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As will be apparent from the Report, notable progress has been accomplished in many directions during the period under review especially in the field of stellar evolution, where some of the most interesting phases are being tackled leading in some cases to the discovery of new forms of instability. Interaction between dynamical instability towards non-radial oscillations and convective instability has been clarified and delicate forms of instabilities in presence of superadiabatic gradients have been explored. Interest in stellar rotation has increased and has brought sometimes unexpected results as the resulting increase of the critical mass of white dwarfs. Neutrino astrophysics raises fascinating questions. Non-linear methods have definitely entered the field of variable star theories. The attack of the supernovae phenomenon takes by and by a quantitative aspect. The fundamentals of magnetohydrodynamics as applied to stellar structure are emerging. A proper theory of stellar convection may still be a long way off but progress is undoubtedly on the way. The discovery of quasi-stellar-sources and the attempts to find an interpretation in terms of very large masses as well as the interest in stars of very high densities have opened a whole new field in the theory of stellar structure where the general relativity terms become important. No doubt, the recent discoveries of cold infrared objects and of concentrated X-ray sources will create new spectacular developments in the activities of our Commission and may already require new sections in the next of its reports to the IAU.

I. NUCLEAR ENERGY GENERATION, NEUTRINO EMISSION AND NUCLEOSYNTHESIS

In this section as in many others of this report, due to the growth of the subject or of its complexities, it seems impossible to try to account for many of the relevant papers beyond repeating essentially the titles, often quite explicit, and which will be found, in such cases, in the bibliography.

Comprehensive review articles attempting to bring up to date previous compilations and presenting a critical survey of the field of nuclear energy generation and low energy nucleosynthesis in stellar interiors have been prepared by Reeves (1).

Experimental results of interest for nuclear astrophysics have been reported by Pearson and Spear (2), Larson and Spear (3), Domingo (4) and McNally (5). New experimental cross-sections as well as the modern theory of nuclear beta-decay have been used to discuss the termination of the proton-proton chain in stellar interiors (6, 7, 8), the results in reference (8) implying an increase of the $\text{Be}^7(p, \gamma)\text{B}^8$ reaction leading to a doubling of the possible solar neutrinos flux. Different points concerning the nuclear reaction rates in stellar conditions are examined in papers (9-13), the last one, in particular, presenting methods for finding

accurate nuclear reaction cross-sections averaged over resonances for astrophysical purposes. The evaluation of these rates has been extended by Wolf (14) and by Van Horn (15) to situations arising in the interior of a white dwarf when the Coulomb energies between neighboring nuclei are large compared to their kinetic energies. Bahcall (16) has studied the rates of capture of free electrons by nuclei in stars and especially their dependence on temperature and density, considering different applications. More recently (17), he has discussed the physical basis for exchange and overlap corrections in these electron capture processes.

As far as nucleosynthesis in stars is concerned, Bashkin (18) has prepared a critical survey of the literature up to 1962–63. A general article on the origin of the elements was also written by Fowler (19). Caughlan (20) has studied in detail the abundances of the nuclei involved in the CNO bi-cycle as equilibrium is approached and the implications for the interpretation of the ratios of C^{13} to C^{12} and of nitrogen to oxygen observed in typical carbon stars. The effects of combined hydrogen and helium burning in the core of a population II red-giant star have also been discussed (21, 22) in an attempt to interpret the anomalous abundances of carbon, barium and the rare-earths in these stars. Wielding (23) reports work in progress on alpha capture reactions at high temperatures expected to exist in supernovae, using recent nuclear data. The problem of nucleosynthesis of elements heavier than the iron group by neutron capture on both slow and fast time scales has been rediscussed by Seeger and Fowler (24) using new cross-section data. On this basis, they find that the solar-system *r*-process material must have been submitted to extreme conditions which could have been realized in an object with a mass of the order of $10^6 M_{\odot}$. On the other hand, Reeves (25) has reviewed the different neutron-emitting processes in ordinary stars and has evaluated the flux of neutrons per metal atom made available at different stages of evolution through these processes which in general do not overlap. The capture of positrons by *s*-elements during the carbon and oxygen burning stages has been considered by Reeves and Stewart (26) as a possible source of the *p*-elements. Shaw, Clayton and Michel (27) have developed the theory of photon induced β -decay in stellar interiors which may also play an interesting role in that respect.

Calculations of nuclear abundances at statistical equilibrium for 333 sets of values of the temperature, density and ratio of total number of protons to total number of neutrons are reported by Clifford and Tayler (28) and detailed results are given for 96 of the cases. The importance of the equilibrium process for the iron group elements has also been discussed by Fowler (29) and Tsuruta and Cameron (30) have investigated the shift of the Fe equilibrium peak to higher mass number at high densities. The approach to nuclear statistical equilibrium has been studied by Truran, Cameron and Gilbert (31) by solving the nuclear rate equations under conditions of advanced stellar evolution and supernova shock waves.

The formation of the light elements D, Li, Be, B in the solar system by spallation reactions in metric-sized planetesimals as suggested first by Fowler, Greenstein and Hoyle has been further discussed by Mitler (32) and by Burnett and Fowler (33). Recently the problem of the formation of these elements and its meaning for the solar system and stars, in particular T Tauri stars, has also been the object of much interest and research in the group around Reeves and Schatzman in Orsay and Paris (34).

A new attempt to explain the anomalous abundances in peculiar stars of type A and B is due to Fowler, E. M. Burbidge, G. R. Burbidge and Hoyle (35). Considering that these peculiar abundances can only be explained by the combined effect of violent surface spallation (underabundance of light elements especially helium) and a process of rapid neutron capture (high abundance of heavy elements in the rare-earth region) followed by mixing, they have suggested that these stars have returned to the neighborhood of the main sequence after a long and complicated evolution. Some of their conclusions have been criticized by Truran and Cameron (36). Brancazio is reported as re-examining the feasibility of surface nuclear reactions to explain the extreme abundance anomalies in magnetic Ap stars. The problem of surface nuclear reactions

has also been reviewed in general by Audouze (37) who finds interesting correlations between abundances and the energy of the processes involved.

The general problem of dating the time of nucleosynthesis in our Galaxy by a comparison of the cosmoradiogenic abundances of daughter species with the abundances of their radioactive parents has been discussed by Clayton (38).

Let us note also that the problem of the origin of helium has reached at present an extremely interesting stage. The difficulties encountered up to now by all schemes attempting to explain the formation in stars of the bulk of the galactic helium (39) have led Hoyle and Tayler (40) to revert to a cosmological origin of this element. The subsequent discovery of the background radiation at 3°K, in providing evidence for the conditions prevailing very early in the expansion of the universe have considerably reinforced the plausibility of such an origin. But, on the basis of an ordinary isotropic relativistic expansion, we are now faced with a very large pre-galactic helium abundance, perhaps too large according especially to some recent observational evidence, and cosmologists are trying to devise anisotropic models capable of reducing it (41). On the other hand, it seems unavoidable that some helium be produced in the galaxy itself and Boursy and Ledoux (42) are investigating the possible effect, in that respect, of large mass stars evolving homogeneously and shedding their excess of mass above the critical mass corresponding to vibrational instability, the latter decreasing rapidly as hydrogen is transformed into helium.

The role of neutrinos in astrophysics and cosmology has been the object of intense activity and quite a few general summaries (43–47) have appeared since 1964 discussing both the nuclear and astronomical aspects of the problem.

Bahcall has pursued his investigations on the emission of solar neutrinos and their possible detection (48, 49) and the problem has been discussed also by Pochoda and Reeves (50) and Kuzmin (51).

Bahcall and Frautschi (52) have also considered the possibility of neutrino emission by strong radio-sources and observational tests are discussed.

Neutrino opacity and scattering in a variety of stellar conditions have been discussed with a view to applications to the detection of solar neutrinos, the escape of neutrinos from stars, neutrino scattering in cosmology and the energy deposition in supernovae explosions (53, 54, 55, 56). Sakashita and Nishida (57) have computed the energy loss rate by neutrino emission from excited nuclei due to the interaction of the type $(n, n) (\nu, \nu)$ which was suggested by Pontecorvo, showing that it is at most comparable with other neutrino processes.

Inman and Ruderman (58) have computed the neutrino pair emission from coherent electron excitation (transverse plasmons) for a range of temperatures and densities relevant to stellar evolution. Correction to the longitudinal and transverse neutrino emissivities of Adams, Ruderman and Woo in a stellar plasma are given by Zaidi (59).

The URCA process has been rediscussed, taking into account the pair production at high temperature, by Pinaev (60) and Wataghin (61) and applications have been considered by Masani *et al.* (62).

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II. RELATIVISTIC STELLAR MODELS (SUPERDENSE OR SUPERMASSIVE STARS)

In the last three or four years, much effort has been devoted to what is now called Relativistic Astrophysics. In the field, the 'Texas Symposia' are becoming institutional (1). From our viewpoint, three main subfields can be distinguished, although some overlapping is unavoidable: study of the very dense configurations, attempts to interpret quasi-stellar objects in terms of huge supermassive stars and the study of gravitational collapse. Very masterful reviews, some of them giving more emphasis on the superdense (neutron) stars, have been published by

Zel'dovitch and Novikov (2, 3), Harrison, Thorne, Wakano and Wheeler (4), Wheeler (5) and Thorne (6, 7). They contain very extensive bibliography. We will give here a good deal of typical titles but without any claim to exhaustivity. We rather refer the reader back to the reviews above.

Let us first turn towards usual stellar evolution; it is currently thought that the final term in a star's life lies in the white dwarf stage, the neutron stage (both stable below some mass) or gravitational collapse, the fate of the star depending on its mass when it reaches the end point of thermonuclear evolution. The neutron star or the hyperon star have been given a great deal of attention, both from the point of view of the structure and of the properties of matter: various equations of state and forms of the interaction potential have been used. Their stability, their oscillations and their energy emissivity in the form of photons (surface X-rays), of neutrinos and of gravitational waves have also been the subject of much research. Some of the works involve rotation and non-zero temperature (for all these points, *cf.* references 2 to 41).

The onset of dynamical instability due to General Relativity effects in supermassive 'stars' is still being contemplated as possibly significant for the quasi-stellar objects and the strong radio-sources. The state of affairs by 1964 was described by Fowler on various occasions (43, 44, 45). Since that time, more investigations on the structure of supermassive stars and relativistic configurations have been carried out, some of them on very general grounds; for instance, Buchdahl (46) has improved the general inequalities he had derived in 1959 (47) for regular fluid spheres.

Some simplified models more easily amenable to mathematical treatment: relativistic polytropic and 'adiabatic' structures have been studied by Gratton and Giannone (49), Van der Borcht (50) and Tooper (51, 52, 53) who also checked the validity of the post Newtonian approximation in the binding energy calculation (54) as compared with its 'standard model' (53). Tooper considers in detail the conditions of dynamical instability for his various models. Some of his work (52) also applies to the superdense stars considered above. As far as the theory of stellar interiors is concerned, more physical considerations have been developed by Hämeen-Anttila and Anttila (55, 56, 57, 58) and are also described briefly in a very intuitive manner in Thorne's reviews (6, 7). Thorne has studied locally the convection criterion in General Relativity establishing in all generality the validity of Schwarzschild criterion (59) already recovered by Chandrasekhar (60) in the post-Newtonian approximation, as a result of his general analysis of radial and non-radial oscillations of spherical gaseous masses. Sato (61) has studied non rotating massive stars and gives conditions for the onset of the instability. Let us also recall the work of Bardeen *et al.* (32) giving methods for studying the radial pulsations of General Relativistic Models, and a talk by Bardeen at the Second Texas symposium (62). The pulsations and evolution of supermassive stars have also been studied by Osaka who finds the expected vibrational instability (63). As far as quasistellar objects are concerned, an oscillatory explanation of the variations of 3C 273 has been attempted by Anand (64) and a model based on relaxation oscillations in supermassive stars was proposed by Fowler (65) but was criticized by Zel'dovitch and Novikov (2). Finally, extra factors have been taken into account, rotation by Chandrasekhar (66), Roxburgh (67), Dubney and Roxburgh (68), and magnetoturbulent energy by Anand (69) and Bardeen and Anand (70). Those factors can significantly delay the setting in of dynamical instability and allow the extension of stable models with energy generation by thermonuclear reactions to much larger masses. However, as has been pointed out on various occasions, these models would then become strongly vibrationally unstable even well before the nuclear energy generation accounts for the total luminosity of the star (63, 71).

As has been indicated, stars more massive than a certain critical mass (corresponding to about twice the baryon content of the Sun) cannot settle down into an equilibrium configuration when they arrive at the end-point of thermonuclear evolution if they are prevented to get

rid of the excess mass. If this is possible, as in the case studied by Colgate and White (72) in their work on supernovae, they end as neutron stars. If matter ejection does not take place, the body should undergo a limitless collapse. On the other hand, supermassive stars, albeit tenuous and containing considerable nuclear energy storage, are also dynamically unstable in the absence of rotation or magnetic fields. All this has led to much current interest in gravitational collapse. Again, the reviews referred to at the beginning of this subsection are of prime interest. A very brief but clear description of the collapse is given at the end of Wheeler's paper (5). An extensive treatment in terms of Kruskal metric (73) is found in Thorne's Les Houches lectures (7). The book (4) by Harrison *et al.* does not limit itself to astronomical considerations (e.g. the quantum probability of a mass however small to undergo gravitational collapse by tunnel effect is taken in consideration). Let us also mention the anticollapse theory, which is not the time reversal of the contraction, briefly reviewed by Zel'dovitch and Novikov (2).

Dynamical calculations of General-Relativistic collapse, ignoring heat transfer by neutrinos, radiation etc., have been made by May and White (74) who do not find very different critical masses, compared with the static case (5). They use equations obtained also by Misner and Sharp (75). Misner (76) and Podurets, Zel'dovitch and Gusseinov (77, 78, 79) have studied collapse with escaping neutrinos. Numerical calculations have been carried out by Imshennick, Nadezhin, Gurevitch and Gusseinov (80, 81). Collapse with radiation has been treated analytically by Vardya (82). Nariai and Tomita have studied several problems appearing in the General Relativistic treatment of collapsing supermassive stars. Nariai (83) has first found the formulation in which Einstein's equations have no singularity at the stellar surface. Using this, Nariai and Tomita (84, 85) have proposed a way to treat the collapse system free from the Schwarzschild singularity. They have also shown (86) that the total energy loss by neutrino emission during the collapse is smaller than one tenth of the mass energy if the stellar mass is inferior to $10^6 M_{\odot}$. The behavior of the central region of a collapsing star with spherical symmetry and a finite pressure gradient has also been studied, touching upon the case of non-adiabatic collapse (87). Possible relevance of the C-field has been investigated (88). McVittie (89) has also treated some aspects of the question.

Rotation (90), magnetohydrodynamics (91), scalar interactions in superdense matter (92) have also been taken into consideration. Bonnor (93) finds that charged interior solutions may be relevant to the problem of the collapse. Harrison (94) discusses the meaningfulness of the study in an external system of the ultimate fate of a gravitationally collapsing body in the frame of different models of the universe.

Synge (95) has discussed the asymptotic behavior of photons escaping from gravitationally intense stars as $2GM/Rc^2$ tends towards unity. In this connexion, the problem of invisible ('hidden') masses in the universe has been brought up again. This has been discussed by Hoyle, Fowler, Burbidge and Burbidge (96), and Gratton (97). Zel'dovitch and Gusseinov (98) suggest the possibility of detecting collapsed stars in spectroscopic binaries.

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III. OPACITY, EQUATION OF STATE, CONDUCTIVITY, ETC.

The important work of A. N. Cox *et al.* on stellar absorption coefficients and opacities has now been published (**1**, **2**, **3**) and contains a great deal of useful information not only on opacities but also on ionization, conductivity, equation of state, etc. A review of the general problem has also been published by Mayer (**4**).

Chiu (**5**) has calculated the Rosseland mean opacity due to Compton scattering in the semi-relativistic, semi-degenerate region. We may also report evaluation of thermal conductivity of a fully ionized gas by Sundaresan and Ta-You Wu (**6**) and by Wyller (**7**) in the case of degenerate stellar cores.

Rouse (**8**) has extended his investigations on ionization equilibrium and equations of state to high densities introducing in Saha equation a factor depending on the density and representing the probability that an ion can exist in a mean atomic volume related to a density-dependent normalization of the isolated atom wave-function. A paper by Stewart and Pyatt (**9**) on the lowering of the ionization potentials in plasma of finite density provides, apart from interesting result, both formal and numerical, a critical survey and significant references for a problem which is particularly important for the very low mass stars [cf. Gabriel (**10**)]. A treatment of the same problem by Harris (**11**) seems to suffer from counting twice the electrostatic corrections.

A general review of the physics of a degenerate gas by Guess (**12**) covers practically all aspects relevant to this section. Kaminishi (**13**) has studied the electron degeneracy at extremely high temperatures including the effect of pair creation and shown that the critical stage of degeneracy is given by $\rho/\mu_e \propto T^2$ instead of $\rho/\mu_e \propto T^3$. In the field of very high density, let us note also the work by Suda (**14**) who has studied the nuclear size correction to the relativistic Thomas-Fermi equation of state.

Kaminishi (**15**) has considered a gas at very high temperature composed of iron nuclei, α -particles, neutrons, electron-pairs and radiation. In particular, he has computed the adiabatic exponent which becomes smaller than $4/3$ over a wide range of the dissociation of iron into helium suggesting the possibility of dynamical instability. However, in stellar models, according to Colgate and White (**16**), it does not mean that the corresponding region is always large enough to destabilize the star.

In recent years, the sensitivity of the internal structure to surface boundary condition especially for cold stars has been emphasized on different occasions and, in some cases, opacities, adiabats, equation of state in the very external layers are badly needed. Auman (**17**) has calculated the opacity in the infrared due to water vapor and found it to be important for $T < 3360^\circ\text{K}$ and dominant for $T < 2520^\circ\text{K}$. Vardya (**18**) has also done a considerable amount of valuable work in this field. Much other work about opacities in the external layers has been done concerning the effect of various negative ions, free-free transitions of electrons, etc., but a review of these investigations will appear in reports of other Commissions.

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IV. CONVECTION AND SURFACE BOUNDARY CONDITIONS

The meaning of Schwarzschild's criterion and its justification have been discussed by Lebovitz (1) in the general frame of the variational and energy methods applied to linear non-radial perturbations and it has been shown that it is both a sufficient and necessary condition in the adiabatic approximation.

This approach through the linear modes (cf. also sect. VII) has also been used by Saslaw and Schwarzschild (2) to show that the penetration of the convective currents in the stable region surrounding a convective core is negligible. This result is also confirmed by physical arguments about the entropy variation of the moving elements. In this respect, one may keep in mind that, in other contexts this kind of argument does not always seem to work (3).

The usual mixing-length theory of turbulent convection which has been submitted to a thorough analysis on different occasions by Spiegel (4) still seems to remain the main tool for the building up of convection zones in stars including sometimes various refinements of the type introduced first by Vitense and Böhm.

The latter are significant mainly in the external layers and there it might still be possible to improve the description of the structure of a convective zone, as well as of the penetration in the stable surroundings, by coupling of the linear modes analysis with some of the most significant non-linear effects (5, 6, 7). In this context, an interesting question is whether the usual homogeneous turbulent convection picture remains valid or whether it has to be superseded by one in which organized large scale temporary circulations prevail instantaneously at least in certain regions (8).

Surface boundary conditions for late-type stellar models and applications to the post-main-sequence evolution of stars with masses of 2.5 and 5 M_{\odot} have been discussed by Henyey, Vardya and Bodenheimer (9). Tanaka (10) has computed boundary conditions for pure helium stars ($1 \leq M/M_{\odot} \leq 15$, electron scattering, radiation pressure included) and has determined the narrow allowed region in the HR diagram using the same procedure as Hayashi and Hôshi. Auman and Bodenheimer (11) in studying the early pre-main-sequence evolution including the effects of the opacity due to water vapor have found, for masses between 0.5 and 8.0 M_{\odot} , effective temperatures as low as 2600°K for reasonable values of the parameters in the mixing-length theory of convection.

The formation of zones with variable mean molecular weight $\bar{\mu}$ and their interaction with convection or 'semi-convection' is a delicate problem which takes many aspects in the course of stellar evolution. Baglin (12) has discussed the conditions for the setting in of convective instability in presence of a gradient of chemical composition due to gravitational stratification. In cases where the external convection zone extends down to a region of increasing $\bar{\mu}$, Unno (13) finds that penetrative convection will wash out the local gradient of $\bar{\mu}$, the edge of the convection zone being determined again by Schwarzschild's criterion. On the other hand, the difficulties connected with the transition between a convective core ($\bar{\mu}_i$) and a radiative envelope ($\bar{\mu}_e < \bar{\mu}_i$) when scattering is the main source of opacity (large masses) have been, as in the past, mainly solved by the introduction of 'semi-convective' zones (cf. however (14) where another type of solution is adopted) and Stothers (15), in particular, has given detailed attention to the problem.

However, in thermohaline convection, Veronis (16) finds that, while the presence of a stabilizing salt gradient deletes considerably the setting in of convective instability towards *infinitesimal* perturbations, overstability and *finite* amplitude convective instability occur practically as soon as the usual criterion of stability is violated. Similar unpublished results have been obtained by Kato and it would certainly be worthwhile to discuss carefully the implications of these results for the stellar case. According to Kato and Stern, the reverse situation with a stabilizing temperature gradient and destabilizing $\bar{\mu}$ -gradient, while stable for small adiabatic perturbation may also develop instabilities. A similar result was found also by Zahn (17) who points out that it may be important in presence of meridional circulations where destabilizing $\bar{\mu}$ -gradients can arise.

Let us note also that direct numerical integrations (18) have been used recently to tackle the problem of convection in various simplified situations. Although we are still a long way of being able to handle the case of turbulent convection in stars, progress in computing techniques and computing machines may open a new interesting approach which should at least allow us to 'experiment' efficiently with the fundamental parameters of the problem.

The influence of rotation on the onset of convective instability through the g -modes of linear non-radial oscillations of polytropes has been discussed by M. J. Clement (19) by means of a variational principle. The effects of overshooting from a convective zone in the presence of rotation have been considered by Roxburgh (20). Demarque and Roeder (21) have attempted to connect the rotation of stars with their chromospheric properties and the existence of an outer convection zone. Smith (22) has studied the behavior of the circulation currents in fast

rotating stellar models close to the surface and he suggests that the non-local nature of radiative transfer might be the most efficient factor in limiting the velocity to a finite value there.

For the interaction between convection and magnetic fields, cf. Sect. VII.

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V. STELLAR STRUCTURE AND EVOLUTION

Apart from a general treatise by Menzel, Bhatnagar and Sen (1) devoted to the physics of stellar interiors, we may mention also detailed review articles on stellar models for main sequence stars and sub-dwarfs (2), on stellar evolution and age determination (3), on the physics and constitution of white dwarfs (4), on the theories of novae and supernovae (5), on mass loss from stars (6) and on the evolution of protostars (7). Many other general articles of interest will be found in the proceedings of the International School 'Enrico Fermi', XVIII course (8) as well as in the proceedings of a NASA meeting (9).

Henyey and his group of associates have pursued the development of their method to compute evolutionary sequences (10, 11) and comments or improvements have also been published by various authors (12, 13, 14, 15).

Meggitt (16) has given the seven first terms of a series development around the center of a convective core. Tayler (17) has extended the criterion of Naur and Osterbrock for the occurrence of convective cores. Sack (18) has presented a method for the adjustment of radiative and convective zones in the external layers. Faulkner *et al.* (19) have discussed the surface boundary conditions in stars with an external convection zone.

Pre-main sequence contraction

Hayashi's theory of convective gravitational contraction has been applied to a large domain of stellar masses and has been generally confirmed. However, due to the difficulty of describing with precision the phases of dynamical instability (cf. sect. VI) preceding the sequence of contracting quasi-static models, the initial luminosity of these models remains in doubt. A first attack (20) shows that it must be quite high which seems also very likely on the basis of work by Bodenheimer (21) and von Sengbusch (22, 23).

Cameron and Ezer (24) who have considered again the problem of the contraction of small mass stars confirm that the latter cannot be stopped by nuclear energy generation if $M \lesssim 0.1 M_{\odot}$. After going through a stage of sub-luminous red dwarfs (25), these stars end as black dwarfs detectable as planetary companions of stars of small mass (26). For masses $0.1 < M/M_{\odot} < 0.3$, main sequence stars are wholly convective.

The gravitational contraction of stars of intermediary masses ($0.4 < M/M_{\odot} < 10$) has been studied by many authors (27-34). Iben (28) has carefully followed the initial evolution of the abundances of the C-N-O group of elements and Bodenheimer *et al.* (29) have specially discussed the influence of these abundances on the end of the contraction phase. Bodenheimer has followed the combustion of Li (30) and of D and Be (31). Ezer and Cameron (32, 33) have more particularly concentrated on the contraction of solar mass stars and find that the final model properties and the contracting time vary appreciably especially with the treatment adopted for the external layers (convection, opacity).

The contraction of massive stars ($10 < M/M_{\odot} < 100$) is little affected by Hayashi's theory as they spend, at most, a few hundred years in the entirely convective stage (34). Tanaka and Sakashita (35) have found that the convective core appears already during the contraction because of the high relative radiation pressure if $M > 40 M_{\odot}$. According to Meggitt (36), the contraction time increases again when $(M/M_{\odot}) > 32$.

Penston (37) has shown that the observations of NGC6530 and NGC2364 are compatible with Hayashi's theory.

McCrea (38) has attempted to explain the stars above the turn off points in some clusters in terms of mass exchange between the components of a close binary while Williams (39) and Dorschner *et al.* (40) attempt to account for their existence in terms of accretion.

Schatzman (41, 42) has studied the influence of a rapid rotation coupled with magnetic activity on stellar contraction.

Penston (43) has suggested an interpretation of the Neugebauer-Martz-Leighton objects as stars in formation dissipating their excess angular momentum.

Main-sequence

Main sequence models are computed essentially in view of testing the influence of various parameters. According to Morris and Demarque (44) models with $X = 0.63$ and $Z = 0.03$ are in good agreement with the observations. Mrs Masevitch and her associates have shown (45, 46, 47) that the homogeneous models on the main sequence are quite sensitive to the initial chemical composition. The results have also been compared with observational data (48) and the possible sources of disagreement, discussed (49, 50). For masses $M < 2.5 M_{\odot}$, the external convective zone is not deep enough, according to Tutukov (51), to allow enough depletion of Li⁷ to account for Herbig's observations (52).

We may also report studies of the effects on models around the mid-main-sequence of different factors such as opacity and isotopic abundances (53, 54), of the generation of energy outside the convective core (55), of the pressure of radiation (56), of the bound-bound transitions (57), of the central convection on energy generation (58). Boury (59) has shown that a variable scattering opacity modifies appreciably the models of massive stars.

A comparison (60, 61) between models computed following Henyey's method (relaxation) and ordinary models shows satisfactory agreement although the first method seems to be more sensitive to the number of mass shells adopted.

Detailed models for $M = 1.48 M_{\odot}$ (61), $0.6 M_{\odot}$ (53), $0.27 M_{\odot}$ (Kr 60 A) and $0.16 M_{\odot}$ (Kr 60 B) (63) have also been constructed.

Post main-sequence evolution

Stothers (64, 65, 66) has followed the evolution of a $30 M_{\odot}$ star of Population I up to the point where He is exhausted at the centre. He has also discussed (67) the development of semi-convective zones in stars with masses between 45 and $1000 M_{\odot}$ as they burn their hydrogen. Evolutionary sequences covering about the same interval have also been constructed by Kotok (68–72) for $M = 15.6, 20$ and $30 M_{\odot}$. The relative contributions of the different energy sources during these phases have been compared (73). Van der Borgh (74) has studied the phase of hydrogen burning in heavy mass stars (40, 60, 70, $120 M_{\odot}$) composed initially of pure hydrogen.

Iben (75–78) has followed in great detail the evolution of Population I stars with masses 3, 5, 9 and $15 M_{\odot}$ up to the exhaustion of He at the centre taking into account all the possible nuclear reactions. He has also pushed the evolutions of 1, 1.25 and $1.5 M_{\odot}$ stars up to the red giant stage (79).

Evolution through the carbon flash has been studied by Kippenhahn *et al.* (80, 81) neglecting the neutrino emission. In taking the latter into account, Weigert (82) has encountered 'thermal' instabilities when He burns in a shell. Hofmeister (83, 84) has followed the evolution of 5, 7 and $9 M_{\odot}$ stars up to and through the He burning phases for $X = 0.602$, $Z = 0.044$ and $X = 0.739$, $Z = 0.021$. She finds that the loops of the evolutionary paths in the HR diagram during He-burning are very sensitive to the initial chemical composition. According to Dluhnevskaya (85) and Varskowsky (86), the comparison between the number of stars observed and the number predicted by theory reveals a serious lack of agreement in the Hertzsprung-gap. The situation improves if the evolutionary loops are removed but agreement could also be obtained by using a variable initial luminosity function. The models so constructed have been used (87, 88, 89) to deduce some properties of the β Cephei stars ($X \simeq 0.7$, $M \simeq 4.5 M_{\odot}$; life-time $\simeq 1.5 \cdot 10^6$ years). It was also found (90, 91), in comparing ordinary models and those given by Henyey's method that a divergence sets in after H-burning in the centre affecting the shape of the evolutionary loops in the HR diagram.

Hydrogen burning has also been the object of a variety of other papers (92–98). Reiz *et al.* (98) find that, for stars of spectral type later than about A7, age determinations could be significant only after the location of the zero-age main sequence has been fixed taking variations in initial chemical composition into account.

Solar models and solar evolution have been studied (99, 100) especially to determine the neutrino luminosity (101, 102, 103) of the Sun in view of its possible detection by appropriate observations (104).

The helium flash has been studied in detail by Härm and Schwarzschild (105) and by Thomas (106, 107) for a $1.3 M_{\odot}$ star of population II. Thomas shows that, if energy losses by neutrinos are taken into account, He-burning starts in a shell around the centre. In general, it appears that the flash is much less violent than anticipated and does not lead to a mixing of

the envelope into regions where nuclear reactions can take place. Sugimoto (108) finds that if conduction is taken into account in the core, the flash leads to the apparition of a convection zone which, under certain conditions, could lead to a real explosion of the core. According to Eggleton (109), the same conduction could, for certain chemical compositions, start the flash outside the core.

A considerable amount of work (110–118) has also been devoted to the interpretation of the horizontal branch of the globular clusters, Faulkner (113) and Faulkner and Iben (114) finding, in particular, that it is facilitated by adopting a much larger initial abundance of He than is customary for population II stars.

The effects of mass loss on stellar evolution have continued to be studied by different authors (119–124) and the possible effects of a variable G on the evolution of the Sun (125, 126) have been considered.

A beginning has been made on the evolution of pure helium stars (127–130) and some models for pure carbon stars have also been built (131, 132).

The very important problem of the evolution of close binary systems taking into account the exchange of mass between the components has received serious attention by Kippenhahn and Weigert (133), by Paczynski (134, 135) and Ziolkowski (136, 137) and by Efrenov (138). In view of tackling the evolution of these systems after the start of helium burning, Giannone (139) has constructed main-sequences for models comprising a helium core and an envelope with high hydrogen content for various values of the ratio of the mass of the core to the total mass.

Last stages of stellar evolution

The idea that planetary nebulae constitute an intermediary stage in the evolution towards the white dwarf stage has received much attention from a theoretical as well as from an observational point of view and has been particularly developed by Savedoff (140). Vila (141) and Chin, Chiu and Stothers (142) have followed the evolution of hot stars of 0.7 to $2 M_{\odot}$ with high initial luminosity after complete exhaustion of the nuclear fuel. In the HR diagram, their models do indeed evolve from the region of planetary nebulae to the white dwarf region. They also find that neutrino losses accelerate considerably the contraction and lead to better agreement with observations. Rose (143) finds that models of small mass stars with a helium envelope in which nuclear reactions take place evolve like planetary nebulae. According to Kahoutek (144) the latter could be produced by novae while Deeming (145) considers that they could issue from infrared supergiants. Let us recall however that some opposition to these views has been expressed by Woolf (146).

Mestel and Ruderman (147) have evaluated the energy content of white-dwarfs and their rate of cooling.

The possible influence of mass loss on the formation of white dwarfs has been considered by Auer and Woolf (148) and by Williams (149). The evolution of stars with masses in excess of the Chandrasekhar limit, during the phases preceding a supernova explosion has been studied by Chiu (150) and by Rakavy and Shaviv (151).

Takarada *et al.* (152) have evaluated the relation between central densities and temperatures in polytropic or isothermal configurations to estimate the general features of advanced stages of evolution.

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VI. STELLAR STABILITY AND STELLAR PULSATATIONS; SHOCK WAVES
 AND NON-STATIONARY PROCESSES IN STARS

Interesting developments have occurred in the field of stellar stability during the period under review and we shall try to situate them with respect to the classical subdivisions of the subject: dynamical stability, vibrational (or pulsational) stability and secular (or thermal) stability, reviewed for instance in (1).

Note. Although probably difficult to enforce, a convention might be useful here. If the time dependence is taken to be of the form $\exp(i\sigma t)$, then:

(a) *dynamical stability* or *instability* might be reserved to characterize the behavior of purely *conservative* systems in *absolute equilibrium* (adiabatic perturbations, real eigenvalue of the form $\lambda = \sigma_a^2$).

(b) *Vibrational stability* or *instability*, to characterize the *damping* or *amplification*, under the influence of *non-conservative factors*, of the oscillations found under (a) when the system is dynamically stable (non-adiabatic perturbations, complex eigenvalues of the form $\lambda = a + ib$, $\sigma = \pm(\sigma'_a + i\sigma')$, σ' (+ or -) due to the contributions of the non-conservative terms.

(c) *Secular stability* or *instability* to characterize the amplification or damping of slow motions (for which the inertial terms are negligible) under the influence of the non-conservative effects (non-adiabatic slow perturbations, σ of the form $i\alpha$, α : + or -); *thermal stability* or *instability*, to characterize the more complicated situation where non-conservative thermal factors (for instance in some cases neutrino losses) can have very strong effects, leading to motions on a time-scale sometimes comparable to that of dynamical motions (inertial terms not necessarily negligible).

(d) In the case of conservative systems in relative equilibrium (for instance, in presence of rotation) it happens that some eigenvalues become complex, due to a failing of the dynamical stability of the corresponding absolute equilibrium, the extra-relative force (for instance, due to rotation) providing the restoring force responsible for the real part of λ . In such cases (stability of rotating fluid masses for instance), the word *overstability* has been used in recent years and it could perhaps be kept to characterize this kind of situations although it was coined originally, and not too happily, by Eddington to denote what is called here *vibrational instability*. A somewhat closer case of instability of relative equilibrium may also occur in more complex circumstances as in the Rayleigh problem for convection in presence of rotation or magnetic field and has generally been called there too, *overstability*.

Dynamical Stability and adiabatic oscillations

In the case of purely *radial* oscillations of 'classical' stars, the essential result (namely that dynamical instability occurs only if $\Gamma < 4/3$ in an appreciable part of the star), has been known for some time but work has continued on interesting applications at the very beginning of stellar evolution (2), when ionization of H, He and dissociation of H_2 reduces Γ below $4/3$ or towards the very end (3) when nuclear equilibrium tends to be established between Fe, α -particles and neutrons or later between He, protons, neutrons and electron pairs lowering again Γ below $4/3$. Equilibrium between electron pairs and radiation may also lead to low values of Γ and recently Rakavy and Shaviv (8) have found that this can render a large mass star dynamically unstable before nuclear equilibrium gets established.

The newly discovered dynamical instability due to General Relativity corrections in large mass or very dense stars has been the object of further investigations reviewed mostly in Section II. Let us add Chandrasekhar's discussion (4) of this factor in ordinary white dwarfs and the investigations of Baglin (5) who has further taken into account the effects of finite temperature and of equilibrium between electrons and nuclei and gives critical masses and radii beyond which dynamical instability occurs for different values of the mean atomic weight.

The overtone radial adiabatic oscillations of massive stars have been studied numerically by Van der Borgh (6) and the results compared with the analytical asymptotic behavior. Bhatia and Kushwaha (7) have studied the radial adiabatic oscillations of a generalized Roche model.

From a methodological point of view, the energy method seems to have become more popular (8, 9) following a very clear unpublished presentation by Dyson. In (8) a beginning is made in the discussion of the third order terms.

Non-radial oscillations have been the object of concentrated numerical attacks by Smeyers (10) and by Van der Borgh and Wan Fook Sun (11) taking into account the full 4th order problem when the perturbation of the gravitational potential is not neglected.

The application (12) of the variational principle derived by Chandrasekhar for non-radial oscillations has led directly or indirectly to some clarifications. As far as the interaction between dynamical instability towards these oscillations and Schwarzschild's criterion for convective stability is concerned we have already referred, in Section IV, to Lebovitz's proof of the existence of unstable *g*-modes as soon as Schwarzschild's criterion is violated in a region however small. Ledoux and Smeyers (13) have discussed the complete spectrum in such cases and shown that the *g*-spectrum splits then into two spectra both converging to zero, one positive (stable) corresponding to oscillations with large amplitudes and nodes in the convectively stable region and rapidly decreasing amplitudes in the convectively unstable region and one negative (unstable) with amplitudes whose behavior is exactly the opposite. The latter are actually the modes studied by Schwarzschild and Saslaw (cf. Section IV) in relation with penetrative convection. Detailed examples by Smeyers (10, b) are in the course of publication. Furthermore, from the work of Chandrasekhar and Lebovitz, it became clear (cf. Robe, 14) that the so-called *f*-modes constitute really the extension to compressible configurations of the unique Kelvin spectrum for the incompressible sphere (one mode only per value of the degree *l* of the spherical harmonics). For $l = 1$, this *f*-spectrum vanishes in compressible as well as in incompressible configurations while in compressible masses the *p*- and *g*-spectra subsist for this value of *l* (10). Robe and Brandt (15) have applied the variational method using a five parameter trial function to polytropes of indexes $n = 1, 2, 3, 4$ and find that the method does not seem to converge for the *f*-mode when $n > 3$. This is probably due to the peculiar behavior, in highly concentrated models, of the eigensolutions for the *f*- and first *p*-modes which acquire 'spurious' nodes as shown in detail by Robe (16).

As an illustration of the possible effects of a finite rotation on non-radial oscillations, Cretin and Tassoul (17) have studied the case of a homogeneous compressible cylinder rotating

uniformly around its axis with angular velocity Ω . They find that the azimuthal degeneracy is lifted by rotation and that, for Ω smaller than some critical value, the g -modes (which in absence of rotation are unstable in this case) have complex frequencies, the real part increasing with Ω . Of course the imaginary part corresponds to the energy available due to the strong convectively unstable stratification in the model. One may expect this result to remain valid for the unstable g -modes of any configuration comprising a region in which Schwarzschild criterion for convective stability is violated and Ledoux (18) has suggested that this might provide an explanation of both the maintenance and the order of magnitude of the periods of the oscillations observed in magnetic variable stars. Robe (19) is extending the work of Cretin and Tassoul and of Ostriker (20) to general polytropic cylinder with and without rotation and tries to disentangle, in these cases, the gravitational instabilities proper from the convectively unstable gravity-modes (g -modes). Zahn (21) is tackling again the more realistic problem of the oscillations of a rotating star especially in view of possible resonance in close binaries.

A review of the principle and of some applications of the higher order virial method has been prepared by Chandrasekhar (22) and new applications have been the subject of different papers (23) (24) which, although somewhat formal, provide interesting examples of detailed analysis of complex situation in presence of uniform or differential rotation and tidal effects.

Let us note also an extension of Chandrasekhar's variational principle to the 6th order problem of the oscillations of an elastic sphere by Ottelet (25) and others to the oscillations of a gaseous mass in presence of a rotation by Clement (26) or of a magnetic field by Kovetz (27).

Vibrational Stability and non-adiabatic oscillations

Boury (28) has shown that the critical mass associated with the vibrational instability of helium stars is about 8 to 10 M_{\odot} as compared to some 60 M_{\odot} for stars on the main sequence burning hydrogen. Gabriel and Bovie-Thirion (29) have discussed the importance of the vibrational instability which manifests itself in a small mass star going through the phase of deuterium burning as a function of the initial abundance of the latter. Perdang (30) has discussed in great detail the behavior and the effects of the 3α reactions in pulsating stars. Gabriel (31) has confirmed, on the basis of fairly realistic models, the significant vibrational instability of small mass stars ($M < 0.27 M_{\odot}$) on or close to the main sequence due to both the energy generation and the low values of the I 's in the external ionisation zones of H and He. Noels, Boury and Gabriel (32) have shown that quasi-static models of pure carbon stars are strongly vibrationally unstable in presence of neutrino emission however small their masses.

Epstein (33) has attacked numerically the general linear problem of the non-adiabatic radial oscillations treated as a complex differential system of 4th order with complex eigenvalues.

Unno (34) is reporting detailed work on the rôle of a convective envelope upon the vibrational stability of a star, based on a generalization of Vitense mixing-length theory of convection to the time-dependent case. In a similar line, Gough (35) finds that energy transport by convection in the external layers can have, depending on the circumstances, either a stabilizing or a destabilizing influence and he suggests that the latter may be related to the variability of the long period variables.

Kopal (36) has studied some aspects of the interactions of matter with radiation in the fundamental mode of radial oscillations and finds that they can be significant only in models with very high central condensation.

An attempt at evaluating the effect of gravitational contraction keeping the terms in the velocity of contraction as well as the time variations of ρ and p was made by Meurice (37) for a star close enough to the main sequence to be wholly radiative but the thermonuclear energy generation being still negligible. A rough evaluation of the very complicated expression of the coefficient of vibrational instability failed to reveal any significant effect. A somewhat similar

problem has been considered by Kato and Unno (38) both for radial and non-radial oscillations around a quasi-equilibrium state in which the nuclear energy generated is not exactly balanced by the heat flux. They find that the region of decreasing entropy (as the star evolves) works in general to excite pulsations.

Let us turn now to investigations directly oriented towards the problem of the excitation of the pulsations of variable stars. Alyoshin (39) concluded on the basis of an approximate model of a cepheid that the ionization zones of H and He I could not lead alone to the excitation of pulsations. Takeuti (40) has written down a boundary condition at the photospheric level of the pulsating radiative atmosphere of a cepheid. Unno (41) has discussed the linear radial pulsation of such an atmosphere to obtain boundary conditions for the pulsation of the interior of a cepheid model. He finds that, very generally, the first overtone as well as the fundamental mode are excited. The paper contains also a discussion of the propagation of running waves in a corona and remarks on the breaking down of the linear theory in the ionization zones as well as suggestions for an approximate non-linear treatment of the external zones. Simon (42) has also analyzed the effects, both from the points of view of dissipation and of phase-shifts, of progressive waves on the non-adiabatic oscillations of an atmosphere surrounded by a hot corona discussing especially the case of resonance between a proper frequency of the atmosphere and that of the exciting mechanism. In an extension of their previous work, containing many improvements, Baker and Kippenhahn (43) have discussed very fully the vibrational stability of actual models along their evolutionary tracks across the cepheid strip confirming the instability due essentially to the second helium ionization although the first helium and hydrogen ionization contribute to it especially for the first overtone. Hofmeister (44) has amplified their investigation of the problem producing a theoretical period-luminosity relation in good agreement with observations and discussing the period changes and the period-frequency relation as they follow from the theoretical evolutionary sequence.

In an interesting paper, dealing with a simple one-zone model, Baker (45) has isolated the essential physics at the basis of the pulsational instability in cepheids and Unno and Kamijo (46) have generalized somewhat his discussion. Baker has also investigated the pulsational properties (47) of outer layer models of RR-Lyrae stars.

The vibrational stability towards non-radial oscillations of large mass stars has been studied by Wan (48) who shows that the coefficient of vibrational stability increases with the mass for the f - and g -modes. It decreases for the p -modes when the mass increases but so slowly that it is not likely that any significant critical mass should exist.

Applications to variable stars

As far as the *excitation* of linear oscillations is concerned, the problem has already been dealt with at the end of the subsection on vibrational stability.

There have been however some other applications of the linear theory mainly concerning the periods given by the eigen-value problem corresponding to the adiabatic approximation. The essential point is to obtain a reasonable model for the internal structure of the variable considered either by direct evolutionary computations or by approximate evolutionary considerations. If the luminosity and radius can be determined more or less directly from the observations, then one can try to adjust the mass and to some extent the model, its chemical composition, etc. in such a way that the theoretical period, corresponding in most cases to the fundamental mode, be just equal to the observed period. Investigations of this type have been reported for η Aquiliae, RT Aurigae, X Cygni, Mira Ceti and some RR Lyrae stars by Masani *et al.* (49) who seem however to have used an unduly complicated approach intended, in principle, to yield also the effects of the non-adiabatic terms. Jørgensen and Petersen (50) have applied the method to derive indications on the masses of RR Lyrae stars of type a , b and c in different clusters.

The same authors (50) have also discussed the secular changes in the periods of β Cephei stars on the basis of actual evolutionary sequences. The smallness of the observed changes leads them to conclude that the β Cephei phase must occur essentially in the slow main-sequence stage a conclusion in agreement with the values of the periods themselves. They also note that the ratio of the periods of the first overtone to that of the fundamental mode does not agree with those derived by van Hoof from the observations. Stothers (51) has also discussed this problem and using Van Hoof's data finds that β Cephei stars of lower mass should lie closer to the initial main sequence. For the same stars Hitotuyanagi and Takeuti (52) find that the pulsation constant should be 0.021.

The interpretation of Fernie's period-radius relation for variable stars has been discussed by Demarque and Percy (53) and by Gough, Ostriker and Stobie (54).

Van der Borgh and Murphy (55) have studied the non-linear adiabatic oscillations of a $10 M_{\odot}$ star up to the third order terms in $(\delta r/r)$ including up to the 5th overtone as spatial coefficients in the development of $(\delta r/r)$. They obtain appreciable skewness of the variations comparable to that observed in cepheids.

Alyoshin (56) has determined the amplitudes of the auto-oscillations of the model whose vibrational instability was discussed in (39) and has discussed the growth of their asymmetry from the interior to the surface as well as the variation of the phase-lag. In another paper (57) taking into account the effects of convective transfer, he finds that, in order to get self-excited oscillations and the right-phase shift, the ratio of the mixing-length to the scale height should be smaller than some upper limit of the order of unity.

The direct computational attack of the full non-linear non-adiabatic problem of stellar pulsation has been reviewed by Christy with emphasis on computational methods and RR Lyrae stars (58) and on the cepheid instability strip (59). In these review articles, as well as in the important paper on RR Lyrae models (60), the author has attempted to clarify as far as possible the physical nature of the effects at the origin of the different characteristics of the computed self-oscillations. A comparison of his own work with the somewhat different approaches due to Zhevakin and Cox and Whitney, reveals that the criteria to which they lead for the location of the Cepheid pulsational strip in the HR diagram are of remarkably similar physical content and require essentially that the depth of He II ionization be such that the heat content of the overlying layers is about 1/4 of the heat radiated in a period. Christy points out also the weaknesses of the present calculations, as for instance the lack of an adequate treatment of time-varying convection in the external instability-layers associated with the ionization of H and He and the limitations of the technique by which they are carried out which may be responsible for the difficulties in relating the finite self-excited modes found (sometimes the fundamental, sometimes the first overtone) to the linear instability which in some cases extends to many overtones. In some ways, this is somewhat reminiscent of the weakly non-linear mechanical systems whose linear modes have uncommensurable periods and in which limit cycles, if they exist, are close to one or more of these linear modes (61). Another aim of Christy's non-linear pulsation computations has always been to derive some of the unknown parameters like total mass or the abundance of He for a given variable star by adjusting these parameters so that the theoretical pulsation fits as well as possible with the observed one and recent extensions in these directions have been made to cepheids (62) and RV Tauri stars (63). Let us note also a new attempt (64) to include a more detailed treatment of radiative transfer and of the development of ionization and shock fronts in the external layers of RR Lyrae models.

A review of some aspects of the non-linear problem insisting more on the growth from linear instability to finite amplitude for a stellar envelope with the position of and the luminosity at the bottom fixed can also be found in a paper by J. P. Cox (65) and a detailed investigation has now been published (66). It is remarked there that the rather small limiting

finite amplitude suggests that the limiting mechanism is determined primarily by a saturation effect of the driving mechanisms (varying opacity and low value of κ in the He⁺ ionization zone). This paper contains also a discussion of the theoretical location of the cepheid strip in the HR diagram. A note by Christy (67) offers some comments and comparisons with his own work.

The thermally excited non-linear oscillator studied by Moore and Spiegel (68) is physically rather directly related to the kind of vibrational instability arising for acoustic modes (or non-radial p -modes) in a super-adiabatic region. This may however be directly relevant for some variable stars and, in any case, as noted by the authors, this oscillator provides a good illustration of the variety of behavior, including irregular variability, which can be generated by a single mechanism. In further work, with Baker (69), they have extended their non-linear analysis to a model more akin to the radial pulsation of cepheids.

We may perhaps add here references to work on the interpretation of the activity of flare-stars (70, 71) and to a review of observational data and theoretical inferences concerning magnetic variable stars (72).

Secular and Thermal Stability

These questions which up to recent times had received relatively little detailed attention have suddenly come to the fore with the numerical discovery of secularly unstable phases in the course of the evolution of models with non-degenerate helium burning shells. On the occasion of the first case discovered, Schwarzschild and Härm (73) developed a linear analysis of the situation of the type suggested by Ledoux (74), but including the terms due to entropy variations, which illustrates the origin and characteristics of the responsible incipient secular instability. Since then, this type of linear analysis has been repeated by Rose (75) for similar models and Gabriel (76) has found that models with an isothermal core followed by a shell burning zone become secularly unstable, when the core reaches the Schönberg-Chandrasekhar limit, towards a displacement corresponding to a contraction of the core and an expansion of the envelope. Quite generally, at least the linear secular instabilities in question appears for perturbations more general (often with nodes inside the star) than the homologous modification which for so long was the only one considered in the discussion of secular stability. The elucidation of some of the mathematical properties of the problem would be very welcome.

Kippenhahn (77) and Weigert (78) on their side, encountered the same kind of problem in following the late evolution of a $5 M_{\odot}$ star but found numerically that the thermal runaway gave rise to some kind of 'relaxation oscillations' of long periods (~ 3000 years). Recently Schwarzschild and Härm (79) found that, in their case also, the secular instability discovered earlier leads indeed to long period 'relaxation oscillations' which, after about a dozen relaxation cycles (at which time the helium burning shell is still entirely non-degenerate), causes mixing of hydrogen into the hot interior. Rose (75/b) was led to similar conclusions in his study of models of helium shell-burning stars and points out that the origin of planetary nebulae and the occurrence of novae may be associated with this instability. It has subsequently been shown (80) that thermal instability can lead to vibrational instability, a circumstance that provides further physical basis for the above suggestion concerning planetary nebulae and novae. Rakavy and Shaviv (9) have also encountered significant thermal instabilities in presence of neutrino emission.

It is obvious that a new and important field of research has been opened here for both the linear and non-linear theories of stellar stability and that a careful choice of descriptive terms will help avoid confusion.

Shock waves, Novae, Supernovae and other sources of extreme energy release

It is quite beyond the present reviewer's ability to cover the abundant literature on shocks in all the various circumstances that have been considered. Many of these refer to conditions

not specially typical of stellar interiors and we shall have to ignore them even if something could be learned from them for the problems of interest to this Commission.

Much of the typical work has been associated with the discussion of novae or supernovae explosions and a general review of the relevant theories is due to Schatzman (81). As to the effects of nuclear reactions started by the collapse of the initial configuration, the problem has been reviewed by Fowler and Hoyle (82) and it has been discussed again by Masani *et al.* (83) who took also into account radiation, pair production, relativity effects and neutrino emission. Colgate and White (84) came to the conclusion that thermonuclear reactions had a negligible effect in circumstances likely to occur in supernovae and that the reversal of the collapsing motion in the envelope and the formation there of strong relativistic shock waves was due to the deposition of energy by neutrinos. Masani *et al.* (85) have extended their computations to the case of very high densities such as might occur in neutron stars. Kalenichenko and Porfiriev (86) have followed numerically the outburst of a massive hydrogen star caused by an instantaneous release of a large amount of energy at the centre and compared the luminosity curve with observational data for supernovae type II.

Finzi and Wolf (87) have found two types of massive (1.4 and $1.2 M_{\odot}$) white dwarfs models which, by implosion at the end of a slow contraction (due to inverse β processes), could account for type I supernovae and their frequency.

Let us also note some papers (88, 89) treating some consequences of the current ideas on supernovae and refer to a general discussion of the subject (90).

Schatzman (91) has considered a mechanism in which resonance could lead to ordinary novae explosions and Kahoutek (92) has also contributed to the problem of the origin of novae and planetary nebulae. Ishizuka, Hashimoto and Ono (93) have generalized the quasi-stationary method of Ono *et al.* to the case of oblique shock propagation and discussed in detail the behavior of a shock in polytropic gases. Simon (94) has discussed the stability of shocks by the normal-modes method. Let us also mention a few investigations (95–98) directed towards applications to the external layers of variable stars or which might be of interest in this respect.

Before leaving this section, let us add a word about some other aspects of oscillations typical of the very external layers but which might be significant for the problems of internal structure either because they might remain significant at greater depth or because they might play a rôle in the determination of surface boundary conditions when the latter become critical for the internal structure. In the first group, I want to mention especially one result of Spiegel (99) who finds that, in a superadiabatic region, acoustic modes can be excited directly by conduction. By analogy, one might expect that non-radial p -modes in the interior could become vibrationally unstable due to the same mechanism. In the second group, which will certainly be reviewed in some other Commission, let us simply cite by title, a few papers (100–104) devoted to the excitation of waves and oscillations in a stable atmosphere by the convection and turbulence in an underlying convection zone.

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VII. EFFECTS OF ROTATION, MAGNETIC FIELDS, AND
EXTERNAL GRAVITATIONAL FIELDS

The effects of rotation on various types of stellar models have been studied extensively but although the information gathered is valuable, the impression persists that one is still some way from a completely satisfactory physical solution.

Roxburgh (1) has built models in which the angular velocity field adjusts itself so as to drive no meridional circulation. Although there is a typical difference in the resulting angular velocity distribution for upper and lower main sequence stars it is not sufficient to explain the observed difference between the two classes of stars.

Polytropes in fast uniform rotation have been discussed by Duheert (2), James (3), Roberts, Hurley and Limber (4) and Stoeckly (5) has constructed, by direct numerical methods, polytropic models in fast, non-uniform rotation. Monaghan and Roxburgh (6) have developed an approximation method based on the use of a first order perturbation technique in the interior and the neglect of the self-gravitation in the outer envelope (Roche model). This method has been extended by Roxburgh, Griffith and Sweet (7) to the construction of more realistic models in fast uniform rotation.

Models for early main-sequence stars in non rigid rotation with an increase in angular velocity towards the surface have also been obtained by Kapylov (8) while Porfirjev (9) has investigated a Roche model with barotropic rotation. Rotational oscillations connected with meridian circulations have been discussed (10) in view of the interpretation of the β CMa stars. Radial velocity curves for eclipsing variables have been computed taking into account an equatorial deceleration (11, 12). A relativistic generalization of the Roche model for applications to neutron stars has also been developed (9).

The question of meridian circulations or the Von Zeipel-Eddington problem has been surveyed by Mestel (13) with special reference to the problem of self-consistent circulation patterns, the effect on stellar evolution and possible observable surface effects. Steenbeck and Krause (14) have studied the differential rotation with equatorial acceleration associated, in a perfect fluid sphere, with a weak meridian circulation flowing on the surface towards the equator and descending there. Mestel (15) has confirmed Öpik's result that, in a uniformly rotating radiative envelope, the meridian circulation pattern breaks up into two zones, the velocity becoming large for low surface densities. Smith (16) suggests that the velocity singularity arising at the surface when the density tends to zero can be removed by taking into account the non local nature of radiative transfer in the external layers. The problem of the effect of rotation in the external layers has also been rediscussed by Osaki (17) using the appropriate equations of radiative transfer. He finds that two steady states are possible depending on whether the viscosity is negligible (redistribution of angular velocity so that the divergence of the radiative flux and the meridional circulation vanish) or not (the non-vanishing divergence of the radiation flux drives meridional circulations which maintain differential rotation).

Anand (18) has also studied polytropic models in uniform rotation but including applications to rotating white dwarfs in which case he finds that, for maximum rotation compatible with stable equilibrium, Chandrasekhar's limiting mass M_3 is increased by 3.5% in agreement with an earlier result of Roxburgh (19) who, furthermore, discussed the possibility for a star of any mass to attain the white dwarf stage through mass loss by rotational instability. This interesting result was already apparent in James' investigation (3) although the critical mass itself was not derived by this author. The discussion of the structure of fast rotating white dwarfs has been refined by Monaghan (20). Differential rotation in white dwarfs has been taken into account by Ostriker, Bodenheimer and Lynden-Bell (21) who find that, in this case, equilibrium models exist with masses considerably greater than M_3 . This important result suggests that, for a star above M_3 , there is an alternative to mass loss or rapid collapse in the

final stages of stellar evolution associated with the redistribution of angular momentum. Papers on the self-consistent field method used numerically (22) and on a number of physically realistic white-dwarf models (23) are in preparation.

A number of investigations were devoted to the evaluation of the effects of rotation on the observable surface characteristic ($L - T_{\text{eff}}$, $M_{\text{bol}} - B - V$) and on the position of the model in the HR diagram (24–26). In the case of an age determination of the Pleiades on the basis of isochrones derived from the evolutionary paths of non-rotating models, it was found necessary to make appropriate corrections for effects of rotation (27).

We may perhaps add a few references to semi-empirical papers presenting or discussing collections of data of direct interest for the theory. A discussion, on a large statistical basis, of interrelations between the distribution of rotational velocities on one side and evolutionary paths, early-type stars variability and ‘peculiarities’ on the other is due to Paczynski (28). A study of the distribution of rotational velocities with mass by Kopylov (29) shows a maximum for stars of 5–9 M_{\odot} , but, on the main sequence, even this maximum is smaller than the critical velocity leading to rotational instability. In the same paper a comparison between observed velocities and predicted ones along early evolution assuming conservation of angular momentum reveals good agreement except for stars B9–A3 (region of the magnetic and peculiar A-stars) for which the observed velocities along the evolutionary paths become rapidly smaller than the predicted ones. Sletteback (30) situates the minimum difference between calculated minimum equatorial breakup velocities and observed velocities at the Be stars.

The problem of the interaction between stellar rotation and magnetic fields is more or less unescapable and the generation of a toroidal magnetic field in an initially non-magnetic rotating star by the battery effect of the electron partial pressure has been discussed in detail by Roxburgh (31) and the discussion has been extended by Roxburgh and Strittmatter (32) to the structure of the rotation and magnetic field inside non-uniformly rotating early main sequence stars where the limit on the growth of the thermally generated toroidal magnetic field comes from the Hall effect rather than the ohmic field. Carr (33) finds that balancing, on the average, the extra-force introduced by general relativity in a non-uniformly rotating star by a magnetic force, yields a reasonable value of the magnetic field.

Equilibrium models of polytropes with magnetic fields of various geometries have been discussed by Monaghan (34) and Roxburgh (35).

In his lectures on the theories of stellar magnetism (36), Mestel has reviewed the general problem of the origin of stellar magnetic field and has discussed especially various questions: rôle of ‘weak’ and ‘strong’ primeval fields during the Hayashi convective contraction phase including magnetic braking; the appearance or non-appearance of a field above the stellar surface; the reduction in the Cowling decay-time of a strong field; the oblique rotator and the magnetic binary-star models of magnetic variables. Cowling (37) has written a very clear account of the fundamentals of the subject and a general review is also given in the article by Ledoux and Renson (38).

The study of the interaction between magnetic field and convection has also been pursued. Parker (39) and Weiss (40) have shown how in a zone which is the seat of laminar convection, a ‘weak’ field is steadily pushed towards the edge or concentrated in localized flux ropes within the zone. Gough and Tayler (41) find that simple local criteria (applicable in absence of intrinsic hydromagnetic instabilities) suggest a high degree of suppression of convection in a sunspot model. This work is being extended by Gough to polytropic atmospheres. This last problem has also been considered by Gusseinov (42) who shows, on the basis of convection criteria for the increase of vortexes of different scales, that small ones are depressed by the magnetic field in high atmospheric layers while, in some cases, the magnetic field adds to the convective instability. The influence of radiation has also been considered (43). Rüdler (44) finds that only completely disordered turbulence intensifies the decay of the mean field. In other cases,

especially when one of the two types of helical motion is preferred, for instance as a consequence of the Coriolis forces in a rotating body (planet or star), dynamo effects can arise (45, 46, 47) yielding different types of solutions, some periodic, depending on the distribution of rotation inside the configuration.

As far as the theory of magnetic variable stars is concerned, one may record a suggestion by Steinitz (48) appealing to the over-stability due to rotation. Although the case discussed by Steinitz is rather far removed from any likely physical model, fundamentally the same over-stability could act in more realistic circumstances and the advantages of an oscillation theory based on this mechanism have been discussed by Ledoux (49). In such a theory, the selection of appropriate modes of oscillation would be greatly helped by the presence of a companion as in the excentric double-star hypothesis suggested and developed by Renson (50). This last theory presents intrinsically many advantages and the possibility to explain, on its own, field-reversals if the angular momentum and magnetic axes make a large angle although, in this case, some of the ordinary physical objections to the oblique rotator would probably persist. A discussion of the comparative merits of variable stars theories can be found at the end of the review article by Ledoux and Renson (38).

We have already covered some of the work on double-stars and their evolution in Section V. Roxburgh, (51) in investigating the pre-main-sequence evolution of a rotating non-magnetic star finds that rotational instability is likely to occur for $M > 0.8 M_{\odot}$ when a radiative core develops and that this could lead to the formation of close binaries for $M < 4 M_{\odot}$ and wide pairs for $M > 4 M_{\odot}$. Apsidal motion in close binary system is discussed by Peraiah (52) who shows that if the components rotate faster than they revolve, the derived central densities could be higher than usually found. Zahn (53) has discussed the principal properties of tides in close binaries in absence of synchronism between rotation and orbital motion and explains three well-known effects on this basis: circularity of the orbits of binaries when at least one of the components possess an external convection zone, the presence of a superficial turbulence zone on star built on the Cowling model, the peculiar evolution of some systems when the secondary is a sub-giant. Kopal (54) has surveyed the general problem in a review paper.

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VIII. GRAVITATIONAL INSTABILITY — ORIGIN OF STARS

The activity in this field has continued to increase probably because of various favorable factors: better and more detailed observational data on the interstellar matter and better knowledge of the physical processes taking place in it as well as the possibility of new significant observations and tests in the near future, the discovery of very cold infrared objects, new information on the structure of galaxies, a better grasp of factors arising in magnetohydrodynamics and their possible interactions with rotation and finally the access to powerful

computers and the resulting enticement to tackle more realistic problems including non-linear ones.

As a result, new factors like thermal instabilities have entered the field and others like magnetic fields and rotation have been the object of new and more precise discussions. One may also note a shift of the attention and interest towards the larger scale problem of the formation of the characteristic structures of galaxies. In this sense, many investigations are strictly outside the competence of this Commission and we shall only survey roughly part of this work when the methods used may be of some interest for the problem of the formation of stars themselves.

The general problem of gravitational instability and of star formation has been reviewed by Spitzer (1), Layzer (2) and Mestel (3). Each author emphasizes different aspects but all voice a certain uneasiness with the pioneering treatment of Jeans. The difficulties arise in defining properly the initial state of the system whose stability should be studied. As a finite configuration which has relaxed completely on its own and in all directions to equilibrium (mechanical and thermal) cannot exhibit gravitational instability at least for infinitesimal perturbations, a certain measure of lack of equilibrium (motions, thermal inequalities, extra gravitational field due, for instance, to pre-existing stars, etc.) should be present and this certainly increases the difficulties. Spitzer insists perhaps more on the physics of the problem. Layzer's article contains, apart from a general critical review, a clear account of his own views which place 'gravitational clustering' at the origin of all cosmical condensations. It is further assumed that interactions among pre-stars in a pre-cluster can inhibit their gravitational contraction for periods comparable to the age of a galaxy so that systems of all levels of the hierarchy are actually born almost simultaneously. In Mestel's article, the emphasis is laid rather on anisotropic factors like centrifugal force and magnetic field, especially the latter, and a number of interesting problems are raised in this connection.

Another general review by Woltjer (4) is directed more at the problem of the formation of large scale structures in a galaxy and especially spiral arms.

In an attempt to elucidate some aspects of Jeans's criterion, Simon has followed the development in time of a local perturbation in Jeans medium (5) showing that, whatever the initial scale, it always leads to instability after a time which is the longer the smaller the initial dimensions. In the case of the isothermal stratified nebulae, he has established the existence of a critical wavelength of maximum instability.

Lynden-Bell (7) and Lin, Mestel and Shu (8) have followed the collapse of a uniform non-rotating pressure-free spheroid showing that any initial excentricity is rapidly amplified an oblate spheroid becoming a disk and a prolate spheroid a spindle. C. Hunter (9) has studied the formation, inside a large spherical cloud contracting as a whole, of gravitational sub-condensations and finds that rotation has a negligible influence on the process.

The effects of cooling on the contraction of interstellar cloud with an initial polytropic density distribution have been discussed by McNally (10) and Gould (11) finds that a molecular hydrogen protostar ($M > 1 M_{\odot}$) would collapse due to the 28μ radiation. The general problem of the thermal stability of a dilute gas in mechanical and thermal equilibrium has been studied exhaustively by Field (12) who finds that, under a wide range of conditions, thermal instabilities exist capable of giving rise, although in times rather long, to astronomically significant condensations of higher density and lower temperature than the surroundings. In a following paper with Saslaw (13), using Oort's model, they find, for the rate of star formation ($\propto \rho^2$) and for the cloudmass spectrum, results in reasonable agreement with observations. The problem of the interaction of this thermal instability with gravitational instability and possibly magnetic fields has been elaborated further by J. H. Hunter (14) who shows that thermal instability at pressure equilibrium followed by gravitational collapse can lead to the formation of significant condensations of the order of $100 M_{\odot}$ in 10^9 to 10^{10} years. Thermal effects have also

been taken into account by Simoda, Kikuchi and Unno (15) in studying the homologous contraction of an interstellar cloud represented initially by a negative polytrope ($n = -3.32$) which ultimately reaches a stage of free-fall collapse except if the density is too high ($> 100 \text{ cm}^{-3}$) in which case oscillations occur. In a paper in preparation however, Kikuchi (16) finds that homologous contraction is not a good approximation and studies the effects of non-homology.

The effects of rotation, in a more or less classical approach, have been discussed in a series of papers (17–20). Some of the results here, as in the investigations of Toomre (21), of Lin and Shu (22) and of Goldreich and Lynden-Bell (23), might be more significant for the formation of large scale structures rather than stars.

Mestel finds that only in special cases, does the spin of sub-condensation in a non-uniformly rotating medium have the same sign as the hydrodynamic vorticity.

The general influence of magnetic fields is taken into account in (24) and (25). Some of the conclusions of the linear analysis are discussed by Strittmatter (26) who shows that, even if the linear criteria is not modified by the presence of a magnetic field, contraction in this case will not go on indefinitely. In another paper (27), the same author discusses the gravitational contraction of a cloud in presence of a frozen-in magnetic field taking into account the possibility of anisotropic flow along the field lines. He finds that a change by a factor 4 in the magnetic flux will make all the difference between complete gravitational collapse and the establishment of an equilibrium state at almost the initial lateral dimensions. Considering an anisotropic plasma, Gliddon (28) has shown that, in that case, hose instability as well as gravitational instability may occur, the criterion for the latter depending now on the magnetic field.

The magnetic field of a contracting gas cloud has been studied by Mestel (29) who finds that, if the flux-freezing constraint is strictly enforced, the non-homologous contraction of a gas to form a cloud yields a distorted field that exerts strong pinching forces just beyond the cloud radius. Relaxation of the flux-freezing allow the field lines to snap, yielding a magnetically isolated cloud. A simple illustration model for this snapping has been worked out (30) while in another investigation (31), the relative importance in this context of the Sweet mechanism ambipolar diffusion and the Petschek mechanism are being discussed. Of course once magnetic detachment has occurred, transport of angular momentum from the cloud ceases and the possibility of magnetic braking vanishes. However the latter will persist as long as the cloud field is undetached and its effects are being studied in two cases: gravitational collapse (32) and slow contraction due to a sufficiently strong centrifugal force (33).

Mestel (34) has also considered the effect of magnetic braking on the evolution of a protostar in the Hayashi phase assuming a strong stellar wind. He shows that, in the case of a double star system with a strong coupling between orbital and spin motions, this effect could lead to the formation of a close binary. The question of magnetic coupling between the stars is also considered.

The question of the ultimate fate of fragments formed by sub-condensation in a contracting cloud has been the object of some controversy, the linear contraction time-scales being of the same order for the fragments and the cloud as a whole. Layzer (35) has advocated other factors favoring the coalescence of the fragments by collisions. Hunter (36) has shown that the effects of non-linear terms invalidate some of Layzer's arguments and Nakano (37) assuming that the initial large cloud is endowed with some angular momentum finds that, despite some coalescence and disintegration of fragments, the theory can lead to mass-functions which are not incompatible with observations in galactic clusters.

Arny (38) has introduced an interesting approach covering the essential non-linear and thermal effects for studying fragmentation and the growth of sub-condensations in large clouds

which may themselves be in internal motion and especially in contraction. He has also considered the same problem as that tackled by Nakano and arrives at more detailed and somewhat different conclusions (39).

Finally, let us mention that the possibility of star formation from superdense matter has been considered by Imshennik, Nadezhin, Pinaev and others (40).

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In closing this report, it is a pleasure to thank all the members of Commission 35 who have kindly communicated information on their work. I am especially grateful to Mrs Massevitch and Dr Hayashi who have prepared very complete surveys of the relevant literature and of work in progress in their respective countries. Finally, I want to acknowledge the efficient help of Drs A. Boury and M. Gabriel in preparing this report. I could not hope to have covered *all* contributions of significant interest for the Commission and emphasis may not always have been distributed rightly but, omissions or bias certainly were involuntary and I hope that I may count on the forbearance of all.

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