THE EVOLUTIONARY STATUS OF POPULATION II CEPHEIDS

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1. Introduction

Schwarzschild and Härm (1970, hereafter abbreviated SH) have suggested that the presence of Cepheids in globular clusters can be explained by the occurrence of 'flashes' in the thermally unstable helium-burning shells of post-horizontal-branch, or 'asymptotic-branch' stars. These flashes in the deep interior of the star produce large changes in the energy flux at the base of the envelope, which in turn cause the position of the model in the $M_{\mathrm{bol}} - \log T_e$ diagram to move. For most values of the total stellar mass and envelope composition studied by SH, the models simply moved up and down the asymptotic branch in response to a helium shell flash. However in a few cases, characterized by low total mass and a relatively high value of the envelope helium abundance, the models left the asymptotic branch and described loops extending into the region of pulsational instability. The Cepheids of Population II were thus tentatively identified by SH as asymptotic-branch stars that had recently undergone a helium shell flash and whose tracks were as a result describing a loop extending into the instability strip.

The primary purpose of the present investigation has been to study the possible occurrence of loops at various stages of the asymptotic-branch phase of a 0.60 $M_\odot$ star of Population II composition, using a more detailed treatment of the physics and the surface boundary conditions than were used by SH. Models were constructed using the Princeton stellar evolution program (Schwarzschild and Härm, 1965) as modified by Sweigart (1972). Radiative opacities were obtained by interpolation in the tables of Cox and Stewart (1970). The surface boundary conditions were derived from explicit envelope integrations. In the surface convective zone the mixing-length theory was used, with the mixing length taken to be one pressure scale height.

2. Preliminary Discussion

Schwarzschild and Härm (1967) discovered that helium shell burning in low mass stars does not proceed smoothly, but rather in a series of relaxation cycles, consisting of alternating active and quiescent phases. Figure 1 shows the total hydrogen- and helium-burning rates and the surface luminosity L as functions of time t for 2 active phases and the intervening quiescent phase of a 0.60 $M_\odot$ star near the end of its asymptotic-branch lifetime. The envelope composition parameters are $(X; Z) = (0.732; 0.001)$. The horizontal scale between $t = 14$ and $t = 20$ has been expanded by a factor of 50. Of particular interest are the decreases in L which occur during the active phases. A fast drop in L, of less than 1000 years duration, occurs immediately

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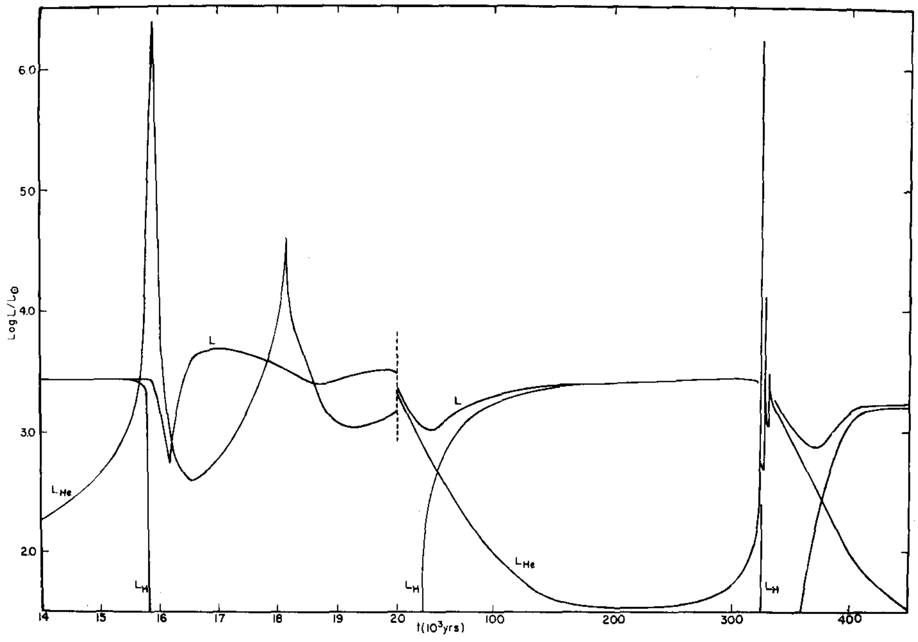

Fig. 1. Hydrogen- and helium-burning rates and surface luminosity as functions of time for two advanced relaxation cycles of a 0.60 $M_\odot$ star. The horizontal scale between $t = 14$ and $t = 20$ has been expanded by a factor of 50. The peaks in the helium burning at $t = 15.9$ and $t = 325$ represent respectively the main peaks from the active phases of cycle C and a post-asymptotic-branch cycle.

following the main helium burning peak. This drop can be seen at $t = 16.2$ in Figure 1. (The corresponding drop for the second active phase cannot be seen in Figure 1 because of the compression of the horizontal scale.) The results of SH indicate that the track in the HR diagram may describe a loop during these drops. A slower reduction in L occurs at the ends of the active phases. These may be seen in Figure 1 for $20 < t < 100$ and for $330 < t < 410$. There are therefore at least 2 stages during an active phase when loops may be expected to occur. ‘Fast loops’, lasting several hundred years, may be associated with the fast drops in L, as found by SH. A slower loop, lasting roughly 100 times longer, may occur at the ends of the active phases. Such loops were not reported in SH, but have subsequently been found by Schwarzschild and Härm (Schwarzschild, 1971).

3. Results for the Initial Relaxation Cycle

The initial relaxation cycle for a 0.60 $M_\odot$ star with the composition (0.732; 0.001) has been computed by Sweigart (1972) using the modified Princeton code. He found a maximum departure from the asymptotic branch of only 0.01 in $\log T_e$. An independent calculation by Demarque and Mengel has similarly found that loops do not occur for the initial relaxation cycle of this star. A calculation of the initial cycle for a 0.60 $M_\odot$

star with the composition (0.600; 0.001) also failed to produce a loop. These results are somewhat surprising because the parameters of the models appear favorable from the findings of SH. The disagreement with SH must arise either from the treatment of the physics and the surface boundary conditions, or from differences in the structure of the models at the time of the initial cycle.

4. Principal Results for Later Relaxation Cycles

The results of SH suggest that of two stars having the same envelope composition, the one having the smaller amount of mass above the center of the hydrogen-burning shell (the 'envelope mass') is the more likely to leave the asymptotic branch in response to a helium shell flash. This implies that the occurrence of loops for a given star will become more likely for later cycles, when the outward progression of the hydrogen-burning shell has reduced the envelope mass.

In the present study the last 4 relaxation cycles for a 0.60 $M_\odot$ star with the envelope composition (0.732; 0.001) have been computed. Of these cycles, the fourth occurred after the star had evolved off the asymptotic branch and will not be discussed here. To reach these advanced cycles without expending an inordinate amount of computer time, the earlier cycles were suppressed using a method developed by Sweigart (1971). The following discussion deals with the last 3 of the approximately 12 relaxation

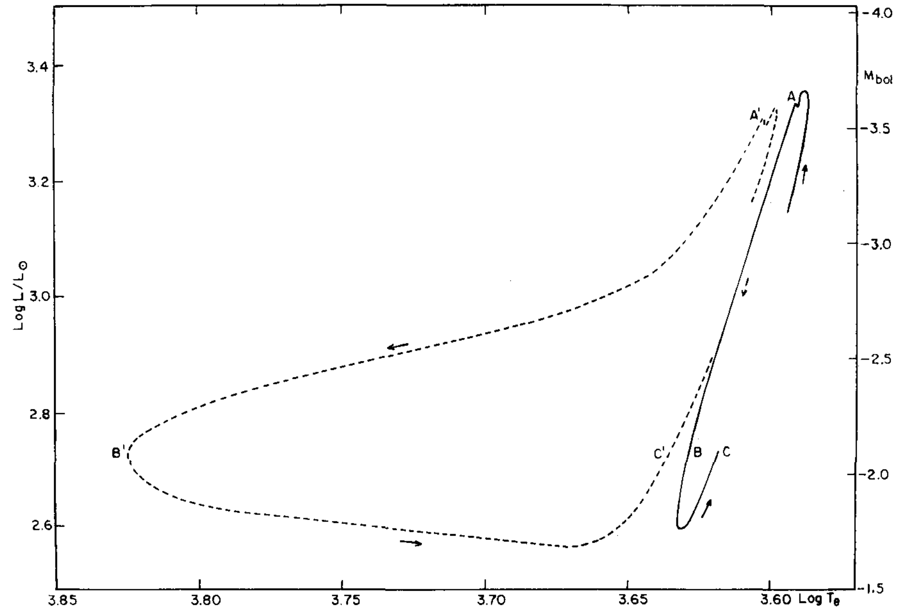

Fig. 2. Tracks in the HR diagram immediately following the main peaks in the helium burning of cycle A (solid curve) and the cycle for a 0.60 $M_\odot$ star with the envelope composition (0.600; 0.001) (dashed curve). The envelope masses for the two cases are nearly identical. Times in years between designated points are: $t_{AB} = 525$, $t_{BC} = 127$, $t_{A'B'} = 1000$, $t_{B'C'} = 360$.

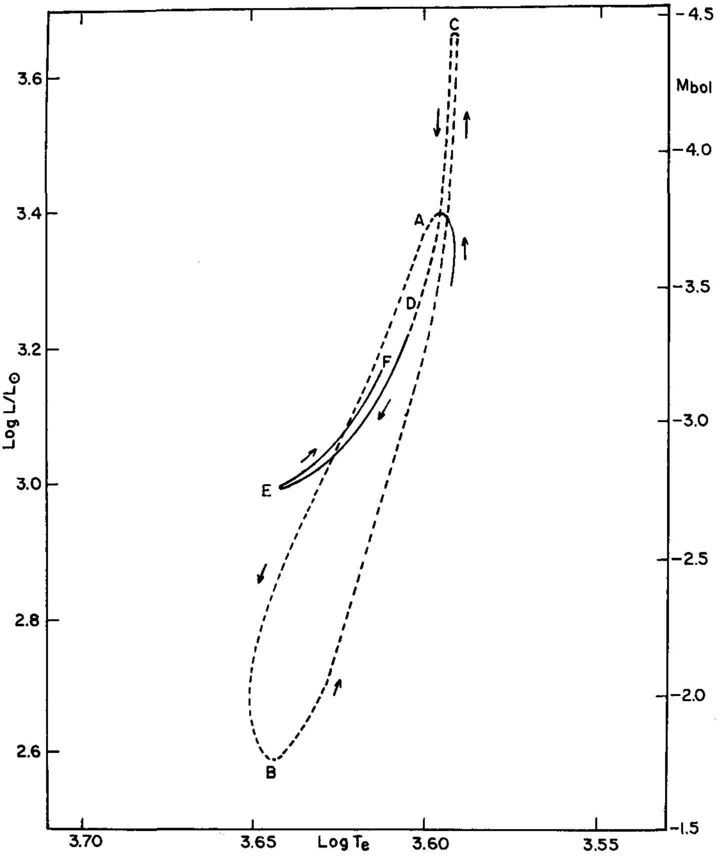

Fig 3. Track in the HR diagram during the active phase of cycle B. Dashed curve (A-B-C) represents a relatively fast phase immediately following the main peak in the helium burning. Solid curve (D-E-F) indicates the relatively slow drop in the surface luminosity at the end of the active phase. Times in years between designated points are: $t_{AB} = 500$, $t_{BC} = 740$, $t_{CD} = 7100$, $t_{DE} = 25000$, $t_{EF} = 34000$.

cycles which this star will undergo before it evolves off the asymptotic branch. These advanced cycles will be denoted by A, B, and C.

Figures 2 and 3 show the evolutionary track for the active phases of cycles A and B respectively. The solid line in Figure 2 represents the track during the rapid drop in L immediately following the main helium-burning peak, at which time the envelope mass was 0.027 $M_{\odot}$. The dashed line shows the corresponding phase for a model with the same total mass and envelope mass, but with the envelope composition (0.600; 0.001). Since no appreciable departure from the asymptotic branch was found at this phase for the case $X=0.732$, the calculations for cycle A were terminated soon after the rapid drop in L. During the active phase of cycle B the envelope mass was 0.019 $M_{\odot}$. Computations were carried out only for the composition (0.732; 0.001), and the entire active phase was followed. Relatively fast and slow stages are indicated in Figure 3 by

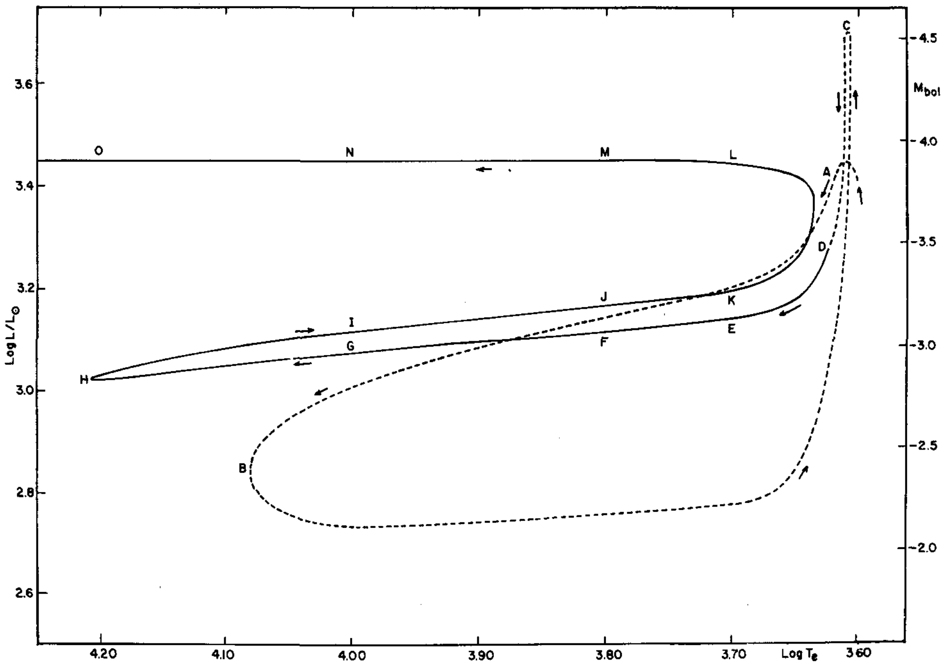

Fig. 4. Track in the HR diagram for the active phase of cycle C. Slow and fast phases are designated by solid and dashed lines respectively. The fast loop following the main peak in the helium burning is given by the segment of dashed curve from A to C. The slow loop at the end of the active phase is given by the segment of solid curve from D to K. Times in years between designated points are: $t_{AB} = 290$, $t_{BC} = 850$, $t_{CD} = 8650$, $t_{DE} = 7000$, $t_{EF} = 1730$, $t_{FG} = 3100$, $t_{GH} = 10700$, $t_{HI} = 16500$, $t_{IJ} = 7500$, $t_{JK} = 5800$, $t_{KL} = 170000$, $t_{LM} = 19300$, $t_{MN} = 13300$, $t_{NO} = 11900$.

the dashed and solid lines respectively. As was found for cycle A, the maximum departure from the asymptotic branch is quite small.

For the active phase of cycle C the envelope mass was 0.011 $M_{\odot}$. The calculations were made for the composition (0.732; 0.001). The evolutionary track is shown in Figure 4. The principal features of the track are a fast loop (points A to C) lasting 380 years, and a slow loop (points D to K) lasting 82000 yr. In the quiescent phase following cycle C the star evolved off the asymptotic branch (points K to O).

5. Discussion

These results indicate that significant departures from the asymptotic branch resulting from helium shell flashes will occur only when the outward progression of the hydrogen-burning shell has reduced the envelope mass to quite low values. For $(X; Z) = (0.732; 0.001)$ this mass must be less than about 0.02 $M_{\odot}$, while for (0.600; 0.001) it may be as large as 0.03 $M_{\odot}$. The requirement of small envelope mass means that loops will occur only for the last few relaxation cycles on the asymptotic branch. The precise number of cycles which produce loops remains somewhat uncertain, since the suppression of the early cycles introduces some arbitrariness into the models. This is because the exact time at which a flash occurs and therefore the envelope mass at that time depend on when the transition from 'flash suppressed' to 'flash permitting' computations is made. For the case (0.732; 0.001) an estimate of 1 or 2 loop-producing cycles is perhaps not unreasonable. A similar result should hold in the case of more massive stars when the envelope mass has been reduced to a value sufficiently small for the occurrence of loops.

The amount of time which the star spends in the instability strip is rather difficult to estimate because of the arbitrariness mentioned above, and because of uncertainties in the position of the edges of the instability strip. For the 0.60 $M_\odot$ star, lower and upper limits for this time may be crudely taken as 50000 and 200000 yr. These estimates include crossings of the instability strip both during loops, and during the final evolution off the asymptotic branch. If the beginning of the asymptotic-branch phase is taken to be the time when core convection ceases following the horizontal-branch stage, then these times represent about 0.2 to 0.8% of the asymptotic-branch lifetime of the star.

The present results predict rates of period change for the slow-loop and post-asymptotic-branch crossings of the instability strip that are about an order of magnitude slower than those found by SH and therefore agree better with observation. Period changes during a fast-loop crossing might be observable, but the probability of observing a star in such a relatively rapid stage is small.

6. Summary

The results of this investigation confirm in general the preliminary results of SH. The principal difference is that SH found that the tracks of asymptotic-branch stars of low mass describe a large number of fast loops, whereas the present results suggest the occurrence of a small number of slow loops. These loops, together with the final evolution off the asymptotic branch, provide a possible explanation for the occurrence in globular clusters of Cepheids with periods greater than about 8 days. The Cepheids with shorter periods can perhaps be identified as stars evolving off the horizontal branch, as has been suggested by Kraft (1972).

Acknowledgements

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DISCUSSION

Schwarzschild: I would like to emphasise two points of Dr Mengel's remarks. First, because Dr Mengel treats the envelope of his stars enormously more carefully than we did in Princeton I believe that he is right in saying that the fast loops in the early cycles probably do not occur to any significant extent, contrary to our earlier and rougher Princeton results. Second, unpublished rough Princeton computations have also given the slow loops in the advanced cycles; this I feel adds some weight to Dr Mengel's discovery of these slow loops and their possible role in forming the long-period group of Population II Cepheids.

Lloyd Evans: There are type *II* Cepheids of the short period ($P \sim 2^{d}$) group in the disk population which show enhanced abundance of *C* and, possibly, *s*-process elements.

Demers: How could we explain the lack of Population II Cepheids with $P > 3^{d}$ in dwarf spheroidal galaxies? Differences in chemical composition? Earlier evolutionary stage of these galaxies?

Mengel: Perhaps the masses of asymptotic branch stars in these systems are a bit higher. The slow loops would be somewhat speeded up in that case, and one might not see stars in this phase. There is also the possibility that the envelope might be lost before the slow loop stage is reached for a star with $M > 0.60\ M_{\odot}$. If that happens, the slow loops may not occur at all, as Prof. Schwarzschild has pointed out.

Dickens: Can you say anything from your models about the possibility of mixing from the hydrogen burning shell into the atmosphere?

Mengel: This has been looked into by Allen Sweigart and myself for a $0.70\ M_{\odot}$ asymptotic branch star. At one point following a helium shell flash the surface convective zone becomes very deep. (This occurs during the rapid rise in the surface luminosity following the burning peak of a helium shell flash.) Surface convection reaches in to the top of the hydrogen-burning shell, but does not penetrate very far into it, even in this favorable case. The hydrogen shell is so tightly bound that the pressure difference from its top to its bottom amounts to many orders of magnitude. Since surface convection did not reach deeply into the shell, we were unable to mix any CNO-processed material to the surface.