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CORRECTION TO 'EXISTENCE AND BOX DIMENSION OF GENERAL RECURRENT FRACTAL INTERPOLATION FUNCTIONS'

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This correction to the paper [1] adjusts the definition of degenerate vertices to avoid some degenerate exceptions to the main theorem. Under the original definition of degenerate vertices [1, Definition 1.4], there could be exceptions to the third sentence in the proof of Proposition 3.8: 'Since V is a nondegenerate strongly connected component, $\Gamma(f|_{I_j})$ is not a line segment for each $j \in V$'. We fix this problem by revising the definition of degenerate vertices and show that the modification does not affect the validity of other statements in the paper involving degenerate vertices.

For $1 \le i \le N$, write $\mathcal{P}(i) = \{1 \le j \le N :$ there exists a path from j to $i\}$. We modify Definition 1.4 as follows.

DEFINITION 1. A vertex $i \in \{1, \dots, N\}$ is called *degenerate* if for all $j \in \{i\} \cup \mathcal{P}(i)$, we have either $d_j = 0$ or points in $\{(x_k, y_k) : \ell(j) \le k \le r(j)\}$ are collinear.

Note that the only difference between the new definition and the original one is the addition of an allowance for $d_j = 0$. For a motivation, please see the later remark. In this new setting, we can prove the following lemma.

LEMMA 2. *Let f be the self-affine RFIF determined by* $\{\omega_i\}_{i=1}^N$, *where*

$$\omega_i(x, y) = (a_i x + e_i, c_i x + d_i y + f_i), \quad (x, y) \in D_i \times \mathbb{R}.$$

Then for $1 \le i_0 \le N$, i_0 *is degenerate if and only if* $\Gamma(f|_{I_{i_0}})$ *is a line segment.*

In particular, [1, third sentence in the proof of Proposition 3.8 and Proposition 3.1] remain valid (in our new setting). Other parts of the paper, including the main theorem, are unaffected by this modification.

PROOF. First we prove the 'if' part. Equivalently, we prove that if i_0 is not degenerate, then $\Gamma(f|_{I_{i_0}})$ is not a line segment. By our new definition, there exists $j \in \{i_0\} \cup \mathcal{P}(i_0)$ such that $d_j \neq 0$ and points in $\{(x_k, y_k) : \ell(j) \leq k \leq r(j)\}$ are not collinear. Since $D_j = [x_{\ell(j)}, x_{r(j)}]$ and $y_k = f(x_k)$, it follows that $\Gamma(f|_{D_j})$ is not a segment. Combining this with $d_j \neq 0$ shows that $\Gamma(f|_{I_j}) = \omega_j(\Gamma(f|_{D_j}))$ is not a segment.

If $j = i_0$, then we are done. Suppose $j \neq i_0$. Then $j \in \mathcal{P}(i_0)$ and we can find $k_0, k_1, \ldots, k_n \in \{1, \ldots, N\}$ with $k_0 = j$ and $k_n = i_0$ such that $I_{k_{t-1}} \subset D_{k_t}$ and $d_{k_t} \neq 0$ for all $1 \leq t \leq n$. Recalling that $\Gamma(f|_{I_j})$ is not a segment, we see that $\Gamma(f|_{D_{k_1}}) \supset \Gamma(f|_{I_{k_0}}) = \Gamma(f|_{I_j})$ is also not a segment. Combining this with $d_{k_1} \neq 0$,

$$\Gamma(f|_{I_{k_1}}) = \omega_{k_1}(\Gamma(f|_{D_{k_1}}))$$

is not a segment. Repeating this argument, $\Gamma(f|_{I_{k_t}}) = \omega_{k_t}(\Gamma(f|_{D_{k_t}}))$ is not a segment for $1 \leq k \leq n$. Since $i_0 = k_n$, $\Gamma(f|_{I_{i_0}})$ is not a segment.

Now we prove the 'only if' part. This direction is an updated version of Proposition 3.1 under the new definition of degenerate vertices, and the proof is very similar. Let $C_*(I) = \{g \in C(I) : g(x_i) = y_i,\ 0 \leq i \leq N\}$ and let T be a map on $C_*(I)$ given by

$$Tg(x) = F_i(L_i^{-1}(x), g(L_i^{-1}(x))) \quad \text{for all } x \in I_i \quad \text{and } 1 \leq i \leq N.$$

Denote by $C^*(I)$ the collection of $g \in C_*(I)$ such that $g|_{I_j}$ is linear for all $j \in \{i_0\} \cup P(i_0)$. Recalling the proof of [1, Theorem 1.3], it suffices to show that T maps $C^*(I)$ into itself.

To this end, fix any $g \in C^*(I)$. Let $j \in \{i_0\} \cup P(i_0)$. If $d_j = 0$, then

$$Tg(x) = c_j L_j^{-1}(x) + f_j = a_j^{-1} c_j (x - e_j) + f_j, \quad x \in I_j,$$

so Tg is linear on I_j. If $d_j \neq 0$, by definition, points in $\{(x_k, y_k) : \ell(j) \leq k \leq r(j)\}$ are collinear. Since $g \in C^*(I)$, this implies that $g|_{D_j}$ is linear. Notice that

$$Tg(x) = F_j(L_j^{-1}(x), g(L_j^{-1})(x)) = c_j L_j^{-1}(x) + d_j g(L_j^{-1}(x)) + f_j, \quad x \in I_j.$$

Since L_j^{-1} is linear on I_j and g is linear on D_j, it follows that Tg is linear on I_j. In conclusion, $Tg \in C^*(I)$. □

REMARK 3. It is not hard to construct a system satisfying the following two conditions:

(1) there is some j_0 such that $d_{j_0} = 0$ and points in $\{(x_k, y_k) : \ell(j_0) \leq k \leq r(j_0)\}$ are not collinear;
(2) there is some $i_0 \neq j_0$ such that $D_{i_0} = I_{i_0} \cup I_{j_0}$, $d_{i_0} \neq 0$ and points in $\{(x_k, y_k) : \ell(i_0) \leq k \leq r(i_0)\}$ are collinear.

Under these conditions and the original definition of degenerate vertices, $\{i_0\}$ is a nondegenerate strongly connected component. However, if we denote by $\widetilde{C}(I)$ the collection of $g \in C_*(I)$ such that $g|_{D_{i_0}}$ is linear, it is straightforward to check that $Tg \in \widetilde{C}(I)$ whenever $g \in \widetilde{C}(I)$. Thus, the corresponding FIF f is linear on D_{i_0} so that $\Gamma(f|_{I_{i_0}})$ is a line segment. This is why we modify the original definition.

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Reference

[1] H.-J. Ruan, J.-C. Xiao and B. Yang, 'Existence and box dimension of general recurrent fractal interpolation functions', *Bull. Aust. Math. Soc.* **103** (2021), 278–290.

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