

# Magnetic helicity and solar activity cycle: observations and dynamo theory

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**Abstract.** We study a simple model for the solar dynamo in the framework of the Parker dynamo, with a nonlinear dynamo saturation mechanism based on magnetic helicity conservation arguments. We find a parameter range in which the model demonstrates a cyclic behaviour with properties similar to that of Parker dynamo with the simplest form of algebraic  $\alpha$ -quenching. We compare the nonlinear current helicity evolution in this model with data for the current helicity evolution obtained during 10 years of observations at the Huairou Solar Station of China. We conclude that, in spite of the very preliminary state of the observations and the crude nature of the model, the idea of using observational data to constrain our ideas concerning magnetic field generation in the framework of the solar dynamo appears promising.

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## 1. Introduction

The solar activity cycle is widely believed to be connected with dynamo action which occurs somewhere inside the solar convective zone or even in the overshoot layer. If the dynamo action is strong enough, a dynamo wave propagating somewhere inside the convective shell is excited. It is necessary to include some saturation mechanism to get a (quasi)stationary wave which can be compared with the observed activity cycle, rather than a dynamo wave with an exponentially growing amplitude. In principle, the phenomenology of the solar cycle can be reproduced using a very primitive  $\alpha$ -quenching model of dynamo saturation, with the energy of the dynamo generated magnetic field achieving approximate equipartition with the kinetic energy of the random motions. A deeper treatment of solar dynamo saturation requires however some ideas concerning the physical processes that give rise to quenching of the generation mechanism. A scenario of dynamo saturation which is now widely discussed is connected with the concept of magnetic helicity. The point is that the weakest link in the dynamo self-excitation chain, i.e.  $\alpha$ , is a pseudoscalar quantity and cannot be directly connected with the magnetic energy, which is a scalar (not pseudoscalar) quantity. A magnetic helicity  $\chi^m$  can however be introduced to describe the level of magnetic field mirror-asymmetry and this quantity can be associated with the magnetic part of  $\alpha$ , i.e.  $\alpha^m$ , which is thought to be responsible for  $\alpha$ -quenching.

The magnetic helicity  $\chi^m$  is an integral of motion for the ideal MHD equation, similar to the hydrodynamic helicity  $\chi^v$  which is conserved in the hydrodynamical case. During the

solar activity cycle, magnetic helicity is redistributed between the large and small scale magnetic field. Based on this concept, a governing equation for  $\alpha^m$  has been suggested (Kleeorin *et al.* 2003 and references therein).

For a long time, it was impossible to observe either the magnetic helicity or the hydrodynamic helicity and these values, crucial for dynamo theory, were taken from theoretical estimates only. In last decade, some basic progress has been made here and the first observations of magnetic helicity in active regions on the solar surface have been obtained (Pevtsov *et al.* 1994; Zhang & Bao 1998). The obvious aim now is to confront predictions of dynamo theory concerning the latitudinal distribution of magnetic helicity and its evolution during a solar cycle with the corresponding observational data.

## 2. Magnetic and current helicity data obtained at the Huairou Solar Observing Station

The averaged value of magnetic helicity, i.e.  $\langle \vec{a} \cdot \vec{b} \rangle$  ( $\mathbf{a}$  and  $\mathbf{b}$  are the small-scale vector-potential and magnetic field respectively), evidently would be a convenient quantity to confront with a dynamo saturation scenario based on a magnetic helicity conservation argument. In practice however, it is the current helicity  $\langle \vec{b} \cdot (\nabla \times \vec{b}) \rangle$  which can be extracted from the observations. The observations are restricted to active regions on the solar surface and we obtain information concerning the surface magnetic field and helicity only. Monitoring of solar active regions while they are passing near to the central meridian of the solar disc enables observers to determine the full magnetic field vector. The observed magnetic field is subjected to further analysis to obtain the value  $\nabla \times \vec{b}$ . Because it is calculated from the surface magnetic field distribution, the only electric current component that can be calculated is  $(\nabla \times \vec{b})_z$  where the direction  $z$  is perpendicular to the solar surface. As a consequence of these restrictions, the observable quantity is  $H_c = \langle b_z (\nabla \times \vec{b})_z \rangle$ .

The observational data used in our analysis were obtained at the Huairou Solar Observing station of the National Astronomical Observatories of China. The magnetographic instrument based on the FeI 5324 Å spectral line determines the magnetic field values at the photospheric level. The data are obtained from a CCD camera with  $512 \times 512$  pixels over the whole magnetogram, whose entire size is comparable with the size of an active region, as well as with the depth of the solar convective zone (i.e. about  $2 \times 10^8$  m). An observational programme to reveal the values of the twist and the current helicity density over the solar surface requires a systematic approach, both to the monitoring of magnetic fields in active regions and to the data reduction, in order to reduce the impact of noise. While this work is still in progress, the largest systematic dataset of active regions presently available consists of 422 active regions over the 10 years 1988-1996 (Zhang & Bao 1998). The averaged quantities of current helicity density are positive/negative over Southern/Northern solar hemispheres respectively, and thus obey the so-called hemispheric rule.

## 3. The dynamo model

Following Parker we consider dynamo action in a thin convective shell

$$\frac{\partial B}{\partial t} = gD \sin \theta \frac{\partial A}{\partial \theta} + \frac{\partial^2 B}{\partial \theta^2} - \mu^2 B, \quad (3.1)$$

$$\frac{\partial A}{\partial t} = \alpha B + \frac{\partial^2 A}{\partial \theta^2} - \mu^2 A, \quad (3.2)$$

where  $A$  and  $B$  represent mean poloidal and toroidal magnetic fields correspondingly. We measure lengths in units of the solar radius and time in units of a diffusion time based on the solar radius and turbulent magnetic diffusivity. The terms  $-\mu^2 B$  and  $-\mu^2 A$  represent the role of turbulent diffusive losses in the radial direction – the value  $\mu = 3$  corresponds to a convective zone with a thickness of about 1/3 of the solar radius.  $g = \partial\Omega/\partial r$  is the radial shear of differential rotation. We use the simplest profiles of dynamo generators compatible with symmetry requirements, i.e.  $\alpha(\theta) = \cos\theta$  and  $g = 1$ . The points  $\theta = 0$  and  $\theta = 180^\circ$  correspond to North and South poles respectively. We take here zero boundary conditions for  $A$  and  $B$ . We are interested in dynamo waves propagating from middle solar latitudes towards the equator. This corresponds to negative dynamo  $D$  numbers provided  $\alpha$  is chosen to be negative in the Northern hemisphere and  $g$  is positive near to the solar equator.

A key idea of the dynamo saturation scenario exploited below is a splitting of the total  $\alpha$  effect into the hydrodynamic ( $\alpha^v$ ) and magnetic ( $\alpha^m$ ) parts. Two types of effect should be taken into account. First of all, the link between  $\alpha$ -effect and the relevant helicities can be modified by the dynamo-generated magnetic field. Correspondingly, we introduce quenching functions  $\phi_v$  and  $\phi_m$ . The second point to be addressed is that magnetic helicity is not a very convenient quantity because it involves a gauge-noninvariant quantity, i.e. the vector potential. We connect magnetic helicity with the current helicity  $\langle \vec{b} \cdot (\vec{\nabla} \times \vec{b}) \rangle$ . Then we need to obtain a quantity of suitable dimension, and introduce the density  $\rho$  to obtain the correctly dimensioned  $\chi^c \equiv (\tau/12\pi\rho) \langle \vec{b} \cdot (\vec{\nabla} \times \vec{b}) \rangle$  ( $\tau$  is the correlation time). Thus,  $\alpha(r, \theta) = \chi^v \phi_v + \chi^c \phi_m$ . The quenching functions  $\phi_v$  and  $\phi_m$  can be found by Kleorin *et al.* (2003).

The function  $\chi^c(\vec{B})$  is determined by a dynamical equation which follows from the conservation law for magnetic helicity (Kleorin & Rogachevskii 1999). A general dimensional form of this equation reads

$$\frac{\partial \chi^c}{\partial t} + \frac{\chi^c}{T} = -\frac{1}{9\pi\eta\rho_*} (\vec{\mathcal{E}} \cdot \vec{B} + \vec{\nabla} \cdot \vec{\Phi}) + \kappa \Delta \chi^c. \quad (3.3)$$

Here  $\vec{\Phi} = C\chi^v\phi_v\vec{B}^2 l^2 \vec{e}_r / \Lambda_\rho$  is a nonadvective flux of the magnetic helicity (here  $\vec{e}_r$  is the unit vector in radial direction,  $l$  is correlation length,  $\Lambda_\rho$  is the density stratification length),  $-\kappa\vec{\nabla}\chi^c$  is the diffusive flux of the magnetic helicity, and  $T = l^2/\eta_0$  is the relaxation time of magnetic helicity,  $\eta$  is the turbulent magnetic diffusivity,  $\vec{\mathcal{E}}$  is the mean electromotive force. The physical meaning of Eq. (3.3) is that the total magnetic helicity is a conserved quantity and if the large-scale magnetic helicity grows with magnetic field, the evolution of the small-scale helicity should somehow compensate this growth. Compensation mechanisms include dissipation and various kinds of transport.

## 4. Results and Discussion

We simulated numerically the model of the nonlinear solar dynamo based on the Parker approximation and conservation of magnetic helicity arguments, as presented in previous sections. We found that the model gives a stable nonlinear wave-type solution similar to the solar cycle phenomenology from the generation threshold of the nonlinear system. It is quite interesting that a slightly stronger generation is needed to get stable nonlinear oscillations with a nonvanishing amplitude than to excite the linear dynamo. The temporal behaviour of nonlinear dynamo waves is quite similar to that with the simple algebraic  $\alpha$ -quenching.

At the present state of observations we can compare latitudinal distributions of  $\chi^c$  averaged over the activity cycle or the temporal behaviour of  $\chi^c$  averaged over a hemisphere. Two types of behaviour are demonstrated. Provided that  $C$  is negative (magnetic helicity inflow) or small and positive ( $C < 0.1$ , moderate magnetic helicity outflow) a cycle is obtained during which the value of  $\chi^c$  is always negative in the Northern hemisphere, in accordance with naive theoretical expectations as well as the available observations. If  $C$  is large and positive (strong magnetic helicity outflow), we obtain a cycle during which  $\chi^c$  changes sign. The available observational data does not recognize changes of the sign of current helicity either latitudinally or temporally.

The results of this comparison look quite promising in spite of the quite limited extent of the observational data, as well as the crude nature of the model. We have been able to choose a set of governing parameters which give helicity properties comparable with the available phenomenology. From another viewpoint, the observational data are rich enough to indicate a disagreement between the available observations and the predictions of the model with other parameter sets.

The parameters which gives an agreement between simulations and observations are quite plausible. However we feel that it is too early to insist that this agreement is more than a coincidence. It must be emphasized that we base our comparison on the 10 year observational data of one scientific team. We stress that an extension of the observational programme to cover several cycles as well the inclusion of data obtained by other scientific teams and from other tracers would be very important. In particular it would be valuable to include the cross-helicity data into the analysis.

We conclude with the expression of a guarded but real optimism that magnetic helicity observations can result in the foreseeable future in a new level of understanding in dynamo theory, which will base the accepted  $\alpha$ -profiles not only on order-of-magnitude arguments and numerical simulations, but also on the observational data.

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