

Preface

We are pleased to introduce this special issue on “*Probability and stochastic modeling in actuarial science and related fields*”, which features a collection of 12 research papers that cover a wide range of topics such as insurance risk processes, control problems, optimal reinsurance, mutual aid, predictive analytics and mortality modeling. We received a large number of submissions for this special issue, and we are grateful to the authors for their efforts and contributions. Each submission went through a rigorous peer-review process, ensuring that the articles selected for publication met the high standards of *Probability in the Engineering and Informational Sciences*.

The first two articles propose different modifications of the classical compound Poisson insurance risk process. In “*Gerber-Shiu analysis in the compound Poisson model with constant inter-observation times*”, Xie, Yu, Zhang and Cui utilize a Laguerre series approach to find the finite-time and infinite-time Gerber-Shiu expected discounted penalty functions when the surplus process is only monitored periodically at constant intervals. This study complements the case of Erlang or exponential intervals in other research works in the literature that are concerned with periodically observed risk processes. The article “*Applications of the classical compound Poisson model with claim sizes following a compound distribution*” by Gao and Sendova considers the Gerber-Shiu function by assuming that the claim distribution itself can be a compound sum. This paper has an equivalent interpretation that the claim counting process follows a compound Poisson distribution.

To allow for small fluctuations in the day-to-day operation of the insurance business, a Brownian motion component can be added to the compound Poisson model. In this context, Cheung and Liu propose a generalization of the Gerber-Shiu function that can be used to study the “*Joint moments of discounted claims and discounted perturbation until ruin in the compound Poisson risk model with diffusion*”. It leads to the analysis of new covariance measures that also include the ruin time. Under the same model, Yang, Xie and Zhang further incorporate a dividend barrier strategy in “*Nonparametric estimation of some dividend problems in the perturbed compound Poisson model*”. They adopt the Fourier-cosine method to approximate not only the Gerber-Shiu function but also the expected discounted dividend until ruin, and they also provide nonparametric estimators for these quantities.

Continuing on dividend problems, the article “*On the dual risk model with Parisian implementation delays under a mixed dividend strategy*” by Hu, Li and Zhou looks into the dual compound Poisson risk model that is suitable for a business that pays constant expense rate over time and faces gains represented by upward jumps. They propose a mixed dividend strategy that has the flavor of both a threshold strategy and a strategy with Parisian implementation delays, and they analyze both the expected discounted dividend and the Gerber-Shiu function.

Control problems involving dividend, reinsurance and/or investment decisions have also been important research topics in actuarial science. In “*Optimal singular dividend control with capital injection and affine penalty payment at ruin*”, the author Xu considers the compound Poisson insurance risk model and allows for singular dividend payments. Utilizing the Hamilton-Jacob-Bellman Quasi-Variational Inequality, it is shown that the optimal dividend and capital injection strategy is band-type. On the other hand, Zhang, Chen, Jin and Li propose “*A class of non-zero-sum stochastic differential games between two mean-variance insurers under stochastic volatility*” and study open-loop equilibrium strategies using forward backward stochastic differential equations.

Optimal reinsurance contract theory and risk sharing have been fundamental topics in actuarial science for decades. In “*Pareto-optimal reinsurance with default risk and solvency regulation*”, Boonen and Jiang study optimal reinsurance in a risk-sharing setting in which the reinsurer can default. Limited liability of the reinsurer and regulation are shown to have an impact on the reinsurance pricing, as

well as on the investment decision of the reinsurer. Layer insurance is a popular reinsurance indemnity, and Wang, Wang, Zhang and Zhao study in “*Optimal allocation of policy limits in layer reinsurance treaties*” the optimal allocation of corresponding policy limits. This article focuses on the impact of dependence and heterogeneity of random losses, and allows for randomization of the policy limits. Risk sharing also plays a crucial role in the design of mutual aid networks as it is related to peer-to-peer networks. The attractiveness of mutual aid schemes is shown in “*Optimal design for network mutual aid*” by Li, Fang, Nie and Chen. They design a fair mutual aid scheme in an expected utility setting that allows for heterogeneous risks and the presence of classical insurance products.

The last two papers in the special issue feature predictive modeling and data analytics with actuarial applications. The first paper, “*On hybrid tree-based methods for short-term insurance claims*”, by Quan, Wang, Gan, and Valdez tackles the challenge of modeling lost costs of nonlife insurance claims due to data imbalance. The authors introduced a tree-based method with a hybrid structure and demonstrated its advantage in lost cost predictions over traditional actuarial and machine learning methods. The second paper, “*Rotation in age patterns of mortality decline: statistical evidence and modeling*” by Li and Kim examines mortality rotation, where mortality decline accelerates at older ages but decelerates at younger ages. The paper proposes a novel approach for modeling and detecting rotation, leading to plausible mortality forecasts.

We hope this special issue will inspire and stimulate further research and discussion on interplay between actuarial science and related fields, and readers will find this issue informative and thought-provoking. Finally, we would like to sincerely thank the authors, reviewers, and admin team for their hard work and dedication in bringing this special issue to fruition.

Guest Editors of the Special Issue on Actuarial Science

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