinals, anti-localisation cardinals, creature forcing, Yorioka ideals, Cichoń's diagram, splitting number, reaping number, independence number.

Abstract

This thesis collects several related results on cardinal characteristics of the continuum, all of which employ the method of creature forcing.

In Chapter A, we use a countable support product of lim sup creature forcing posets to show that consistently, for uncountably many different functions the associated Yorioka ideals' uniformity numbers can be pairwise different. (For an introduction on Yorioka ideals, see [1].) In addition, we show that, in the same forcing extension, for two other types of simple cardinal characteristics parametrised by reals (localisation and antilocalisation cardinals), for uncountably many parameters the corresponding cardinals are pairwise different. The proofs are based on standard creature forcing methods and Tukey connections.

In Chapter B, we disassemble, recombine, and reimplement the creature forcing construction used by Fischer/Goldstern/Kellner/Shelah [2] to separate Cichoń's diagram into five cardinals as a countable support product with more easily understandable internal structure. Using the fact that it is of countable support, we augment the construction by adding uncountably many additional cardinal characteristics, namely, localisation cardinals. The proofs use both creature forcing and combinatorial methods.

In Chapter C, we introduce nine cardinal characteristics related to the splitting number $\mathfrak{s}$, the reaping number $\mathfrak{r}$ and the independence number $\mathfrak{i}$ by using the notion of asymptotic density to characterise various intersection properties of infinite subsets of ω. We prove