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STRONG DOUBLE LAYERS, EXISTENCE CRITERIA, AND ANNIHILATION: AN APPLICATION TO SOLAR FLARES

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ABSTRACT

We present some conditions for the stability of a strong double layer in a warm, current-carrying plasma, which can be extended into the relativistic regime. We apply a model for plasma heating by the electron beam emitted from the double layer and show that this leads to a finite life time of the double layer. We also show that the radio emission accompanying this process can well describe the observed phenomena in Type I radio bursts using a direct emission mechanism, not involving Langmuir waves.

Subject headings: plasmas — Sun: flares — Sun: radio radiation

1. INTRODUCTION

Electrostatic Double Layers (henceforth DLs) are of potential significance in the dissipation of solar and stellar flares. However, because of their peculiar boundary conditions, they may well be intrinsically unstable under astrophysical conditions and show a transient behavior. DL stability has been discussed at various places in the literature (Levine & Crawford 1980; Carlqvist 1979; Knorr & Goertz 1974), but not yet directly applied to astrophysical observations. This report combines the stability of DLs with their radiation properties as discussed by Kuijpers (1990) and Volwerk & Kuijpers (1993) to explain bright radio spikes.

We will discuss a cold plasma approach which leads to the Langmuir condition in § 2. We will study a plasma of finite temperature in § 3 which leads to the existence criteria (Bohm criteria) for strong DLs. In § 4 we will discuss, respectively, the heating of the ambient plasma by the emanating electron beam and the annihilation of the DL. In § 5 we apply the mechanism to the radio emission of the sun.

2. STRONG DLs IN A COLD PLASMA

We will derive existence criteria for stationary, one-dimensional strong DLs. The general way of finding these is by solving Poisson's equation for the plasma containing a DL. In a steady state and in the absence of pair creation the charge densities of electrons and ions in the plasma can be described by their respective current densities i_e and i_i, giving rise to the following form of Poisson's equation:

$$\frac{1}{4\pi}\frac{d^2\phi}{dx^2} = \left(\frac{i_e}{c}\right)\frac{\gamma_{0,e}m_e c^2 + e\phi}{\sqrt{(\gamma_{0,e}m_e c^2 + e\phi)^2 - m_e^2c^4}} - \left(\frac{i_i}{c}\right)\frac{\gamma_{0,i}m_i c^2 + Ze\phi_1}{\sqrt{(\gamma_{0,i}m_i c^2 + Ze\phi_1)^2 - m_i^2c^4}}\,, \tag{1}$$

where $\phi_1 = \phi_{DL} - \phi$ and Ze is the charge of the ion. The symbol $\gamma_{0,e}$ ($\gamma_{0,i}$) is the Lorentz factor of the electrons (ions) at the low (high) potential side corresponding to the current drift velocity. Using equation (1) we define a new function $P(\phi)$ (Levine & Crawford 1980) so that

$$\frac{d^2\phi}{dx^2} = -\frac{dP}{d\phi}\,. \tag{2}$$

A simple integration over ϕ of equation (2) using equation (1) shows:

$$-P(\phi)/4\pi = \left(\frac{i_e}{ec}\right)\left[\sqrt{(\gamma_{0,e}m_e c^2 + e\phi)^2 - m_e^2c^4} - \sqrt{(\gamma_{0,e}^2 - 1)m_e^2c^4}\right] + \left(\frac{i_i}{Zec}\right)\left[\sqrt{(\gamma_{0,i}m_i c^2 + Ze\phi_1)^2 - m_i^2c^4} - \sqrt{(\gamma_{0,i}m_i c^2 + Ze\phi_{DL})^2 - m_i^2c^4}\right]. \tag{3}$$

The integration constant is determined by the condition that $P(0) = 0$. Since $P(\phi)$ is monotonic in ϕ, P is an implicit function of x,

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and it follows from equation (2) that P can be written in terms of the electric field $E = -d\phi/dx$:

$$P(x) = \frac{1}{2}[E^2(0) - E^2(x)] = -\int_0^x 4\pi\rho(x')E(x')dx',$$

with $\rho(x)$ the electric charge density.

The electric field must vanish at the boundaries of the DL which implies that $E(0) = E(x_1) = 0$, where x_1 is the boundary of the DL. We therefore have the extra boundary condition $P(x_1) = P(\phi_{DL}) = 0$. The condition at ϕ_{DL} gives the Langmuir condition:

$$Z\frac{i_e}{i_i} = \frac{\sqrt{(\gamma_{0,i}m_i c^2 + Ze\phi_{DL})^2 - m_i^2c^4}}{\sqrt{(\gamma_{0,e}m_e c^2 + e\phi_{DL})^2 - m_e^2c^4} - \sqrt{(\gamma_{0,e}^2 - 1)m_e^2c^4}}. \tag{4}$$

Equation (4) has two specific limits, in the case of a nonrelativistic DL (i.e., $e\phi_{DL} \ll m_e c^2$) we find for singly ionized particles that $i_e/i_i = (m_i/m_e)^{1/2}$, whereas for relativistic DLs (i.e., $e\phi_{DL} \geq \gamma_{0,i}m_i c^2$) we find for singly ionized particles that $i_e/i_i = 1$.

The above is based on the fact that at the boundaries of the DL there are cold ($T = 0$) reflected particles species to maintain charge neutrality in the ambient plasma. We will now study a finite temperature plasma using a Boltzmann distribution for the reflective particles over the DL.

3. WARM, REFLECTED PARTICLES: BOHM CRITERIA

If the ambient plasmas are not cold, but have finite electron (T_e) and ion (T_i) temperatures, there will be reflected electrons at the high potential side and reflected ions at the low potential side of the DL. We further assume that the DL is strong, $e\phi_{DL}/k_B T_{e,i} \gg 1$, so that the respective particle species cannot penetrate the potential barrier from the "wrong" side. We will describe the distribution of these particles over the DL by a Boltzmann distribution, which modifies Poisson's equation (1) and the function P in equation (3). We now have to solve

$$\frac{1}{4\pi}\frac{d^2\phi}{dx^2} = -\frac{1}{4\pi}\frac{dP}{d\phi} = +\left(\frac{i_e}{c}\right)\frac{\gamma_{0,e}m_e c^2 + e\phi}{\sqrt{(\gamma_{0,e}m_e c^2 + e\phi)^2 - m_e^2c^4}} - \left(\frac{i_i}{c}\right)\frac{\gamma_{0,i}m_i c^2 + Ze\phi_1}{\sqrt{(\gamma_{0,i}m_i c^2 + Ze\phi_1)^2 - m_i^2c^4}} + \delta_e \exp\left\{-\frac{e\phi_1}{k_B T_e}\right\} - \delta_i \exp\left\{-\frac{Ze\phi}{k_B T_i}\right\}, \tag{5}$$

where δ_e and δ_i are the electron and ion densities of the reflected particles just outside the respective boundaries of the DL. They can be determined by the boundary condition that the ambient plasmas are neutral on either side of the DL, so

$$\rho_e(\phi = 0) = \delta_i = \left(\frac{i_e}{c}\right)\frac{\gamma_{0,e}m_e c^2}{\sqrt{(\gamma_{0,e}m_e c^2)^2 - m_e^2c^4}}, \tag{6}$$

$$\rho_i(\phi = \phi_{DL}) = \delta_e = \left(\frac{i_i}{c}\right)\frac{\gamma_{0,i}m_i c^2 + Ze\phi_{DL}}{\sqrt{(\gamma_{0,i}m_i c^2 + Ze\phi_{DL})^2 - m_i^2c^4}}. \tag{7}$$

To determine whether the DL can exist or not we have investigated the behavior of P at the boundaries. Both the function P and its derivative to ϕ vanish at the boundaries. As P has to be negative within the DL we investigate the Taylor expansion of P to second order:

$$P(\phi) = \frac{1}{2}(\phi - \phi_0)^2\left.\frac{d^2P}{d\phi^2}\right|_{\phi=\phi_0}, \quad \text{with} \quad \phi_0 = 0, \phi_{DL}. \tag{8}$$

From equation (5) we find the second derivative:

$$-\frac{d^2P}{4\pi d\phi^2} = -\frac{em_e^2c^4}{[(\gamma_{0,e}m_e c^2 + e\phi)^2 - m_e^2c^4]^{3/2}}\left(\frac{i_e}{c}\right) - \frac{Zem_i^2c^4}{[(\gamma_{0,i}m_i c^2 + e\phi_1)^2 - m_i^2c^4]^{3/2}}\left(\frac{i_i}{c}\right) + \frac{\delta_e e}{k_B T_e}\exp\left\{-\frac{e\phi_1}{k_B T_e}\right\} + \frac{\delta_i Ze}{k_B T_i}\exp\left\{-\frac{Ze\phi}{k_B T_i}\right\}. \tag{9}$$

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Requiring $d^2P/d\phi^2$ to be negative at $\phi = 0$, $\phi_{\rm DL}$ we find that there is a limit to the temperatures of the reflected electrons and ions.

$$\frac{\delta_i Ze}{k_B T_i} \geq \left(\frac{i_e}{c}\right)\frac{em_e^2c^4}{[(\gamma_{0,e}^2-1)m_e^2c^4]^{3/2}} + \left(\frac{i_i}{c}\right)\frac{Zem_i^2c^4}{[(\gamma_{0,i}m_ic^2+Ze\phi_{\rm DL})^2-m_i^2c^4]^{3/2}}, \tag{10}$$

$$\frac{\delta_e e}{k_B T_e} \geq \left(\frac{i_e}{c}\right)\frac{em_e^2c^4}{[(\gamma_{0,e}m_ec^2+e\phi_{\rm DL})^2-m_e^2c^4]^{3/2}} + \left(\frac{i_i}{c}\right)\frac{Zem_i^2c^4}{[(\gamma_{0,i}^2-1)m_i^2c^4]^{3/2}}. \tag{11}$$

If we define that $\mu = m_e/m_i$, $\alpha = e\phi_{\rm DL}/m_ec^2$ and $\tau_{e,i} = k_BT_{e,i}/m_ec^2$ we can rewrite the above limits, neglecting the terms that are of $\mathcal{O}(m_e/m_i)$. We find:

$$\tau_i \leq \frac{Z(\gamma_{0,e}^2-1)[(\gamma_{0,i}+Z\mu\alpha)^2-1]^{3/2}}{[(\gamma_{0,i}+Z\mu\alpha)^2-1]^{3/2}+(i_i/i_e)Z\mu(\gamma_{0,e}^2-1)^{3/2}}, \tag{12}$$

$$\tau_e \leq \left(\frac{i_i}{i_e}\right)\frac{(\gamma_{0,i}+Z\mu\alpha)[(\gamma_{0,e}+\alpha)^2-1]^{3/2}}{\sqrt{(\gamma_{0,i}+Z\mu\alpha)^2-1}}. \tag{13}$$

In this calculation we have made the approximation that the exponentials on the right-hand side of equation (9) can be neglected for the reflected electrons if $\phi = 0$ and for the reflected ions if $\phi = \phi_{\rm DL}$.

4. TIME SCALE FOR HEATING AND DL ANNIHILATION

We will first investigate the electron beam emanating from the DL. It can be shown that in the case of cold ambient plasmas the current density through the strong (nonrelativistic) DL can be described by the Langmuir-Child law (Carlqvist 1982; Raadu 1989):

$$j_{\rm tot}d^2 = \left(1+\sqrt{\frac{m_e}{m_i}}\right)\frac{C_0}{9\pi}\sqrt{\frac{2e}{m_e}}\,\phi_{\rm DL}^{3/2}, \tag{14}$$

where d is the thickness of the DL, $10\lambda_D \leq d \leq 1000\lambda_D$, with λ_D the Debye length of the ambient plasma and $C_0 \approx 1.865$. We will use this relation to find the total number of particles which are accelerated into the ambient plasma to explain the footpoint brightening.

The beam emanating from the DL is completely specified by the voltage drop over the DL, i.e., α, and the current density in the plasma, i.e., $\gamma_{0,e}$. Using the continuity equation for the current we can express the beam particle density and the beam Lorentz factor as

$$n_b = n_a\frac{\gamma_{0,e}+\alpha}{\gamma_{0,e}}\sqrt{\frac{\gamma_{0,e}^2-1}{(\gamma_{0,e}+\alpha)^2-1}}, \tag{15}$$

$$\gamma_b = \gamma_{0,e}+\alpha, \tag{16}$$

where n_a and n_b are the electron densities of the ambient plasma and of the emitted beam, respectively.

In order to describe heating of the plasma by the beam we now use the results obtained by Cromwell, McQuillan, & Brown (1988). They calculate the interaction of a highly energetic electron beam with the ambient plasma. They solve the following equations for the heating of the plasma, taking into account collisions of the beam particles with the ambient plasma particles, collisional temperature equilibration between electrons and ions, and classical and anomalous ion-accoustic resistivity:

$$1.5n_ak_B\frac{dT_e}{dt} = 1.5n_ak_B(T_i-T_e)\tau_{\rm eq}^{-1} + 2\pi e^3\ln(\Lambda)n_aj_pW_0^{-1} + \eta_{\rm cl}j_p^2 + \chi_{\rm i.a.}\eta_{\rm i.a.}j_p^2, \tag{17}$$

$$1.5n_ak_B\frac{dT_i}{dt} = 1.5n_ak_B(T_e-T_i)\tau_{\rm eq}^{-1} + (1-\chi_{\rm i.a.})\eta_{\rm i.a.}j_p^2, \tag{18}$$

where

$$\tau_{\rm eq} = \sqrt{\frac{9}{128\pi}}\left(\frac{m_em_i}{\ln(\Lambda)n_ae^4}\right)\left(\frac{k_BT_e}{m_e}\right)^{1.5}, \tag{19}$$

$$\eta_{\rm cl} = \sqrt{\frac{8}{9}}\sqrt{m_e}e^2\ln(\Lambda)(k_BT_e)^{-1.5}, \tag{20}$$

are the thermal equilibration time for electrons and ions and the classical Spitzer resistivity, respectively. $\chi_{\mathrm{i.a.}}$ and $(1-\chi_{\mathrm{i.a.}})$ are the fractions of anomalous resistive dissipation for the electrons and the ions, respectively; j_p is the return current induced by the beam, $\ln(\Lambda)$ the Coulomb logarithm of the ambient plasma and W_0 is the energy of an electron emitted by the DL.

The results of Cromwell et al. (1988) are applicable to our DL model, since they assume a constant initial energy of the beam particles, with energy W_0, so that a constant energy input rate is obtained. We now only need to specify the rise time of the DL in the plasma. We can take this from laboratory and computer experiments or from a theoretical model on DL formation.

Bohm and Torvén (Bohm 1991; Bohm & Torvén 1991) find from their experiments and computer simulations that the response of a plasma to an applied voltage drop for $\Delta V = 100$ V takes place in $\sim t_{\mathrm{rise}} \approx 100\ \mu$s, or of the order of 10^5 times the electron and 10^3 times the ion plasma frequency (experiments were performed with argon).

Galeev et al. (1981) have calculated the nonlinear development of the Buneman instability into a DL. The condition for the occurrence of the Buneman instability is that the drift velocity of the current-carrying electrons exceeds the thermal velocity of the electrons. For simplicity they take

$$\frac{m_e I_e^2}{2n_e^2} \gg T_e + T_i \,. \tag{21}$$

The creation of DLs takes place in two distinct phases: first a gradual increase in the potential over one wavelength of the instability, which grows asymmetrically. This phase lasts for approximately on the order of 10 μs, or 10^5 times the electron plasma periods for solar plasma parameters, in which potential drops are reached up to 10 $k_B T_e/e$. After this, an explosive phase occurs in which potential drops are reached up to 300 $k_B T_e/e$ on a similar time scale.

We first consider a rise time of $t_{\mathrm{rise}} = 2$ s as considered by Cromwell et al.: their case *b* close to the onset of catastrophic anomalous ion-acoustic resistivity with $T_0 = 5 \times 10^6$ K, $n_a = 10^{11}$ cm^{-3}, $W_0 = 25$ keV for the energy of the beam particles (implying that $\alpha = 0.05$). We find from equation (13) that the DL (and hence the beam) will exist as long as the temperature has not risen above $T_e = 5.93 \times 10^8$ K, which means that $T_e/T_0 = 118$. Following Cromwell et al. (their Fig. 5*b*) this temperature ratio would be reached after $\sim$2 s. However, this result is obtained for a rise time of 2 s, which we already know is far too long for the creation of the DL.

A rough estimate from the equation describing the electron heating shows that short rise times of the current (of the order of 10–100 μs) result in a violation of the Bohm criterion within milliseconds. However, we should bear in mind that with such small rise times of the current density the regime of catastrophic heating applies, case *b* of Cromwell et al., for which they give no definite time scales, as they do for the marginal stability hypothesis (case *a*) in their model. Also cooling and relativistic effects have not been included in this model. Cooling and relativistic effects will prolong the life time of a DL. This is clear for the cooling, the relativistic effects will lead to the fact that the beam will lose less of its energy in a region around the DL and more further on in the plasma tube. We therefore assume that for a rise time between 10 and 100 μs the annihilation time will be of the order of milliseconds.

After the DL has been annihilated, a high-temperature, current-carrying plasma remains. The Debye length has increased by a factor $\sqrt{118}$.

5. BRIGHT RADIO BURSTS AT THE SUN

For solar flares we assume a thickness *d* of a DL of $100\lambda_D$ and $\alpha = 0.05$, which results in a current density of $j = 2.2 \times 10^9$ statamps cm^{-2}. A total cross section of 10^{18} cm^2 of a collection of DLs with equations (15) and (16) leads to a production of high-energy electrons by the DL of 4.5×10^{36} electrons s^{-1} of 25 keV. To explain the foot point brightening seen during the impulsive phase of the largest solar flares (van den Oord 1990) a number of $\geq 10^{36}$ electrons s^{-1} is needed.

It has already been proposed that some radio spikes (Allaart et al. 1990) observed during solar flares and type I radio bursts are produced by transient DLs (Kuijpers 1990). Since DLs heat the plasma this would result in a frequency drift of the spikes.

In the maser mechanism operating inside a DL (Kuijpers 1990) the emitted frequency is given by the resonance condition:

$$\omega_{\mathrm{DL}}(\boldsymbol{k}_{\mathrm{DL}}) - \boldsymbol{k}_{\mathrm{DL}} \cdot \boldsymbol{v} = \pm[\omega_t(\boldsymbol{k}_t) - \boldsymbol{k}_t \cdot \boldsymbol{v}] \,, \tag{22}$$

where $\boldsymbol{v}$ is the velocity of the electrons in the DL, ω_t, $\boldsymbol{k}_t$ are the frequency and the wavevector of the emitted radiation and ω_{DL}, $\boldsymbol{k}_{\mathrm{DL}}$ are the frequency and wavevector of the DL, where $k_{\mathrm{DL}} = 2\pi/L_{\mathrm{DL}}$. For a stationary DL $\omega_{\mathrm{DL}} = 0$ and in the nonrelativistic case $\boldsymbol{k}_t \cdot \boldsymbol{v} = \boldsymbol{0}$ so that $\omega_t = |\boldsymbol{k}_{\mathrm{DL}} \cdot \boldsymbol{v}|$. At the low-frequency end there is a cutoff at the electron plasma frequency. As the ambient plasma is heated the cutoff shifts to lower frequencies and so does the spectral peak of the maser.

The result in this case is that radiation emitted in the gigaHertz band will rapidly decrease in frequency during the lifetime of the DL.

6. CONCLUSIONS

In this paper we have derived conditions for the stability of DLs in a current-carrying plasma which can be extended into the relativistic regime.

We have shown that due to heating of the ambient plasma by the emanating electron beam the Bohm criterion will be violated within milliseconds after the creation of the DL. This leaves behind a heated plasma. Elsewhere in the loop, where the plasma is not "overheated" by the electron beam, a new DL can be created. Indeed, such a process is observed in laboratory plasmas, where DLs move through the plasma column until they move out of the plasma chamber, at which moment a new DL is created at the anode (Lindberg 1987).

Furthermore, we show that as a direct result of the appearance and annihilation of the DL there will be transient radio emission, which drops rapidly in frequency due to the plasma heating, as is observed. The simplicity of this model lies in the fact that the radio emission is direct (linear acceleration emission), which creates the observed radio emission instead of being produced by interaction of two or more (Langmuir) waves.

How DLs in solar flares are formed is not solved. The nonlinear development of a Buneman instability can cause the fast creation of a strong DL. Also a density minimum in the ion component of the plasma is favorable for DL creation (Carlqvist 1972; Raadu & Carlqvist 1981).

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