Correspondence

DEAR EDITOR,

Re: Dicing decimal digits (*Math. Gaz.* July 1997)

The Blest method of producing decimal digits by throwing two dice can be improved by throwing a die and a coin together. We add 4 to the die score if the coin is heads, and -1 if the coin is tails. As before, the die is thrown again if it comes up as 6.

This removes the problems of distinguishing the two dice or confusing the two rules.

Yours sincerely,

JEREMY KING

Tonbridge School, Tonbridge, Kent TN9 1JP

DEAR EDITOR,

I offer a couple of quick thoughts triggered by the articles in July 1997's excellent issue of the *Gazette*.

1. There is a surprising connection between Keith Lloyd's article on the Pell equation $x^2 - 3y^2 = 1$ and H. W. Bitton's quest for nice cubics in Note 81.25. Let u, v be integers satisfying $u^2 - 3v^2 = 1$. Then the cubic $f(x) = (u - 2v)x^3 + 2x^2 + (u + 2v)x$ is readily shown to have the nice rational roots $\frac{v-1}{u-2v}$, 0, $\frac{v+1}{2v-u}$ and its derivative has the nice rational roots $\frac{u-2}{3(u-2v)}$, $\frac{u+2}{3(2v-u)}$.

2. I heartily agree with J. A. Scott (note 81.33) that convexity arguments deserve all the publicity they can get: as Walter Rudin once observed, "Many of the most common inequalities in analysis have their origin in the notion of convexity". Just as the AM–GM inequality has a neat proof using convexity, so the Cauchy-Schwarz inequality has a similarly succinct one. Let $X = \sum x_i^2$, $Y = \sum y_i^2$ where (without loss of generality) none of the x_i are 0. Then

$$\left(\sum x_i y_i\right)^2 = X^2\left(\sum \frac{x_i^2}{X} \cdot \frac{y_i}{x_i}\right)^2$$

$$\leqslant X^2\left(\sum \frac{x_i^2}{X}\left(\frac{y_i}{x_i}\right)^2\right)$$

by convexity if $f(x) = x^2$ with weights $\frac{x_i^2}{X}$.

$$= XY$$

Moreover, there is equality if all the points involved, $\frac{y_i}{x_i}$, are equal.

I also enjoyed the article [1] in the November 1997 *Gazette* and offer two brief observations: