A Giant Step: from Milli- to Micro-arcsecond Astrometry
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Dynamics of particles in slowly rotating black holes with dipolar halos

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Abstract. In general, the model of galaxy assumes a central huge black hole surrounded by a massive halo, disk or ring. In this paper, we investigate the gravitational field structure of a slowly rotating black hole with a dipolar halo, and the dynamics and chaos of test particles moving in it. Using Poincaré sections and fast Lyapunov indicator (FLI) in general relativity, we investigate chaos under different dynamical parameters, and find that the FLI is suitable for detecting chaos and even resonant orbits.

Keywords. Galaxy: kinematics and dynamics, black hole physics, galaxies: halos

1. Introduction

The main motive of our research on the dynamics characteristics of test particles in the relativistic core-shell models is their realistic significance in astrophysics, such as the study of motion of stars in a galaxy with a model of a central bulge surrounded by a halo (Binney and Tremain, 1987), and other astrophysics objects involving active galactic nuclei, black holes, or neutron stars with axisymmetric surrounding sources (for examples accretion disks, massive halos, shells and rings).

In this paper, following Letelier and Vieira (1997), we briefly discuss the spacetime structure of superposition of a slowly rotating Kerr black hole with a dipole along its rotating axis. Furthermore, the dynamics and chaos of test particles in the Weyl field is also studied.

2. Gravity field structure and the stability of test particles

The metric representing the superposition of a Kerr black hole and a dipole along the rotation axis is a stationary axially symmetric spacetime. We only consider slowly rotating black holes. The metric can be simplified by keeping the first order terms in the rotation parameter a (Letelier and Vieira, 1997). The particles moving in it can be written as a four dimensional dynamical system. There are three integration constants: the energy E, the z angular momentum L and the 4-velocity conservation. The last constant is used to check numerical errors, with a precision reaching 10^{-13}.

First we emit five particles along the radial direction with different polar angles. We study the motion of these particles at three different dipole strengths D. Fig. 1 shows the case with D=0.005. The gravity field does not possess reflection symmetry with respect to the black hole's equatorial plane. It can be viewed as if another gravity source is possibly located upon the black hole.

We find that with a larger D, the orbits of particles are prone to instabilities (Fig. 2). For every energy and angular momentum, there exists a critical value of D, beyond which we cannot find a stable area.

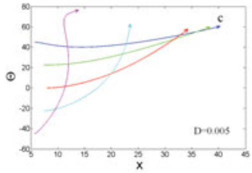

Figure 1. $E = 0.965$, $L = 3.75$ and five particles have different initial inclination: $45°$, $22.5°$, $0°$, $-22.5°$, $-45°$, respectively. The superposition field is not symmetrical about the equator.

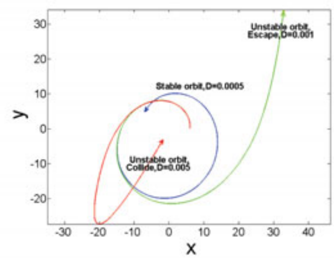

Figure 2. There different orbits of particle with same initial values at varying halo parameter.

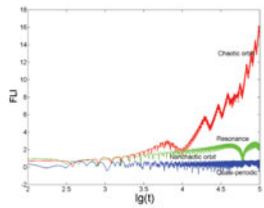

Figure 3. FLIs of three orbits. FLI not only can distinguish chaos, but also resonant orbit.

3. The chaos in varying parameters

Obviously, Poincaré sections are suitable for qualitatively characterizing dynamical systems. With a fixed angular momentum larger energy means emergence of chaos. But if the energy is constant, a bigger angular momentum can enlarge the regular degree of the dynamical system (corresponding Poincaré sections being omitted here).

In Fig. 3, we use FLI (Wu *et al.*, 2006) to study three different aspects: hyperbolic point, resonant trajectory, and quasi-periodic orbits. We find that the first one is chaotic as compared to the last two that are non-chaotic. The FLI also can distinguish between resonant and quasi-periodic orbits.

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