

MAGNETIC EFFECTS ON SOLAR p-MODES

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ABSTRACT. Both analytical and numerical studies of the effect of a magnetic field at the base of the convection zone on p-modes are presented. It is argued that the recently reported changes in the low degree p-mode frequencies, from 1980 to 1984, may result from corresponding changes in the magnetic field strength. A lower limit of some $5 \times 10^5 - 10^6$ gauss is implied for the field strength at the base of the convection zone.

1. INTRODUCTION

The rapid accumulation of ever more accurate data on the frequencies of p-mode oscillations raises the question of the influence of the solar magnetic field on those frequencies. At the photospheric level in the solar atmosphere the magnetic field is confined to isolated flux tubes. The field is concentrated to equipartition pressures with the confining external gas, yielding field strengths of around 1.5kG (compared with 2-3kG in sunspots). Above the photosphere these isolated intense flux tubes rapidly fan out to fill the chromosphere and corona, where they dominate the force balance (see Parker, 1979; Spruit and Roberts, 1983 for recent reviews). Below the photosphere the structure and, more specifically, the field strength is less certain, though there are arguments (Parker, 1975; Spiegel and Weiss, 1980; Galloway and Weiss, 1981; van Ballegooijen, 1982 a,b) for the existence of a relatively stable magnetic layer, evolving over the solar cycle, located just below the base of the convection zone.

Here we examine the role played by a field anchored at the base of the convection zone. For mathematical simplicity we consider a plane stratified atmosphere within which is embedded a horizontal magnetic field. In such a geometry we are able to present both an explicit dispersion relation (albeit a complex one) and a supporting, more approximate, WKB treatment, thus elucidating the role of the

field on the p-modes.

As an application of our analysis we consider the possibility that a magnetic field, evolving over the solar cycle, is responsible for the systematic frequency shifts in low degree p-modes recently reported by Woodard and Noyes (1985) and Elsworth *et al.* (1986). These reports refer to two separate data sets, one from ACRIM and the other from ground-based sites. Woodard and Noyes report a $0.42\mu\text{Hz}$ frequency decrease, over the period 1980 to 1984, in the $\ell=0$ and $\ell=1$ modes; by contrast, Elsworth *et al.* report an increase of $0.72\mu\text{Hz}$ in the $\ell=0$ mode over the same period, and no significant shift in the $\ell=1$ mode.

The analysis we present here (see also Roberts and Campbell, 1986) allows us to argue that the p-mode frequency would decrease (increase) if the field at the base of the convection zone decreased (increased) over the period 1980–1984, corresponding to moving from solar maximum towards minimum. In either case, a field strength of some $5 \times 10^5 - 10^6$ gauss is implied.

2. MAGNETIC EFFECTS

2.1. The Dispersion Relation

We model the solar interior as a field-free polytrope above an isothermal magnetic 'core', and consider linear adiabatic motions of frequency σ and horizontal wavenumber k . The dynamics of the field-free region ($0 \leq z \leq z_c$) is governed by Lamb's equation (Lamb, 1932; Sect. 312)).

In the magnetic region ($z > z_c$) the vertical velocity u_z , satisfies the equation (Adam, 1977; Nye and Thomas, 1976)

$$u_z'' + A_1(z) u_z' + A_0(z) u_z = 0, \quad (1)$$

where

$$A_1(z) = \frac{\gamma g \sigma^2}{(c_e^2 + v_A^2(z))(\sigma^2 - k^2 c_T^2(z))},$$

$$A_0(z) = \frac{(\sigma^2 - k^2 c_e^2)(\sigma^2 - k^2 v_A^2(z)) + (\gamma - 1)g^2 k^2}{(c_e^2 + v_A^2(z))(\sigma^2 - k^2 c_T^2(z))},$$

for constant sound speed c_e , Alfvén speed $v_A(z)$ and cusp speed $c_T(z) = c_e v_A / (c_e^2 + v_A^2)^{1/2}$.

The solution of equation (1) in the 'local approximation' is

$$\hat{u}_z(z) \sim e^{\lambda(z-z_c)}, \quad (2)$$

where λ , satisfying $\lambda^2 + A_1 \lambda + A_0 = 0$, is chosen so that the solution is vertically evanescent. This requires that $A_1^2 - 4A_0 > 0$.

Matching across the magnetic interface $z = z_c$ by imposing the continuity of u_z and the total pressure perturbation gives the dispersion relation

$$\left\{ (c_e^2 + v_A^2)(\sigma^2 - k^2 c_T^2)\lambda + gk^2 c_e^2 \right\} \left[gk^2 c_e^2 + \sigma^2 k c_e^2 - \gamma \sigma^2 g - \sigma^2 c_e^2 \frac{M'(-a, m+2, 2kz_c)}{M(-a, m+2, 2kz_c)} \right] = \frac{\rho_0(z_c^-)}{\rho_0(z_c^+)} c_e^2 (\sigma^2 - k^2 c_e^2) (\sigma^4 - k^2 g^2). \tag{3}$$

Here m is the polytropic index, M is Kummer's function of the first kind, and $\rho_0(z)$ the gas density. Equation (3) is solved numerically for various values of the magnetic field strength B_0 , and exhibits the usual $\sigma^2 - k$ parabolae, changed only slightly by the presence of the field. Figure 1 shows the behaviour of $\Delta\nu$, the difference in cyclic frequency $\nu (\equiv \sigma/2\pi)$ in the presence of a field B_0 and in its absence. We note that a field of order 10^6 G is required for $\Delta\nu$ to be of order $0.5\mu\text{Hz}$.

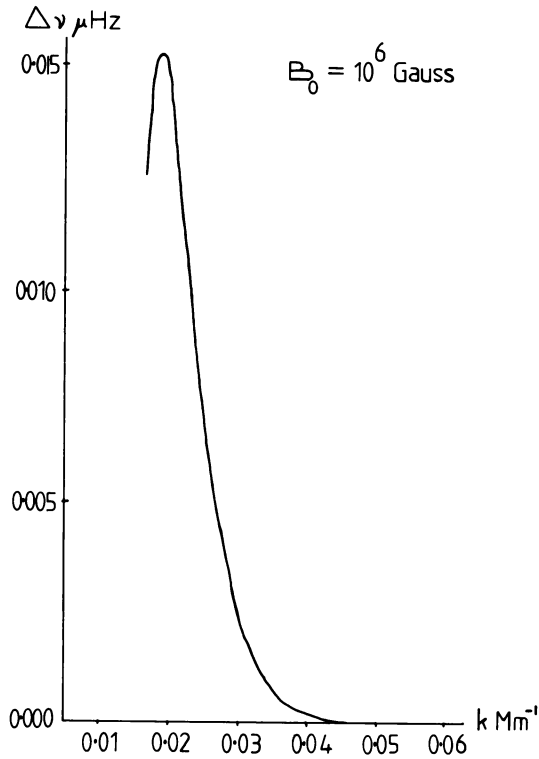


Figure 1. The increase $\Delta\nu$ in cyclic frequency $\nu \equiv \sigma/2\pi$ of the p-mode brought about by a field $B_0 = 10^6$ G.

2.2 WKB Analysis

Complementary to the analysis of a dispersion relation, it is of interest to consider a WKB analysis. To do this it is convenient to derive the magnetic form of Lamb's equation and express it in canonical form. Of particular interest is the high frequency ($\sigma \gg kv_A$) case, which yields

$$\phi'' + \kappa^2 \phi = 0, \quad (4)$$

where

$$\kappa^2 = -k^2 + \frac{\sigma^2}{c_B^2 + v_A^2} + \frac{k^2 c_B^2 v_A^2}{(c_B^2 + v_A^2)^2} - \frac{\omega_C^2}{c_B^2} + \frac{k^2 \omega_G^2}{\sigma^2} \left(\frac{c_B^2}{c_B^2 + v_A^2} \right). \quad (5)$$

In the above, ω_C and ω_G are the usual Lamb and buoyancy frequencies (see, e.g., Deubner and Gough, 1984) and $\phi \sim \rho_0^k (c_B^2 + v_A^2) \Delta$, for sound speed $c_B(z)$ and Alfvén speed $v_A(z)$. Expression (5) is exact if k, v_A or g are zero.

Applying the usual WKB arguments to (4) and taking a mean value β for c_B^2/v_A^2 we obtain (see also Roberts and Campbell, 1986).

$$\nu_B - \nu_0 = \frac{1}{2\beta} \nu_0, \quad (6)$$

where ν_B is the frequency in the presence of a magnetic field B and ν_0 its value in the field's absence. Hence, the presence of a field increases the frequency above its zero field value in proportion to the square of the field strength (see also Bogdan and Zweibel, 1985).

Suppose, now, that the field changes from B to B' , with a corresponding frequency change of $\nu_B - \nu_{B'}$. Then (6) yields

$$B^2 - B'^2 = \frac{8\pi c_B^2}{\nu_0} (\nu_B - \nu_{B'}). \quad (7)$$

As an illustration, taking $\rho_0 = 0.2 \text{ g cm}^{-3}$ and $c_B = 2 \times 10^7 \text{ cm s}^{-1}$ as typical of conditions near the base of the convection zone we obtain $B^2 - B'^2 = 6.5 \times 10^{17} (\nu_B - \nu_{B'}) \text{ gauss}^2$. Thus, to reproduce frequency changes of the magnitude reported we require a peak field of at least $5 \times 10^5 \text{ G}$, a result consistent with the analysis in Section 2.1.

3. DISCUSSION

The analysis of the previous sections makes clear that a peak field at least $5 \times 10^5 \text{ G}$ must reside at the base of the convection zone if it is responsible for the recently reported (Woodard and Noyes, 1985; Elsworth *et al.*, 1986) phase shifts in p -mode frequencies. The actual change in field strength, over the solar cycle, necessary to reproduce phase shifts of the order reported depends upon the magnitude of the peak field. For example, a peak field of $7 \times 10^5 \text{ G}$

declining to 5×10^5 G, from 1980 to 1984, reproduces the frequency decrease reported by Woodard and Noyes, while a peak field of 10^6 G would need to decline to 8.6×10^5 G. If the magnetic field below the base of the convection zone is in fibril form (e.g. van Ballegoijen, 1982 a,b) then somewhat stronger fields are necessary; fields of at least 10^6 G are required.

Altogether, then, we have pointed out that variations of magnetic field strength at the base of the convection zone produce frequency shifts in p-modes. Measurement of such shifts in frequency offer the possibility of determining magnetic field strengths deep below the solar surface. The frequency shifts between 1980 and 1984 reported by Woodard and Noyes (1985) and Elsworth *et al.* (1986) offer an intriguing beginning, though clearly a more extensive data set taken over a larger fraction of the solar cycle is necessary before any firm conclusions can be drawn.

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