the main discussion of objects in complex hyperbolic space and on its boundary begin. There are sections on the two main models of complex hyperbolic space and their boundaries, followed by an extensive discussion of bisectors and spinal spheres, automorphisms and numerical invariants. The amount of material it contains makes the book rather daunting, particularly to those learning the subject. Furthermore, much of the material is treated from a very refined point of view. This can make parts of the book rather terse and it may seem unmotivated. In fact, this is not the case: there are many applications contained in the bibliography. For example, one of the main motivating forces behind the book is the study of discrete groups of complex hyperbolic isometries and fundamental polyhedra for such groups. Though isometries are mentioned throughout the book, there is relatively little material about discrete groups, and fundamental polyhedra only make it into the last section.

The field of complex hyperbolic geometry is wide open and is currently enjoying more interest than for many years. This book will certainly be of paramount importance in future progress. In the various preprint versions the book has become a standard reference and, now that a definitive version has been published, it is a necessary item for the library of everyone working in this field. It makes a rather challenging introduction to the subject but is an invaluable source of useful facts. I strongly recommend it to all those who work on related fields from differential geometry to several complex variables and from symplectic topology to discrete groups.

J. R. PARKER

DONKIN, S. *The q-Schur algebra* (London Mathematical Society Lecture Note Series vol. 253, Cambridge, 1998), x + 179 pp., 0 521 64558 1 (paperback), £24.95 (US$39.95).

The aim of this book is to present q-analogues of the results on the classical ($q = 1$) Schur algebra which appear in J. A. Green's seminal monograph *Polynomial representations of* GL_n [**2**]. The Schur algebras, symmetric groups and general linear groups which appear in Green's work are respectively replaced by q-Schur algebras, Hecke algebras of type A and the 'quantum GL_n' introduced by the author and R. Dipper [**1**].

The book started life as the sixth in the author's well-known series of papers 'On Schur algebras and related algebras', and evolved into its current form as more topics were added. In contrast to Green's treatment of the classical case, many of the main methods used here come from homological algebra and from the theory of quantum groups. There are various definitions of the term 'quantum group' in the literature, but here, the statement 'G is a quantum group over k' means that the author has in mind a Hopf algebra $k[G]$ which is dual to G in the sense that a morphism between two quantum groups $G_1 \to G_2$ is identified with a homomorphism of Hopf algebras $k[G_2] \to k[G_1]$. This approach allows mysterious objects such as 'quantum GL_n' to be studied by means of their dual objects.

The material is organized as follows. Chapter 0 is an introductory section which defines the main objects of study. Chapter 1 is devoted to the study of q-analogues of exterior algebra and of bideterminants. In Chapter 2, the q-analogue of the Schur functor is introduced; this is a functor from modules for the q-Schur algebra to modules for the Hecke algebra. This is a very useful tool which links the representation theory of Hecke algebras and q-Schur algebras, and is used in the same chapter to study the representation theory of the q-Schur algebra at $q = 0$. The latter develops the work of P. N. Norton on the 0-Hecke algebra [**3**], and a character formula is obtained for the irreducible modules. In Chapter 3, the author develops an infinitesimal theory for quantum GL_n for q a primitive l-th root of unity, analogous to the infinitesimal theory for reductive groups in prime characteristic. The main results include q-analogues of Steinberg's tensor product theorem and the theory of tilting modules for quantum GL_n (concentrating on the case $n = 2$).

Chapter 4 consists of a number of miscellaneous topics in the representation theory of the q-Schur algebra. These include the Ringel dual of the q-Schur algebra, truncation to Levi subgroups, components of tensor space, connections with the Hecke algebra, a description of the modules $\nabla(\lambda)$ in terms of bideterminants, Levi subalgebras of q-Schur algebras, quotients of Hecke algebras arising from saturated and cosaturated sets of dominant weights and, finally, global dimension of q-Schur algebras at roots of unity. The appendix gives an exposition of the theory of quasihereditary algebras and their tilting modules.

This is a well-written book which should be accessible to a graduate student with a background in homological algebra. It will be of particular interest to researchers working on the representation theory of the general linear group and quantum groups.

R. M. GREEN

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PIETSCH, A. AND WENZEL, J. *Orthonormal systems and Banach space geometry* (Encyclopedia of Mathematics and its Applications, vol. 70, Cambridge University Press, Cambridge, 1998), ix + 553 pp., 0 521 62462 2 (hardback), £55 (US$85).

The type of 'Banach space geometry' presented in this book is essentially that which can be described in terms of ideal norms. Indeed, 'Orthonormal systems and ideal norms' would have served well as an alternative title. An *ideal norm* is a norm α defined for a suitable class of linear operators which satisfies

$$\alpha(BTA) \leqslant \|B\|\, \alpha(T)\, \|A\|$$

for all bounded operators A, B, where $\|\ \|$ denotes the usual operator norm. Familiar examples of ideal norms are the p-summing norms and the type and cotype norms. If α is an ideal norm and I_X denotes the identity operator in a space X, then it is usually non-trivial to determine $\alpha(I_X)$ (or even to determine whether it is finite): this quantity can therefore be regarded as a parameter describing in some sense the geometry of the space X. The notion of ideal norms runs through the book from beginning to end, and almost every chapter contains the words in its title.

The most basic way in which an ideal norm is derived from orthonormal systems is as follows. Suppose that $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$ are given orthonormal systems in $L_2(M, \mu)$ and $L_2(N, \nu)$, respectively. Let $T : X \to Y$ be an operator between Banach spaces. The corresponding 'Riemann ideal norm' is the least constant C such that for any $x_1, \ldots, x_n \in X$, we have

$$\int_N \left\| \sum_{k=1}^{n} b_k(t)(Tx_k) \right\|^2 \mathrm{d}\nu(t) \leqslant C^2 \int_M \left\| \sum_{k=1}^{n} a_k(s) x_k \right\|^2 \mathrm{d}\mu(s).$$

If (a_k) is replaced by a trivial system, the quantity on the right-hand side becomes simply $\sum_{k=1}^{n} \|x_k\|^2$ and we are left with the 'type' norm for T corresponding to $(b_1, \ldots, b_n)$. A similar substitution on the left-hand side gives the 'cotype' norm corresponding to $(a_1, \ldots, a_n)$. The powers 2 could of course be replaced by other indices. The classical type and cotype norms are obtained by letting the orthonormal system be either the Rademacher or Gaussian system and allowing n to vary.

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