The dissertation is motivated by the $\Sigma_1$-elementary substructure problem: Can one structure in the following structures $\mathcal{R} \subsetneqq \mathcal{D}_2 \subsetneqq \cdots \subsetneqq \mathcal{D}_\omega \subsetneqq \mathcal{D}_{\omega+1} \subsetneqq \cdots \subsetneqq \mathcal{D}(\leq \mathbf{0}')$ be a $\Sigma_1$-elementary substructure of another? For finite levels of the Ershov hierarchy, Cai, Shore, and Slaman [***Journal of Mathematical Logic***, vol. 12 (2012), p. 1250005] showed that $\mathcal{D}_n \not\preceq_1 \mathcal{D}_m$ for any $n < m$. We consider the problem for transfinite levels of the Ershov hierarchy and show that $\mathcal{D}_\omega \not\preceq_1 \mathcal{D}_{\omega+1}$. The techniques in Chapters 2 and 3 are motivated by two remarkable theorems, Sacks Density Theorem and the d.r.e. Nondensity Theorem.

In Chapter 1, we first briefly review the background of the research areas involved in this thesis, and then review some basic definitions and classical theorems. We also summarize our results in Chapter 2 to Chapter 4. In Chapter 2, we show that for any $\omega$-r.e. set $D$ and r.e. set $B$ with $D <_T B$, there is an $\omega+1$-r.e. set $A$ such that $D <_T A <_T B$. In Chapter 3, we show that for some notation $a$ with $|a|_o = \omega^2$, there is an incomplete $\omega+1$-r.e. set $A$ such that there are no $a$-r.e. sets $U$ with $A <_T U <_T K$. In Chapter 4, we generalize above results to higher levels (up to $\varepsilon_0$). We investigate Lachlan sets and minimal degrees on transfinite levels and show that for any notation $a$, there exists a $\Delta^0_2$-set $A$ such that $A$ is of minimal degree and $A \not\equiv_T U$ for all $a$-r.e. sets $U$.

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ALEJANDRO POVEDA, ***Contributions to the Theory of Large Cardinals through the Method of Forcing***, Doctorate program in Mathematics and Computer Science—Universitat de Barcelona, Barcelona, Spain, 2020. Supervised by Joan Bagaria. MSC: Primary 03Exx, Secondary 03E50, 03E57. Keywords: Large Cardinals, Forcing, Prikry-type forcings, Singular Cardinal Combinatorics.

**Abstract**

The dissertation under comment is a contribution to the area of Set Theory concerned with the interactions between the method of Forcing and the so-called Large Cardinal axioms.

The dissertation is divided into two thematic blocks. In Block I we analyze the large-cardinal hierarchy between the first supercompact cardinal and Vopěnka's Principle (Part I). In turn, Block II is devoted to the investigation of some problems arising from Singular Cardinal Combinatorics (Part II and Part III).

We commence Part I by investigating the Identity Crisis phenomenon in the region comprised between the first supercompact cardinal and Vopěnka's Principle. As a result, we generalize Magidor's classical theorems [2] to this higher region of the large-cardinal hierarchy. Also, our analysis allows to settle all the questions that were left open in [1]. Finally, we conclude Part I by presenting a general theory of preservation of $C^{(n)}$-extendible cardinals under class forcing iterations. From this analysis we derive several applications. For instance, our arguments are used to show that an extendible cardinal is consistent with "$(\lambda^{+\omega})^{\mathrm{HOD}} < \lambda^+$, for every regular cardinal $\lambda$." In particular, if Woodin's HOD Conjecture holds, and therefore it is provable in ZFC + "There exists an extendible cardinal" that above the first extendible cardinal every singular cardinal $\lambda$ is singular in HOD and $(\lambda^+)^{\mathrm{HOD}} = \lambda^+$, there may still be no agreement at all between V and HOD about successors of regular cardinals.

In Part II and Part III we analyse the relationship between the Singular Cardinal Hypothesis (SCH) with other relevant combinatorial principles at the level of successors of singular cardinals. Two of these are the Tree Property and the Reflection of Stationary sets, which are central in Infinite Combinatorics.

Specifically, Part II is devoted to prove the consistency of the Tree Property at both $\kappa^+$ and $\kappa^{++}$, whenever $\kappa$ is a strong limit singular cardinal witnessing an arbitrary failure of the

SCH. This generalizes the main result of [3] in two senses: it allows arbitrary cofinalities for $\kappa$ and arbitrary failures for the SCH.

In the last part of the dissertation (Part III) we introduce the notion of $\Sigma$-Prikry forcing. This new concept allows an abstract and uniform approach to the theory of Prikry-type forcings and encompasses several classical examples of Prikry-type forcing notions, such as the classical Prikry forcing, the Gitik-Sharon poset, or the Extender Based Prikry forcing, among many others.

Our motivation in this part of the dissertation is to prove an iteration theorem at the level of the successor of a singular cardinal. Specifically, we aim for a theorem asserting that every $\kappa^{++}$-length iteration with support of size $\leq \kappa$ has the $\kappa^{++}$-cc, provided the iterates belong to a *relevant* class of $\kappa^{++}$-cc forcings. While there are a myriad of works on this vein for regular cardinals, this contrasts with the dearth of investigations in the parallel context of singular cardinals. Our main contribution is the proof that such a result is available whenever the class of forcings under consideration is the family of $\Sigma$-Prikry forcings. Finally, and as an application, we prove that it is consistent—modulo large cardinals—the existence of a strong limit cardinal $\kappa$ with countable cofinality such that $\mathrm{SCH}_\kappa$ fails and every finite family of stationary subsets of $\kappa^+$ reflects simultaneously.

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Pierre Touchard, ***Transfer Principles in Henselian Valued Fields***, Westfälische Wilhelms-Universität Münster, Germany, 2020. Supervised by Martin Hils. MSC: Primary 03C45, 03C60, Secondary 12J10. Keywords: model theory of valued fields, classification theory, stable embeddedness.

**Abstract**

In this thesis, we study transfer principles in the context of certain Henselian valued fields, namely Henselian valued fields of equicharacteristic 0, algebraically closed valued fields, algebraically maximal Kaplansky valued fields, and unramified mixed characteristic Henselian valued fields with perfect residue field. First, we compute the burden of such a valued field in terms of the burden of its value group and its residue field. The burden is a cardinal related to the model theoretic complexity and a notion of dimension associated to $\mathrm{NTP}_2$ theories. We show, for instance, that the Hahn field $\mathbb{F}_p^{\mathrm{alg}}((\mathbb{Z}[1/p]))$ is inp-minimal (of burden 1), and that the ring of Witt vectors $W(\mathbb{F}_p^{\mathrm{alg}})$ over $\mathbb{F}_p^{\mathrm{alg}}$ is not strong (of burden $\omega$). This result extends previous work by Chernikov and Simon and realizes an important step toward the classification of Henselian valued fields of finite burden. Second, we show a transfer principle for the property that all types realized in a given elementary extension are definable. It can be written as follows: a valued field as above is stably embedded in an elementary extension if and only if its value group is stably embedded in the corresponding extension of value groups, its residue field is stably embedded in the corresponding extension of residue fields, and the extension of valued fields satisfies a certain algebraic condition. We show, for instance, that all types over the power series field $\mathbb{R}((t))$ are definable. Similarly, all types over the quotient field of $W(\mathbb{F}_p^{\mathrm{alg}})$ are definable. This extends previous work of Cubides and Delon and of Cubides and Ye.