



Aerodynamic Aspects of Helicopter Design

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PH D , B SC , D I C , A F R A e S ,
A M I MECH E

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DR G S HISLOP (*Chairman of the Executive
Council*)

in the Chair

INTRODUCTION BY THE CHAIRMAN

The CHAIRMAN, in introducing the Author, said that Dr ROBERTS was very well known by everybody in the helicopter field in this country and abroad. He had taken his B Sc Engineering Degree at Queen Mary College and his Ph D at Imperial College. He had served with General Aircraft Ltd, and had also been a lecturer at Imperial College. Subsequent to this appointment he had been with Saunders-Roe as Chief Project Engineer (Helicopters), and was now on the staff of Fairey Aviation where initially he had been Chief Aerodynamicist and was now Chief Project Engineer. Dr Roberts was a Founder Member and a member of the Council of the Association.

DR H ROBERTS

INTRODUCTION

The scope of a lecture of this kind tends to be somewhat on the large side, and indeed in time we shall see at least one book dedicated to this very important design feature. In the past however, although papers have been produced bearing somewhat similar titles (Ref 1-3) the accent has been on performance rather than on aerodynamic design in general.

The reasons for this pre-occupation with performance are not very difficult to discover. The difficulties which beset the early pioneers and designers in persuading their helicopters to leave the ground were primarily due to the arduous task of designing to a practical power-weight ratio (we now realise the importance of growth factors but the significance of this concept has taken a long time to get home) and to the current lack of appreciation of the need, or the cost, of torque counter-action. No great knowledge of helicopter stability would have helped in the early days. When the stage was reached that power-weight ratios became marginal and refined design was needed to convert "marginal" to "adequate," then the aerodynamicist found his place in the helicopter field—but only to help in the achievement of higher lifting efficiencies.

This stage ended only a few years ago and today the tasks of the aerodynamicist cover a much wider field—performance, stability and control, handling, flight analysis, wind tunnel testing are all subjects of intimate concern to the practicing aerodynamicist. A review of some of the remaining outstanding problems has been given recently by Hislop (Ref 4). Unfortunately established methods do not exist for determining the complete behaviour of a helicopter under the conditions which are the most interesting and so a large part of the tasks of an aerodynamicist is concerned with establishing techniques and it will be some time before the cut and dried methods which one associates, say, with low speed aeroplanes, are with us.

In this paper an attempt is made to present some of the more important aspects of helicopter aerodynamics as seen through the eyes of an industrial aerodynamicist rather than those of a research worker in an establishment devoted to theoretical study. The difference is an important one for the former is concerned to a far greater extent with numbers. He is not satisfied to call the drag coefficient δ he needs to know the magnitude of δ . He is not satisfied to state that rivets on a blade increase the drag. He needs to know by how much. Each number he uses must be justified, certainly to himself—and more often than not to others.

An anonymous letter in one of our weekly aeronautical journals about a year ago, made some very disparaging remarks about the efforts being made in industry to cover, even in a limited way, the calculations required to ensure that, from the aerodynamic view point, the helicopters of today are safe flying vehicles. As I believe that no-one has ever put on record the coverage of a typical helicopter aerodynamics office I make no apology for starting off by listing some of the work undertaken by one that I know pretty well. Quite apart from establishing a true perspective of the task, it provides a useful starting point for discussing some of the specific details involved.

(1) OUTLINE OF THE WORK OF THE INDUSTRIAL AERODYNAMICIST

There are three broad headings into which the work falls

- (a) Prediction and analysis
- (b) Wind tunnel and spinning rig testing
- (c) Flight testing and analysis

I have omitted project design since the methods of prediction used are directly applicable.

Item (a) consists primarily of

- (a1) Performance analysis
- (a2) Stability (including static, dynamic longitudinal, dynamic lateral, blade stability or “weaving”)
- (a3) Control (including control forces and moments, response to control application and determination of the flight envelope)
- (a4) Provision of stressing data (including gust loads, loads on fixed surfaces, etc.)
- (a5) Miscellaneous calculations and fundamental investigations

Item (b) is much more difficult to pin down because wind tunnel techniques are still in their infancy while spinning rig tests are used far too much for

structural investigations at the expense of aerodynamic needs. The measurements taken are basically—

- (b1) Six component measurements without rotor in the tunnel
- (b2) Repeat tests with the rotor
- (b3) Deceleration or constant speed tests on the rig

Item (c) is only partially within the scope of an Aerodynamics Department, in so far as the actual testing and recording are rather too specialised and would normally be carried out by a separate group. The analysis of performance flight tests is so far the only work in this sphere which can be regarded as being a common feature of the work in a normal Aerodynamics Office. This deficiency is likely to be made good in the near future.

(2) PERFORMANCE ANALYSIS AND PREDICTION

In spite of the enormous amount written about performance, I want to spend a little time giving some comments on this aspect.

There is a plethora of methods for estimating the performance characteristics of a helicopter. The most popular are the well known ones of Wald, Lichten, Castles and Squire as well as the various methods based on the energy equation. Having used at some time or other all of these, the accuracy achieved by refined methods leaves me somewhat unimpressed in view of the vagueness of the initial design parameters. The energy method by virtue of the simple physical concepts involved, is probably in widest use.

The most rudimentary form of the energy equation is —

$$P = P_i + P_o + P_p + P_c \quad (1)$$

where P is the total power available, allowing for gear losses, torque counteraction where applicable, etc

P_i is the induced power

P_o is the power due to blade friction, and includes the power absorbed by the H force as well as the rotational power

P_p is the power due to fuselage drag, wing drag, etc

P_c is the power due to climb

Each of these terms although representing a familiar quantity also represents an interesting problem when it comes to assigning numbers rather than symbols. For example, the value of P_i must depend on the accuracy of the current knowledge of the distribution of the thrust and velocity across the effective lifting disc as well as the degree of off loading onto fixed or secondary rotating surfaces.

Similarly the value of P_o depends on the blade profile, the surface finish, radial flow effects, discrete excrescences such as mass balance, etc. Equally the value of P_p is very difficult to assign in view of the somewhat aerodynamically horrible shapes now being extensively used.

At normal operating speeds the two major influences on the performance of the helicopter are P_o and P_p . At low speeds the induced power is the most important. It follows that to get best operating efficiency—and it is this which helps to sell the helicopter—the rotor blades, and most particularly the blade sections, should be efficient. This means that the section, apart from having small centre of pressure travel for purely structural reasons, must be thin to give low drag coefficients and high lift/drag ratios, must have

high critical Mach Numbers to allow high rotational and forward speeds to be used without adverse compressibility effects, must have as few excrescences as possible, must have properly designed tip units (if used) and the blade should preferably be both tapered and twisted. The final requirement listed involves production difficulties which rule it out in many cases. The other points will be mentioned later. As far as fuselage design is concerned, the shapes in current use are dictated in the main by specialised use envisaged by the operator, both civil and military. What is quite certain is that it is time the designers took a firm stand against the requirements that lead to the monstrosities we see to-day—monstrosities that lead to bad efficiency, bad stability, bad control, bad helicopters.

The value of P_i

Many workers in the field of helicopter aerodynamics have made attempts to investigate the induced velocity distribution due to a rotor (see Ref. 10). The work of Mangler and others has led to a closer understanding of the velocity distribution while many others (*e.g.*, Stewart, Lock, Hafner) have tried to present the mean induced velocity in the form of characteristic curves. Considering first the hovering condition, there is no inherent difficulty in analysing the flow theoretically, apart from assessing the magnitude of the tip loss factor B . The induced velocity pattern is substantially parabolic radially for $r/R < B$. In the absence of blade twist there is about 6% power loss due to this cause. In spite of the large number of formulae for estimating B (Ref. 5-8) there would appear to be little merit in any particular one since the spread of answers is between 0.96 and 0.99. The figure 0.96 is my own choice and this corresponds to a further loss of 4% power. A good rotor should show no more than 5% loss in power over the 10% due to the above sources (including slipstream rotation) but a total loss of 17½% is a good overall figure for a constant chord blade without twist. (The savings due to blade twist are given by Squire and by Gessow and Myers, Ref. 9)

The simplest case then of a hovering rotor yields the formula

$$P_i = 1.175 T v_i \tag{2}$$

where T is the rotor thrust and v_i is the uniform induced velocity over a disc of radius R for a thrust T .

The thrust is not, of course, equal to the weight. In fact the vertical drag of the fuselage is a serious problem particularly on winged helicopters. A rough estimate for the Sikorsky S 56 indicates a vertical drag of about 1,000 lb. and for the Rotodyne a figure of the same order. It is misleading to compute these figures on the assumption of uniform induced velocity across the disc (since most fuselages are in fact near the disc centre where the induced velocity is small) or to ignore the rapidly developing slipstream below the disc.

A formula which has come in useful for this purpose is —

$$T = W + K_1 D v_i^2 \tag{3}$$

where D_v = vertical drag on the aircraft in the absence of the rotor at a rate of climb $v_{T O}$

$$v_{T O} = [T/2\pi\rho B^2R^2]^{\frac{1}{2}} \quad (4)$$

$$f = 1 + h/[h^2 + B^2R^2]^{\frac{1}{2}} \quad (5)$$

h = height of the rotor above the fuselage

K_1 = constant

By assuming a parabolic distribution of velocity along the disc radius it can be shown that K_1 is in general roughly equal to the fraction of the radius in which the plan area is concentrated (usually about $\frac{1}{2}$)

In the absence of wind tunnel tests D_v can be estimated by taking a flat plate coefficient of about 0.6 on the plan area of the fuselage and about 1.0 on wings and tailplane (See also Ref. 14)

The case of vertical ascent or descent is theoretically more difficult to analyse. From the practical point of view the simple analysis by Oliver (Ref. 11) gives a good working method. However, the value for the ratio of induced power in hovering to the theoretical minimum is given as 1.2 and as stated earlier, I believe this value to be pessimistic, so that a slight modification is necessary to Oliver's curves before full use can be made of them. The downwash effect on the vertical drag must again be included, the appropriate formula being

$$T = W + K_1 D_f \quad (6)$$

$$\text{where } D_f = D_v \left[f^2 + \frac{V_c^2}{v_{T O}^2} \left(1 - f + \frac{f^2}{2} \right) + f(2 - f) \frac{V_c}{v_{T O}} \left(1 + \frac{V^2}{4v_{T O}^2} \right)^{\frac{1}{2}} \right] \quad (7)$$

Oliver's analysis has been found to give a good working method for low forward speed providing the modification mentioned earlier is made. A good alternative for this regime is to write

$$P_i = P_{i,HOV} \cos \left(40 \frac{V}{v_t} \right)^\circ$$

where V is the forward speed and $P_{i,HOV}$ is the magnitude of the induced power when hovering. This formula is quite accurate for forward speeds of less than 1.2 times v_t .

In forward flight the problem is more complicated. Considering first an isolated rotor, the usual assumption is that the mean induced velocity v is given by

$$v = T/(2\pi R^2 \rho V^{\frac{1}{2}}) \quad (9)$$

The logic behind this formula has for a long time gone unquestioned. The origin is due principally to the analogy between fixed and rotating wings

in forward flight. In fact however there is some doubt as to whether the formula holds at all, for since low aspect ratio wings are known to have high induced drag factors (which may be interpreted as high downwash velocity), it follows that by similarity the value of v should be correspondingly increased. Recent unpublished work by Winny on the effects of vortex sheaths gives a value of v of double the generally accepted figure. It is too soon to give a final answer on whether to accept this doubled value, and it is perhaps fortunate that in forward flight the term P_1 is small. I have so far not used the doubled value and have been quite happy to take the somewhat superficial estimate

$$P_1 = 1.05 T v \quad (10)$$

where the 1.05 allows for non uniformity of distribution of the induced velocity (fuller details of which may be found in Ref. 12). The rotor wing combination is difficult to analyse accurately but a useful approximation is to regard the rotor disc as a disc for computing its own P_1 but as a fixed surface for computing interference powers.

On this basis

$$P_1 = 1.05 z W v + \frac{(1-z)^2 W^2}{2\pi s_1^2 \rho e V} + \frac{2\sigma_1(1-z)z W^2}{2\pi R^2 s_1 \rho v} \quad (11)$$

the symbols being defined in the section on Notation.

The effect of tandem rotors can be evaluated on the same basis. Thus the downwash angle at the front rotor, regarding it as a wing, is—

$$\frac{w}{V} = k_1 C_{L1} / \pi A \quad (12)$$

$$\text{where } C_{L1} = W_F / \frac{1}{2} \rho V^2 \pi R_F^2 \quad (13)$$

where k_1 is the low aspect ratio correction ($1 < k_1 < 2$) and at the rear rotor, the downwash angle due to the front rotor is

$$\frac{w}{V} = \frac{k_1 k_2 C_{L1}}{2\pi A} = \frac{k_1 k_2 C_{L1}}{8} \quad (14)$$

where k_2 is the correction for distance between the rotors ($1 < k_2 < 2$). The value of P_1 is thus increased by

$$\Delta P_1 = k_1 k_2 C_{L1} W_R \frac{V}{8} \quad (15)$$

The effect of the rear rotor on the front can be ignored. The order of magnitude of ΔP_1 is large, being obviously the same as that of the value of P_1 for the isolated rear rotor. This result is in line with that of the comprehensive analysis of Ref. 12 where the same general conclusion is reached. Incidentally in that reference the rear rotor is shown to reduce the P_1 of the front rotor by some 7%.

The value of P_o

The conventional expression for the power absorption by the rotating blades is

$$P_o = \frac{1}{8} \rho \delta \sigma \pi V_T^3 R^2 (1 + 4.65\mu^2) \quad (16)$$

In addition the power absorption must be increased to cover the extra drag of tip jets, mass balances and any other particular excrescences

Data on the value of δ is sparse and indeed very little has been done systematically to determine δ at all accurately. Indeed for some reason the effect of Reynold's Number has always been ignored in the usual values for δ (e.g., Bailey)

Three methods can be used to determine δ . A wind tunnel test can give the value at a characteristic section (by using a small section cut off a blade for the purpose). Alternatively the Jones pitot traverse method can be used without cutting the blade. The third involves the use of deceleration tests on a rotor stand and this method has the advantage of giving the integrated value.

All three have been used by my colleagues and myself to obtain some more concrete results than would be otherwise available. In the wind tunnel methods it is necessary to select a typical section. Since the drag coefficient reduces with Reynold's Number, we may write for a constant chord blade,

$$C_d = C_{d0} \left(\frac{r}{R} \right)^{-\epsilon_1} \quad (17)$$

where ϵ_1 is a small quantity

$$\delta = 4 \int_0^1 C_d \left(\frac{r}{R} \right)^3 d \left(\frac{r}{R} \right) \quad (18)$$

$$\text{If the characteristic section is at } \left(\frac{r}{R} \right) = x_1 \quad (19)$$

use of the formulae leads to the value for x_1 in the form

$$x_1^{-\epsilon_1} = \frac{4}{4 - \epsilon_1} \quad (20)$$

$$\text{so that } x_1 = \left(1 - \frac{\epsilon_1}{4} \right)^{\frac{1}{\epsilon_1}} \quad (21)$$

$$\text{and as } \epsilon_1 \rightarrow 0, x_1 \rightarrow \epsilon_1^{-1} \quad (22)$$

Thus if Reynold's Number effect is to be included the characteristic section is at 77.8% of the radius

Fig 2 shows the results of a Pitot traverse behind the outer portion of a particular rotor blade. This particular blade was a very early prototype blade and had a particularly bad finish having both exposed rivet heads as

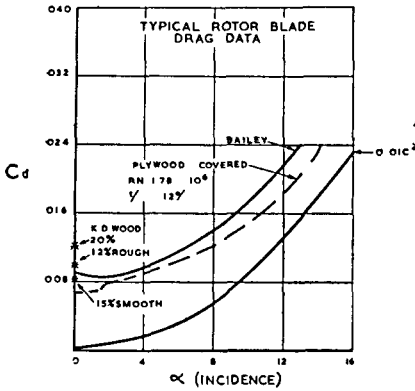


Fig 1

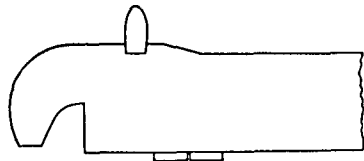
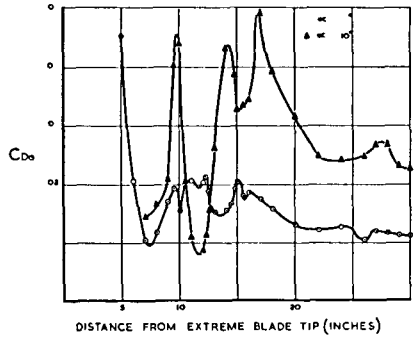


Fig 2

well as a dirty mass balance and a badly welded tip jet unit. Fig 3 shows a comparable blade after partial cleaning up.

Deceleration tests are a little difficult to analyse because of the inaccuracy in differentiation unless the results are presented in a suitable form. Re-writing the power equation in the form (hovering only)

$$\frac{P}{\Omega} = -I \Omega = \frac{1}{8} \delta \rho \sigma \pi \Omega^2 R^5 \tag{23}$$

and allowing for a variation of δ with angular velocity in the form

$$\delta = \delta^1 \Omega^{-\epsilon_1} \tag{24}$$

it is easily shown that

$$\frac{I}{\Omega} \frac{1}{(1 - \epsilon_2)} = \frac{1}{8} \delta^1 \rho \sigma \pi R^5 t + K \Omega^{-\epsilon_1} \tag{25}$$

where K is a constant

It follows that plotting the reciprocal of the angular velocity against

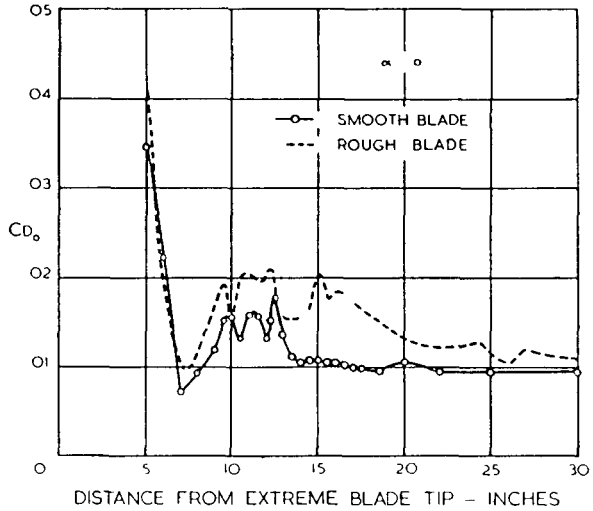


FIG 3

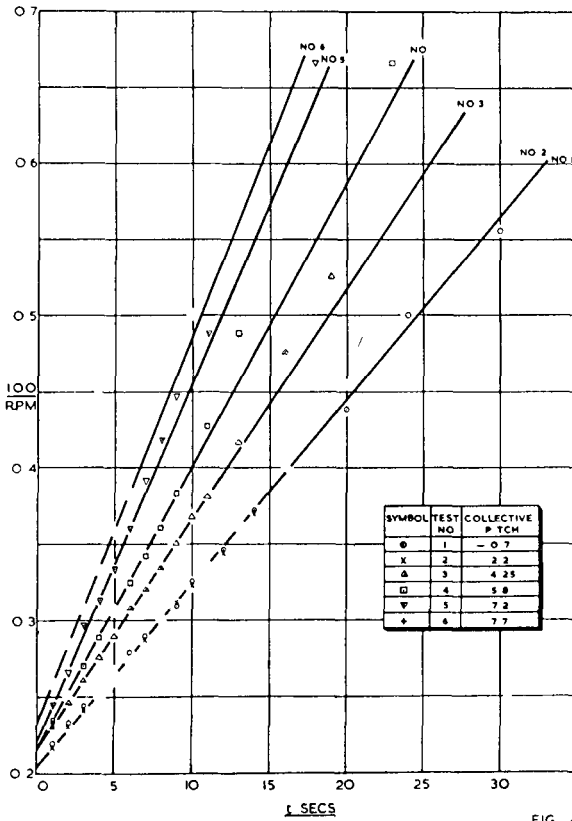


FIG 4

time should give near straight lines, each incidence giving its own particular line. The deviation from the straight can be used to find ϵ_2 and the effect of incidence on δ .

A typical set of curves derived from a deceleration test is shown in Fig. 4. The slopes are easy to measure and no differentiation of curves of angular velocity versus time is needed. It will be seen that in this case the lines are almost straight indicating that the variation of drag coefficient with angular velocity is negligible. The ground effect which of course is present during these tests has a very small effect on the curve of power versus blade angle (the ground increases the thrust at constant blade setting but reduces the induced velocity giving almost no net effect) so that conversion of the curves to free air conditions is simple.

Excrescences are always a source of considerable power loss. Apart from the drag of the excrescences themselves there is as well their additional effect on the position of transition from laminar to turbulent flow. This position is also affected by the degree of local yaw at each section of the blade and although under hovering conditions the transition may be at up to 40% of the blade chord (except near the tip where there is considerable out flow) in forward flight the transition may well move forward to no further back than 10% of the chord. Contamination of the surface which is normally worst near the leading edge over the outer portion of the blade—where the resulting power loss is the most severe—has a similar disconcerting effect. Unfortunately ducted blades which have essentially higher thickness/chord ratios and therefore start off at a disadvantage are somewhat worse in this respect than the thinner blades which are characteristic of the direct drive (see Ref. 13). Additive drag due to rivets is calculable from data given in the various standard references (*e.g.*, Hoerner or the various R.A.E. Reports by Williams on protruberances).

The effect of compressibility on blade drag is a matter for dispute. Liptrot (Ref. 15) gives a method for allowing for such effects. The best that can be said for this method is that if the answers it gives are correct, a number of helicopters now flying are doing so under false pretences! The essential feature of Liptrot's analysis is that compressibility effects are significant at well below the section critical Mach Number. Although it is true that incidence reduces this critical speed, I cannot in fact accept the thesis that all the observed effects are not calculable by direct integration of the section characteristic allowing for the effect of incidence on critical Mach Number without assuming large sub-critical increases. Beyond this I do not visualise any further correction. The method outlined below is one which although laborious is probably more accurate than Liptrot's. It can be seen from inspection of test results that at supercritical speeds ($\delta - \delta_{M=0}$) is approximately a function F of $(M - M_{crit})$ independent of incidence and that M_{crit} is a function of incidence and t/c ratio. The increase in overall δ due to compressibility is given by

$$\Delta \delta = \frac{\int_0^{2\pi} \int_0^1 M^2 \frac{r}{R} F d \left(\frac{r}{R} \right) d\psi}{\frac{\pi}{2} (1 + \mu^2) \frac{V_T^2}{a^2}} \quad (26)$$

The labour comes in to determine the distribution of incidence and local Mach Number for the derivation of F Fig 5 shows the particular data used in one particular calculation Fig 6 shows the region affected by

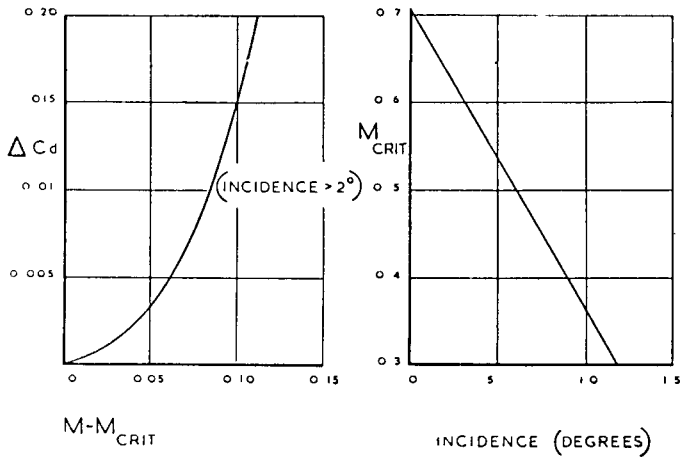


Fig 5 Supercritical drag of NACA 0015 Section
(Ref NACA Report No 832)

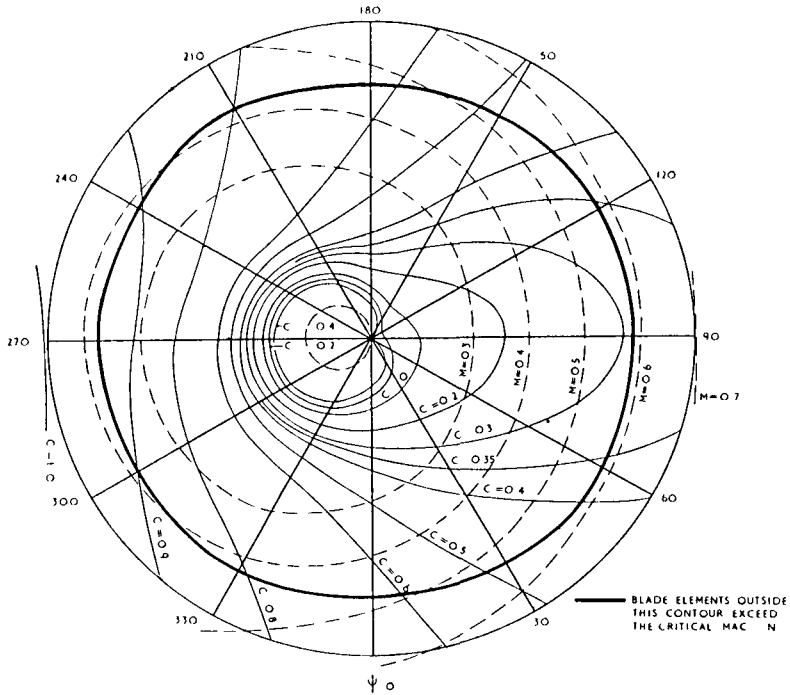


FIG 6 DISTRIBUTION OF C_L OVER DISC $\mu = 0.2$ $v_t = 650$ /SEC

compressibility effects at $\mu = 0.2$ and $650'$ /sec rotational tip speed. The interesting result is that while Liptrot's method would give factors of 1.2 for hovering and 1.74 for flight at $\mu = 0.2$, the figures obtained by this method are only 1.14 and 1.26 respectively, a considerable reduction. On the Gyrodyne the correction factors come out in the region of 1.1 and are quite consistent with flight results, Liptrot's factors again are prohibitively high.

Equally imponderable is the drag due to tip jet units. When the jets are functioning the flow round the unit fairing should be smooth and the drag negligibly low. When the blades are in a state of autorotation the drag assessment is made difficult by the effect of yaw (particularly at reasonable values of μ) and the somewhat unpredictable afterbody effects.

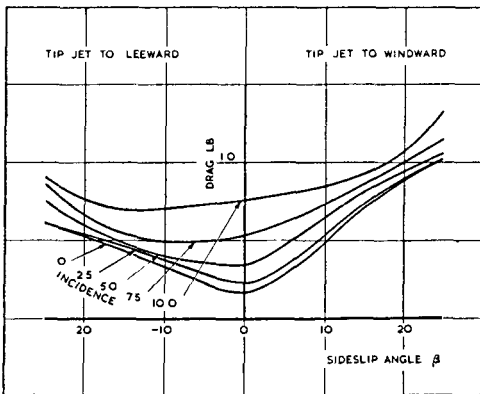


Fig 7

Fig 7 shows the sort of thing that happens. The moral is to design the tip jets for high critical Mach Number at high incidence—a practice that cannot be too strongly pressed.

Summarising some of the results obtained, a rational estimate of δ_0 is obtained for calculation purposes by assuming transition at about 10% chord, allowing at least a 10% margin over the drag so obtained to allow for surface finish, and adding about double the estimated drag of excrescences and tip jets. The increase of δ due to lift is best obtained from the expression

$$\delta = \delta_0 + \frac{\overline{C_L}^2}{100} \quad (27)$$

where $\overline{C_L}$ is the mean blade lift coefficient, related to the thrust by the further relation

$$\overline{C_L} = \frac{6}{B^3} \frac{C_T}{\sigma} \frac{1}{1 + \frac{3}{2} \frac{\mu^2}{B^2}} \sim 5.5 \bar{a} \quad (28)$$

A comparison of this growth rate with lift and the variation given by other authorities is shown in Fig 1

The value of P_p

Helicopter fuselages are so poor aerodynamically that the only effective way of assessing their drag characteristics is to test them in a wind tunnel. Bluff rear ends give not only high drag but queer pitching characteristics which influence the stability considerably. They also give rise to an oscillating wake which apart from the obvious possibility of fatigue makes the provision of fixed wing type control surfaces somewhat inefficient and uneconomic. Incidence effects are large and, paradoxically are worse for aerodynamically good shapes than they are for aerodynamically bad ones. This last effect is to be seen in Fig 8. At high incidences—and these are produced when the aircraft climbs rapidly or descends rapidly since the fuselage attitude hardly changes during such manoeuvres—the magnitude of P_p is increased considerably due to the incidence effect and allowance must be made in performance estimates or predictions.

The value of P_c

The climb power is given quite simply by

$$P_c = W V \sin \gamma \tag{29}$$

For most methods of analysis this form is adequate. It can however be recast into many alternative forms, all more complicated, but possibly more suited for graphical and chart methods. I would however issue a warning that care should be taken to note the definitions used and to check all formulae. Thus for example the climb equations written as

$$\begin{aligned} \frac{C_{Pc}}{C_T} = \sin \gamma \left[- \sin \gamma \frac{C_{Pp}}{C_T} \frac{\cos \alpha}{\mu} \right. \\ \left. + \sqrt{1 - \cos^2 \gamma \left(\frac{C_{Pp}}{C_T} \right)^2 \frac{\cos^2 \alpha}{\mu^2}} \right] \frac{\mu}{\cos \alpha} \end{aligned} \tag{30}$$

in a recent report by two well known authorities (Ref 16). This equation is certainly derivable from the simple one, but either the H force has been omitted in the derivation, or else the parasite drag coefficient includes a correction for the H force which would not immediately be obvious unless care were taken to check the formulae.

Typical Curves

The power versus speed curves are now so familiar that there is little point in going into them in detail. One curve of interest is however illustrated in Fig 9 which shows a grid of power required versus speed, collective pitch and normal acceleration on the helicopter. Such a grid is of great value in showing a picture of the overall level speed performance, as well as immediately providing the data for designing the synchronisation mechanism between the collective pitch and throttle settings. In addition it provides

the necessary data for analysing the turn performance of the helicopter It would appear odd at first sight that the question of restrictions on forward speed due to tip stall has not been mentioned in the discussions above This is quite deliberate Having seen so many text-books and research documents giving learned discussions as to whether 14° , 16° or even 20° is the blade incidence limit and whether the limit is due to vibration or increase of flapping angles or what have you, I am becoming increasingly convinced that we have to a great extent been blinded by too much science

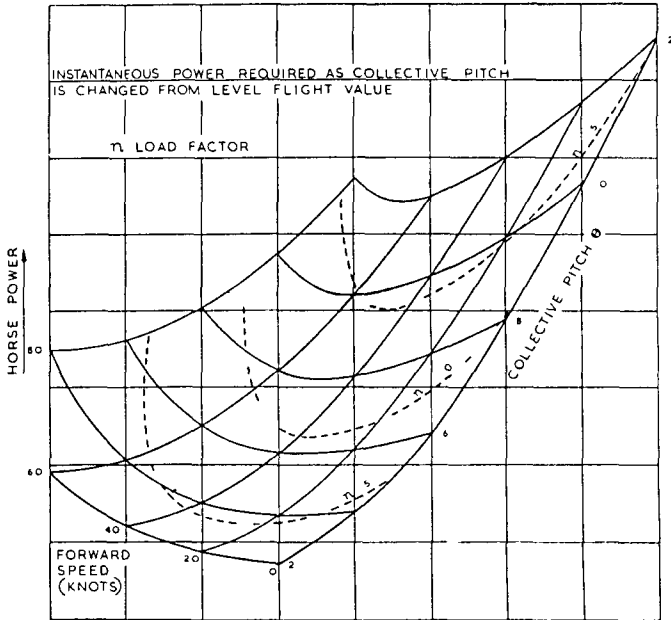


Fig 9

and too little experimenting, and until the effects of number of blades, blade inertia, and offset hinges are fully investigated, there is little point in stressing this aspect of flight performance

(3) STABILITY AND CONTROL

When discussing stability problems on helicopters it is easy to talk generalities—far more difficult to be specific There are in fact three questions to be answered

- (1) What precisely do we need in the way of stability and control ?
- (2) How can we achieve the necessary standards ?
- (3) How can we predict the stability characteristics of a new design in order to ensure it will meet the requirements ?

Answering the first question, there are certain characteristics that we should obviously like to have Every pilot demands crisp positive but not excessively sensitive control to fine limits—no lumpiness, no sogginess He

prefers the control to work in the conventional sense, forward stick to go forward when hovering, and stick moving increasingly forward as the speed is increased. This is often referred to as speed stability or static stability and obviously enables the pilot to maintain correct and positive control of the helicopter.

The aircraft when disturbed from steady flight should preferably be stable, *i.e.*, not begin a series of increasingly divergent oscillations. The aircraft oscillations should decay fairly quickly and the frequency of the oscillations should lie within reasonable limits. What limits? On fixed wing aircraft the background of experience is greater yet we find it difficult to ascribe numbers. Perhaps two cycles to half amplitude might be taken as a rough guide to the degree of damping to be arrived at but I have not yet come across any direct data on the maximum tolerable frequency.

More recently a new dynamic stability criterion has grown up. O'Hara (Ref. 17) gives an analysis of the longitudinal stability of a helicopter and discusses in detail this particular criterion. Broadly it is that the curve of normal acceleration (or *g*'s) against time must become concave downwards within two seconds of the initiation of a pull out and remain positive until the maximum acceleration is obtained. The handling characteristics are presumed to be acceptable providing this is achieved.

Now this is an interesting criterion partly because there is no comparable fixed wing aircraft parallel and partly because it shows some light on the missing figures for frequency and damping. O'Hara gives as the disturbance equation

$$\lambda^2 + B^1 \lambda^1 + C^1 = 0 \quad (31)$$

$$\text{where } B^1 = - \left[\frac{M_q}{I} + \frac{Z_w}{m} \right] \quad (32)$$

$$\text{and } C^1 = \left[\frac{M_q Z_w}{mI} - \frac{M_w V}{I} \right] = \frac{RT_\alpha H_m}{I} \quad (33)$$

He then states that for the range B^1 between 0.8 and 2.0 a value of H_m of 0.01 is adequate to meet this criterion. Now the undamped frequency in cycles per second is given by

$$f = \frac{\sqrt{C^1}}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{RT_\alpha H_m}{I}} \quad (34)$$

and for a typical helicopter for which

$$R = 24 \text{ ft} \quad gT_\alpha = 600,000 \text{ lb. — ft/sec}^2$$

$$I = 7,000 \text{ slug — ft}^2$$

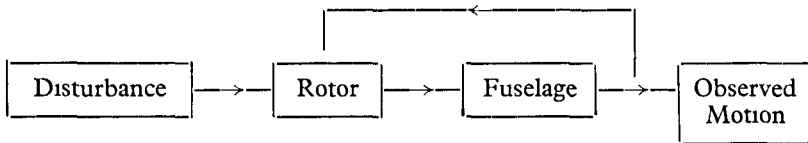
$$f = 1.27 \sqrt{H_m}$$

it follows that a frequency at zero damping of 0.127 c.p.s. would be satisfactory. Such a low frequency would be unacceptable for a fixed wing aircraft—probably double that amount being necessary—and even then over a limited range of damping factors. The number of cycles to half amplitude

comes out using the above figures at about 1.2. It would thus appear that either this new criterion gives too low a value for the undamped frequency (or for the damped frequency since damping reduces the frequency) or we must get accustomed to our helicopters being somewhat less dynamically stable than our fixed wing aeroplanes. Of course there is the other alternative of working to manoeuvre margins of some 0.04 instead of 0.01 advocated by O'Hara!

Analysis of Blade Motion

Before any attempt can be made to understand the behaviour of a helicopter, it is important to consider the behaviour of the rotor. This is because the system is a very simple combination of two independent sub-systems with feed back. This is illustrated by the simple diagram —



Any disturbance will produce an effect on both rotor and fuselage. The fuselage "output" acts as an additional input to the rotor so the rotor output to the fuselage modifies the subsequent fuselage motion. The final result is the motion observed by the pilot and external observer.

The rotor, when subjected to a fixed set of conditions, will take up a so-called steady state although the blades will oscillate both in azimuth and flap. More precisely each blade takes up a state of undamped periodic motion. Any disturbance will tend to disturb the "steady conditions" and there is then a superimposed non-periodic damped oscillation. The damping is usually quite large but decreases as the forward speed increases (ϵ , at large values of μ). The governing equation for the blade motion is a linear one with periodic coefficients. In various forms it has been derived by many workers in this field, perhaps the earliest simple form was derived by Glauert and Shone and some of the most recent additions have been made by Payne (Ref. 18). However, I believe that the general theory given in Ref. 19 is in some respects the most complete even though it omits any allowance for variation of velocity across the disc. Taking the shaft axis as the reference axis, the governing equation, in the absence of δ_3 effects and blade lag ζ is given in Ref. 19 (omitting the terms containing δ_3 and ζ) as

$$\begin{aligned}
 F(\beta) = & \frac{I_b}{R^4} \frac{d^2\beta}{d\psi^2} + [\bar{A}_1 + \bar{E}_1 e + \bar{E}_1 \mu \sin \psi] \frac{d\beta}{d\psi} \\
 & + \beta \left[\frac{I_b}{R^4} + \frac{seW_b}{R^2} - \frac{W_b sg}{V_1^2 R} \sin \chi_0 \cos \psi + \frac{K}{V_1^2 R^2} \right. \\
 & \left. + \bar{B}_1 \mu e \cos \psi + \bar{B}_1 \frac{\mu^2}{2} \sin 2\psi + \bar{E}_1 \mu \cos \psi \right] = 0 \quad (35)
 \end{aligned}$$

When the blade is disturbed the resulting motion is given by

$$\begin{aligned}
 & F(\beta) + F_1 \chi_d + F_2 \frac{d\chi_d}{d\psi} + F_3 \frac{d^2\chi_d}{d\psi^2} + F_4 \left[\frac{U_d}{V_T} \cos \chi_o - \frac{W_d}{V_T} \sin \chi_o \right] \\
 & + F_5 \left[\frac{U_d}{V_T} \sin \chi_o + \frac{W_d}{V_T} \cos \chi_o \right] + F_6 \left[\frac{dU_d}{d\psi} \sin \chi_o + \frac{dW_d}{d\psi} \cos \chi_o \right] \\
 & + F_7 [\theta_{od} - A_{1d} \cos \psi - B_{1d} \sin \psi] = 0 \quad (36)
 \end{aligned}$$

The definitions of the quantities in the equation are given in the "Notation" and the $F_1 - F_7$ functions given in Appendix 1

One method of solution is given in Ref 19, others (unpublished) have been given by Rees. Some typical calculations for a large rotor gives the results shown in Fig 10-12. Fig 10 shows the effect of a change in the collective pitch on the coning angle, Fig 11 shows the effect on α_1 . It is seen that the motion is highly damped in terms of azimuth angle the steady

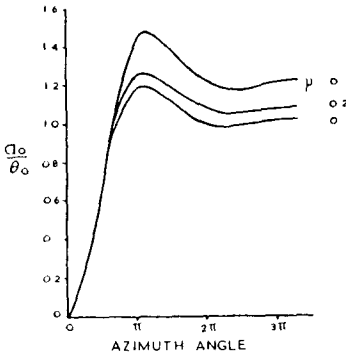


Fig 10 Response to application of collective pitch

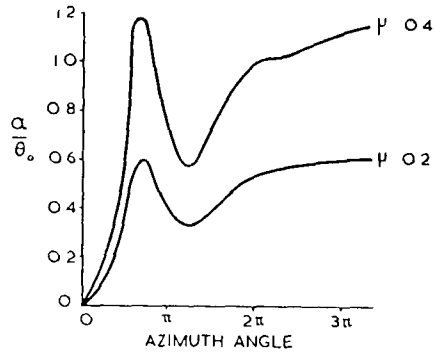


Fig 11 Azimuth angle Response to application of collective pitch

position being reached after about $1\frac{1}{2}$ turns. Since a change in coning angle can be related to a lift change it follows the new lift is established quickly (in about $\frac{1}{2}$ second at 200 r.p.m.) The response to a forward velocity disturbance is shown in Fig 12 and follows the same pattern.

It is evident that a disturbance produces a tilt of the disc and in general brings into play forces and moments at the rotor head. The components of the transverse forces are approximately equal to the force on the rotor times the angle of tilt. Not exactly however. For example, under hovering conditions the lift vector tilts not by b_1 laterally but by b_1' where

$$b_1' = b_1 \left[1 + \frac{1}{2 + \frac{4}{3} \frac{B\theta}{\lambda}} \right] \sim b_1 \left[1 - \frac{B^2 a \sigma}{8\sqrt{2C_T}} \right] \quad (37)$$

and the difference is large for low thrust coefficients. Ignoring these effects however, the forces brought into play are proportional to the disturbances

Similarly the moments due to offset hinges are proportional to the disturbances (e.g., the nose up moment is given by $M = SeRsa_{1s}/2 (s + e)$ and since a_{1s} is proportional to the disturbance so is the pitching moment change) In view of this proportionality the degrees of freedom of the resulting helicopter motion are reduced in number to the body degrees of freedom and the equations of motion for the helicopter become analogous to those for a fixed wing aircraft This is fortunate since we have a ready made

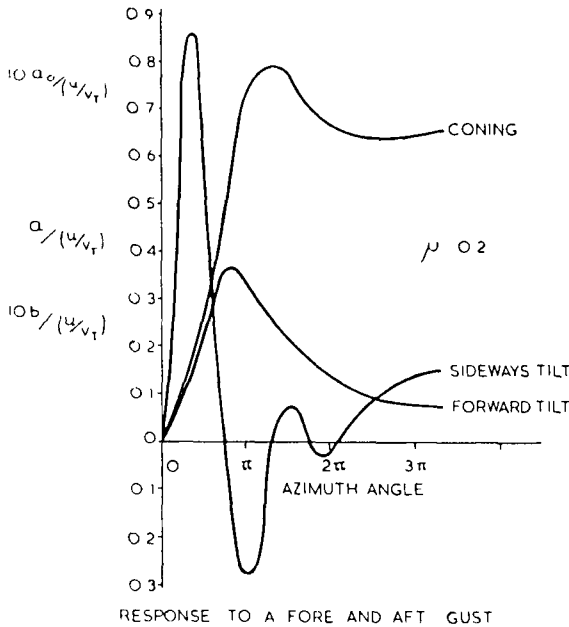


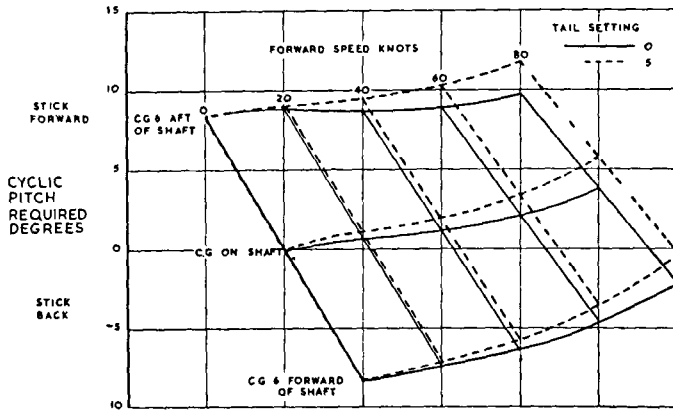
FIG 12

theory immediately available and we are left only with the task of evaluating the derivatives The concepts of static and manoeuvre margins can be carried over and we are able to apply theory to determine the behaviour of the helicopter in relation to the requirements set out to which reference was made at the beginning of the section on stability and control

Static Stability

The static stability of conventional helicopters presents little of great interest except for the influence of the pitching moments of the fuselage Tests on helicopter fuselages in wind tunnels are very limited in number, and suffer from the disadvantage that it is virtually impossible to make full allowance for the interference effects of the rotor The tests with which I am familiar—and I obviously cannot give specific details—show that the pitching moments behave in a most peculiar fashion Short fuselages are inherently the worst Over the working incidence range they tend to be unstable At larger positive and negative incidences there is a region of stability, then they go unstable again Putting on a tailplane has, of course,

a stabilising effect but again not entirely as expected. Over the working incidence range the combination is stable. At larger positive and negative incidences the question of tailplane stall comes in and whereas the fuselage itself is stable, the combination becomes unstable. The size of the tailplane is no remedy. Possibly a floating tailplane is the answer to this—as well as some dynamic response problems. This effect may also be seen from trim data on the helicopter. Fig 13 shows a typical grid of cyclic pitch versus speed and C G position, for two tail plane settings. It will be observed that, since the incidences are small, the behaviour is regular and the helicopter statically stable. Fig 14 shows the same helicopter in autorotation when the incidences become large. It is obvious that even with the C G well



POWERED FLIGHT - CYCLIC PITCH FOR TRIM LEVEL FLIGHT

FIG 13

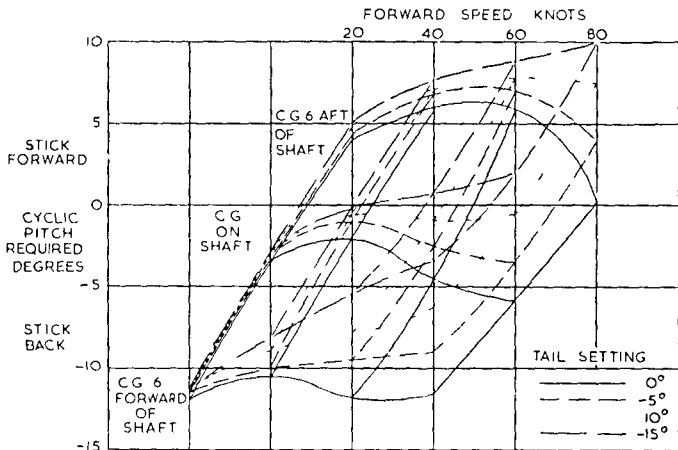


FIG 14

Fig 14 Steady autorotative descent cyclic pitch for trim

forward the machine is statically unstable for small tailplane settings while with the C G on the shaft axis or aft of this axis the instability is present at settings of less than 10° . This gives a warning to would-be exponents of fixed tailplanes for helicopters

The analysis of winged helicopters follows similar lines as for the unwinged ones. The equations of equilibrium are similar except for the extra terms, and in particular the partition of lift between wing and rotor. The fuselage attitude, the disc tilts and the cyclic pitch for trim all come out of the equations quite simply. It is found that winged helicopters with controllable tail surfaces present an unusual problem in so far as there is no unique curve of stick position against speed. On the face of it this would appear to be a contravention of our normal conceptions of static stability—but this is not so. What it really means is that the pilot by deciding the off load ratio he chooses to use at a given speed, automatically fixes the position of the tail adjustment. The definition of static stability (in terms of forward stick movement with increase of speed) still holds providing the tail adjustment is then maintained fixed over a small speed range. There is at each speed a wide range of values of the static margin by virtue of this change in off loading and the pilot by choosing the datum balance point at each speed determines the variation of static margin versus speed.

The choice of off-loading is not entirely a free one since the division

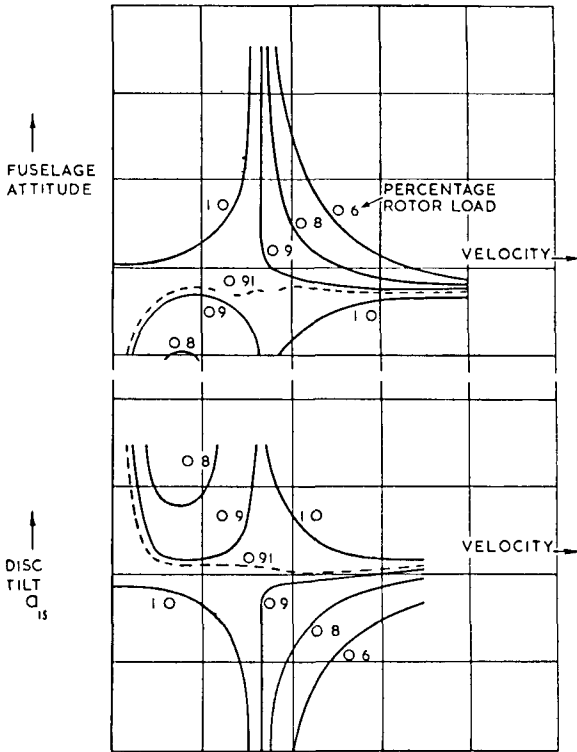


FIG 15

of lift affects both the flapping angles of the blades and the fuselage tilt. When there are offset hinges, there appears to be one singular speed for each loading condition (weight and C G) at which there is no choice at all. In this condition, if steady flight is attempted at the wrong off-load ratio the fuselage tilt and flapping angles tend to become infinite. The physical interpretation is very obscure and what happens in practice should be interesting.

The cause appears to be that although the aerodynamic loads on the wings (and hence pitching moment from that source)

are functions of the speed and incidence, the opposing moments from the offset hinges are to a large degree dependent on incidence alone. Since the difference dictates the angle of the fuselage datum, at one speed the incidence must go to infinity to achieve balance. Fig 15 illustrates the sort of variation of incidence with speed one might anticipate as well as the variation of the disc tilt. For the reasons given earlier, one cannot interpret the curves in terms of static stability. In fact one would be faced with the fallacious result that if the usual practice of reducing the rotor load with forward speed were adopted, the machine would be statically unstable.

Dynamic Longitudinal Stability

As explained earlier, the equations of motion of the helicopter can be reduced to a form similar to those for a fixed wing aircraft. In the notation of and with the axes chosen as in R & M 1801 the equations for longitudinal, normal to flight path, and angular motion are

$$W\theta + \frac{W}{g} u - uX_u - wX_w - qX_q = 0 \quad (38)$$

$$\frac{W}{g} w - \frac{W}{g} V\theta - uZ_u - wZ_w - qZ_q = 0 \quad (39)$$

$$I\dot{\theta} - uM_u - wM_w - qM_q = 0 \quad (40)$$

The characteristic equation is given by the determinant

$$\begin{vmatrix} \frac{W}{g} \lambda - X_u & -X_w & (W - \lambda X_q) \\ -Z_u & \left(\frac{W}{g} \lambda - Z_w \right) & - \left(\lambda Z_q + \frac{W}{g} \lambda V \right) \\ -M_u & -M_w & I \lambda^2 - \lambda M_q \end{vmatrix} = 0 \quad (41)$$

which in turn gives

$$\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0 \quad (42)$$

At speeds away from the hovering condition (and there is some doubt about the need for stability when hovering, although of course such stability would obviously simplify the difficulty of flying particularly under instrument conditions) the equation can be factored approximately into the short period and phugoid terms

$$\left(\lambda^2 + B\lambda + C \right) \left(\lambda^2 + \frac{CD - BE}{C^2} \lambda + \frac{E}{C} \right) = 0 \quad (43)$$

The conditions for stability are B, C, D, E all positive and $\bar{R} > 0$ where

$$\bar{R} = BCD - D^2 - B^2E \quad (44)$$

Quite obviously this last requirement is substantially the requirement for positive damping of the phugoid since D^2 is generally small. The phugoid motion of a helicopter does not seem to be of great importance—possibly because the short period characteristics are in general somewhat deficient. Interest in the longitudinal stability field is therefore centred on the short period behaviour. In particular, although a helicopter may be statically stable, due to rotor instability with angle of attack—which deteriorates with speed—the short period oscillation may lead to a divergence when control movements are applied. The most recent work in this direction has been tied to the step input manoeuvre to which reference has been made earlier. The theory of this manoeuvre is presented in Appendix 2. It will be observed that the aerodynamic parameters that enter the problem are

$$Mq, Zw, Mw, M_{B_1}, Z_{B_1}, I, m$$

The methods of improving the manoeuvre stability must obviously be designed around the effects of variation of these parameters. A fixed tailplane increases the damping (by changing Mq) as well as the frequency. As such it improves the stability. A tailplane linked to the cyclic pitch lever does the same as the fixed tailplane as well as changing M_{B_1} . It can therefore be made even more effective. Since offset hinges contribute to both Mq and Mw , variation in this feature coupled if wished with C.G. variation is also possible to improve the characteristics. Other possible stabilisation methods are possible utilising controlled oscillations of the blades about the radial blade pitching axis. The theory of these is due to Miller (Ref. 20) and to McCabe and McCaskill (Ref. 21). I have grave doubts about the practicability of utilising such techniques—based on the fear of adding further possibilities of resonances on an aircraft already subject to excessive resonance difficulties.

The somewhat limited experience of my colleagues and myself on the effect of the various possibilities listed is confined to fixed tailplanes and offset hinges—as well as of course the effect of centre of gravity. The fixed tail is very effective—although it introduces many other problems—but there are doubts about the efficacy of offset hinges for other than static stability improvement purposes. Calculations show that as the offset hinges are moved outwards the damping may be reduced—to quite low values at some speeds. Since the response is only slightly improved by variation in manoeuvre margin and to a far greater extent by the damping the net result is disappointing. As far as C.G. travel is concerned, as with fixed wing aircraft the stability deteriorates as the C.G. moves aft, the band of speed for stability being reduced (the aircraft is obviously unstable at zero speed unless auto-stabilisation is used) progressively until with a far aft C.G. there is no positive stability at any speed.

One facet which has received little attention so far and on which I feel a lot more work is required is the response not to a step input but to a pulse input. The theory is given in Appendix 3. The longitudinal control sensitivity is very much tied up with this concept and it may become of equal importance to the step input in due course.

Directional Stability

The equations governing the lateral behaviour of a helicopter, like those for the longitudinal behaviour, are analogous to those for fixed winged aircraft. In the usual notation they are

$$\frac{W}{g} v - W\phi + \frac{W}{g} Vr - Y_v v - Y_{pP} - Y_{rR} = 0 \quad (45)$$

$$Ap - L_v v - L_{pP} - L_{rR} = 0 \quad (46)$$

$$Cr - N_v v - N_{pP} - N_{rR} = 0 \quad (47)$$

The characteristic equation is given by the determinant

$$\begin{vmatrix} \left(\lambda \frac{W}{g} - Y_v \right) & - (\lambda Y_p + W) & \left(\frac{W}{g} v - Y_r \right) \\ - L_v & (A\lambda^2 - \lambda L_p) & - L_r \\ - N_v & - \lambda N_p & (\lambda C - N_r) \end{vmatrix} = 0 \quad (48)$$

which in turn gives

$$\lambda^4 + B' \lambda^3 + C' \lambda^2 + D' \lambda + E' = 0 \quad (49)$$

Under hovering conditions there is generally a pair of complex roots of an unstable nature. The period is fairly long (of the order of 8 seconds) but the time to double amplitude can give concern. As the forward speed increases the stability improves and then deteriorates, the complex roots becoming real and the oscillations becoming divergences.

There is a very strong case for autostabilisation of high speed helicopters but there is insufficient data available at present to indicate whether "desirable" rather than "essential" is applicable.

Static Directional Stability and Control

Adequate directional stability is essential over the upper end of the speed range (certainly at cruising) and adequate control essential over the lower end of the range.

The problems involved in producing a helicopter with good directional behaviour are different depending on whether fixed or rotating surfaces are used to supply the damping at the rear end. Tail rotors give directional stability by virtue of the variation of the axial force on the rotor when the effective axial velocity is changed.

To a first approximation the thrust coefficient on the tail rotor in sideslip referred to its own dimensions, is given by

$$\frac{\Delta C_{Tt}}{\beta} = \frac{\mu a_t \sigma_t}{4} \left(B^2 + \frac{\mu^2}{2} \right) \quad (50)$$

It will be observed that the force is approximately linear with forward speed. The thrust coefficient to the same approximation when the rotor is in a state of yaw is given by

$$\frac{\Delta C_{Tt}}{r} = \frac{l_t a_t \sigma_t}{4 V_{Tt}} \left(B^2 + \frac{\mu^2}{2} \right) \quad (51)$$

This now shows a substantial independence of forward speed. This is the big advantage of the tail rotor—it provides yaw damping even under hovering conditions. However it must be borne in mind that these equations are approximate. If the full equations are used it becomes evident that the direction of turn is important—in one direction instability can develop due to the development of the vortex ring state.

Turning to the fixed tail surfaces which have given cause for concern on many designs, the question of direction or rotation ceases to be important. Further the sideslip contribution to fin force increases as speed squared, while the force in yaw increases linearly with speed. The difficulty here is two-fold. First at low speeds the air loads on the fin becomes negligibly small. Second even at high forward speeds the magnitude of the air loads is limited by the small fin surface usually available. This is because there are severe physical limitations on the maximum size possible, the rotor disc (and particularly the blades when drooped on their stops) representing an upper bound, and the ground line a lower bound.

The function of the tail rotor (or fins) is two fold—apart from providing stability it must provide directional control. It is difficult to provide enough control at low speeds with fixed surfaces, while with a tail rotor of high solidity it is essential to ensure that the system is not excessively sensitive. The two well known solutions with tail fins utilise the rotor downwash (over rudders with sloping hinge lines) or the efflux from the propulsive turbines (as in the *Djinn*). In the latter case the rotor downwash can be an embarrassment leading to a deflection of the turbine efflux away from the tail surface(s).

A rough assessment of the final rate of yaw following a change in tail rotor collective pitch is very simply assessed by taking the 2/3 radius as the typical section and writing down the condition for constant incidence. This gives

$$\Delta \theta_t = \frac{l_t r}{\frac{2}{3} V_{Tt}} \quad \text{or} \quad r = \frac{2}{3} \frac{V_{Tt}}{l_t} \Delta \theta_t \quad (52)$$

A more rigorous analysis gives (taking tip loss factor $B = 1$)

$$r = \frac{2}{3} \frac{V_{Tt}}{l_t} \left[1 + \frac{3}{2} \mu^2 \right] \Delta \theta_t \quad (53)$$

It follows that excessively rapid rates of yaw can be reduced only by variations of pedal gearing or reduction in the tail rotor rotational speed.

Roll Sensitivity

The major design consideration concerning the lateral stability and control of helicopters other than the directional stability and control is the

sensitivity in roll. Light helicopters in particular suffer from this defect. Speed seems to have only a secondary effect on this characteristic and in general the provision of adequate roll damping at low speeds gives desirable roll handling over the whole speed range.

The damping is dependent on the flapping response coefficient. This derivative is given in Ref. 19 and in many other reports by

$$b_{1p} = a_{1q} = \frac{16}{\Omega\gamma} \text{ at } \mu = 0 \quad (54)$$

The actual tilt of the disc force, as noted earlier, is given by

$$b'_{1p} = b_{1p} \left[1 - \frac{B_2 a \sigma}{8 \sqrt{2C_T}} \right] \quad (55)$$

These relations indicate that the damping can be increased by reducing the angular velocity or the inertia number γ . Thus high inertia, slowly rotating blades give good roll damping. A further possibility for increasing the damping is to increase the hinge offset since the moments due to offset hinges are proportional to b_1 as well as the offset length. It is also evident that lightly loaded rotors can actually give negative damping and it is a serious error to ignore the effect of the force vector tilt in relation to the flapping or tip path plane displacements. The condition for positive damping is

$$C_T > \frac{B^4 a^2 \sigma^2}{128} \text{ or } \sigma < \frac{\sqrt{128C_T}}{B^2 a} \quad (56)$$

and these provide a bound for the disc loading or solidity as required. A lightly loaded rotor requires solidities below about 0.1 and heavily loaded rotors below about 0.15. This limitation is not likely to represent a very serious design problem.

(4) FLIGHT LOADS

The lack of systematic flight records of the normal accelerations to which helicopters are subject makes stressing requirements rather a matter for synthesis of response and possible flight loading conditions rather than a matter of experience. The work involved in building up a complete picture of the loading conditions is not difficult but tends to be somewhat laborious.

The Normal Flight Envelope

Normal accelerations can be induced in two ways—either by the application of cyclic pitch or of collective pitch. The mechanism is quite different and the respective effects are most important at opposite ends of the speed scale.

As is clear from the discussion of the two second response criterion, application of cyclic pitch gives a small immediate increase in g 's followed by a steady build up to a maximum value in a few seconds.

The g's per degree of cyclic pitch applied decrease with speed and at low speed this is an inefficient method of producing normal accelerations

Collective pitch application gives a nearly instantaneous increase in g's and is most effective at low speeds. The maximum g's in this case are determined either by the maximum collective pitch angle available or by the stalling lift coefficient of the blades. In the latter case the maximum g's are given by

$$n_{\max} = C_{L \max} / \bar{C}_L$$

where C_L is the mean blade lift coefficient. It should be remembered that the value of $C_{L \max}$ is a dynamic quantity and depends on the rate of application of pitch, being up to 25% in excess of the static value.

When the relevant calculations have been performed it is found that the flight envelope is substantially rectangular, something like 2.5 g being attainable over the whole speed range.

Of course the worst possible loads that can be achieved occur when full cyclic pitch is applied, followed a few seconds later by maximum collective, arranging the two maxima to occur simultaneously. Such a case is more theoretical than actual and I am somewhat doubtful whether there is a case for stressing to this condition. The winged helicopter gives results somewhat similar to the ordinary case except that the air loads on the wings can give quite high g's in their own right.

Gust Effects

The effect of gusts is not of primary importance on modern helicopter rotors. As speeds increase the effects of gusts on winged helicopters are, on the other hand, quite serious because the wing lift increment at constant vertical gust velocity increases linearly with forward speed. This characteristic is quite different from the normal rotor one in which the g's are substantially constant independent of speed (apart from the effect at very low forward speeds). As would be expected, the effective low aspect ratio of a rotor gives low values of lift variation with incidence and this effect, apart from the change of form of variation with speed, tends to give low gust load factors.

The response equations for the helicopter subjected to a gust are identical with those for a cyclic pitch application. The results are directly applicable, a change in derivatives (from M_B and Z_B to M_{wg} and Z_{wg}) being required. The new derivatives are easy to evaluate and require a detailed knowledge of the blade response to a gust. The method of Ref 19 is directly applicable and is preferred to the one published some time ago in *Aircraft Engineering* owing to a number of errors in the latter.

(5) MISCELLANEOUS INVESTIGATIONS

Apart from the various investigations considered so far there are in effect two major groups about which I want to make a few somewhat restricted remarks. In the first group are such things as response to engine failure, ground effects, system assessments, dynamic investigations (in particular, general flapping and resonant behaviour).

In the second group comes the planning and testing of wind tunnel

models, investigation of special devices, etc. The difference is that we know a fair amount about the first group while the second group involves the outstanding problems of today

Considering the first group, the present state of our knowledge of ground effects is only partially satisfactory. As a result of the work of Cheeseman we have a semi-empirical theory which gives reasonable answers, but I find little comfort in the use of methods which replace rotors by aerodynamic sinks or sources of a peculiarly directional kind. We have done a fair amount of work on the replacement of the rotor by vortex ring systems but so far we have not reached a very satisfactory answer. I believe that eventually the "complete" solution will be found in that direction rather than the source one.

The response to engine failure is again a field in which we have contributed little new light on the subject, being concerned more with evaluation than with new techniques. We have used the "Boscombe Down" methods and find these very satisfactory. Our general conclusions were expressed in a lecture I gave a year or two back to the students at Cranfield and as this gives a useful background to the problem and its relation to present and future helicopters I am quoting the following extract:

"First it is self evident that the behaviour of the aircraft following complete or partial power failure is dependent on the pilot's course of action. Various possibilities are open. With a high energy rotor, such as I feel is becoming more common, he may adjust his collective pitch continually to maintain rotor speed. He may change his collective pitch to a new constant value or leave it well alone. He may apply cyclic pitch to attain enough forward speed to take advantage of the decreasing power demand with forward speed or he may do this and then flare out to reduce his descent speed at the expense of his rotor energy. Quite obviously his course of action will depend on his height at the time, and on his rate of climb. In this connection, it is worth mentioning that the so called rate of climb at sea level in vertical flight is non-existent. In fact, with modern helicopters an altitude of some 750 ft. is reached before full rate of climb is developed.

A normal helicopter, following power failure during vertical climb first overshoots and then reaches a steady rate of descent which is prohibitively high. This can be reduced by flaring out just before contact. High values of the collective pitch would reduce the rate of descent, but the price is rapid loss of tip speed, high coning angles and tendency to stall the rotor. In general values of about 15° collective pitch angle are reasonable compromises.

A high energy rotor enables the simple flare out technique to be used at any height appreciably less than the height required to produce steady rate of descent. A low energy rotor cannot be used above a low altitude because the drop in tip speed makes control impossible.

Thus forward tilt of the rotor and the development of forward speed is a much more important procedure in the low energy rotor case. Within the practical range increase in the disc tilt reduces the height loss in reaching zero descent rate. Limitations are imposed by fuselage altitude and controllability. Such tilt eventually enables a climb to be achieved if only a part of the power is lost, or a lower descent rate to be achieved, particularly if a flare out is made. A premature flare out would result either in an excessive

descent rate due to a rapid fall off in tip speed or a climb away at reduced tip speed

The effect of pilot reaction time, and lag in boosting up any remaining power supply may not be greatly significant with a high energy rotor but would be much more serious on a low inertia rotor (it is rather interesting to note that at high speed a failure of the engine followed by a reaction time of about 2 secs could wreck most helicopters flying today)

Without going into the figures for an actual helicopter the actual height losses using the various manoeuvres mentioned are rather problematical but it looks as if the multi-engine aircraft does stand a good chance of making a safe landing from any altitude, and that the unsafe altitude band is on its way out ”

Reverting to the second group there is a great deal I should like to say about wind tunnel testing of helicopters but this is such a wide and important subject that I shall leave it in the hope that a future lecture to the Association might be devoted to it I shall, of course, make any information desired available during the discussion should any be wanted

The outstanding problems of the present time in the aerodynamic field may perhaps be listed broadly as

- (a) need for more data on blade and fuselage drag ,
- (b) need for more data on induced velocity—including the interference effects between multiple rotors ,
- (c) need for more data on blade stall and critical Mach Numbers—including the effect of vibration levels on the acceptable degree of permissible stalling ,
- (d) need for more data on desired handling characteristics and auto-stabilisation ,
- (e) need for more data on stability derivatives and the correlation of wind tunnel and full scale results on these ,
- (f) collection and collation of data on flight and gust envelopes ,
- (g) more accurate analysis of the off-loading wing as an aid to the achievement of high speeds
- (h) specialised wind tunnel problems—such as scale effects, tunnel constraint effects ,
- (i) investigation of special devices (*e g* , the jet flap)

The only item of these I would like to comment on is the last I believe that in this country we are very backward when it comes to such devices Yet I am equally convinced that it is only by actual development of flight and performance aids that the helicopter will ever be a successful competitor in the transport field This does not mean indiscriminate frittering away of our development effort on every scheme that is put up—it so happens for example that I do not believe in the feasibility or the technical merits of the jet flap as applied to helicopters—but an acceptance of the principle that facilities should be made available for making proper assessments of novel schemes

CONCLUSION

I have tried in this lecture to bring home the fundamental truth that a present day Aerodynamics Office should not be spending all its time on

finding the maximum number of ways in which it is possible to present performance data. The true field of work is very wide and, if anything, is growing. I look forward to the day when the performance side is sufficiently standardised to develop pocket book nomograms and be relegated to a purely secondary role. That does not mean secondary from the customers point of view but from the viewpoint of the technician who is anxious to turn what tends to be a vehicle of questionable breed into a pure thoroughbred. The helicopter has now been quite some time growing up and the promised land in view at one time seems to be as illusory as ever. By concentrating greater effort on the aerodynamic handling aspects we can probably do much to offset the growing feeling that the helicopter has been oversold. In this country in particular the emphasis is turning away from helicopters to other forms of low speed aeroplanes. Such a change will be halted when a few successful British helicopters are eventually produced—and in the achievement of that desirable objective the helicopter aerodynamicist has an enormous part to play.

Finally, I should like to make the usual disclaimer. The word "I" has been used intentionally in a number of places since what I have said (and the opinions I have expressed) does not in any way represent the opinions of the Company on whose staff I serve. I wish to thank my colleagues for assistance received in the course of preparation of this paper and to thank the Fairey Aviation Company for making available information without which my task would have been impossible.

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NOTATION

SECTIONS 1 — 3

A	Lifting Surface Area πR
B	Tip loss factor
C_D	Drag Coefficient
C_L	Rotor Mean Lift Coefficient
CL_F	Lifting Surface Lift Coefficient, applying to the forward rotor of a tandem configuration
C_P	Power coefficient $P/\rho AV_T^3$
C_T	Thrust coefficient $T/\rho AV_T^2$
D_V	Vertical drag of fuselage
F	Increment of blade section drag coefficient due to compressibility
I	Rotational Inertia of Blade
M	Mach No
P_1	Induced Power Required
P_o	Rotor Parasite Power Required
P_p	Fuselage Parasite Power Required
P_c	Power Required for Climbing
R	Blade Radius
T	Rotor Thrust
V_C	Rate of Climb
V	Flight Speed
V_T	Tip Speed
W	A U W of Aircraft
W_F	Load on forward rotor of a tandem configuration
W_R	Load on rear rotor of a tandem configuration
a	speed of sound
b	number of blades
c	blade chord
e	Oswald wing efficiency factor
f	a function of the relative size and position of rotor and fuselage
h	height of rotor above fuselage
k_1	low A R correction factor
k_2	tandem rotor separation correction
r	radial position of blade element
s_1	wing semispan

v	blade section thickness
v_t	downwash velocity
v_{t_0}	ideal uniform downwash velocity (hovering)
x	ideal uniform downwash velocity (hovering) allowing for tip
z	r/R
	fraction of A U W supported by rotor in a wing/rotor combination
α	incidence of control plane
γ	angle of climb
δ	blade section drag coefficient
δ_0	blade section drag coefficient (zero lift)
ϵ_1, ϵ	index of variation of blade drag coefficient with Reynolds No
μ	tip speed ratio
ρ	air density
σ	blade solidity
σ_1	interference factor for wing/rotor combination
ψ	blade azimuth angle
Ω	Rotor angular velocity

SECTIONS 3 AND 4

A	Rolling Inertia of aircraft
$\bar{A}_1 \quad \bar{B}_1 \quad \bar{E}_1$	Functions of blade dimensions defined Ref 19
B_1	Cyclic pitch application
C	Yawing Inertia of aircraft
$F_1 - F_7$	Functions defined in Appendix 1
H_m	Manoeuvre margin
I	Pitching Inertia of aircraft
I_b	Blade Inertia
$L_v \quad L_p \quad L_r$	Rolling Moment derivatives
$M_u \quad M_w \quad M_q \quad M_{B_1}$	Pitching Moment derivatives
$N_v \quad N_p \quad N_r$	Yawing Moment derivatives
R	Blade Radius
\bar{R}	Roth's Discriminant
S	Centrifugal force on a blade
T_a	Thrust Derivative
V	Flight Speed
W	A U W of aircraft
W_b	Blade Wt
$X_u \quad X_w \quad X_q$	Longitudinal Force Derivatives
$Y_v \quad Y_p \quad Y_r$	Lateral Force Derivatives
$Z_w \quad Z_u \quad Z_q \quad Z_{B_1}$	Vertical Force Derivatives
a	Two dimensional $\frac{dC_L}{dz}$ for blade section
a_0	Coning angle
a_1	Longitudinal Disc Tilt
b_1	Lateral Disc Tilt
$a_{1q} \quad b_{1p}$	Derivatives of a_1 and b_1
eR	Hinge offset
f	Frequency of oscillation
l_t	Tail rotor arm
m	Mass of aircraft W/g
n	Load factor
p	Rolling angular velocity
q	Pitching angular velocity
r	Yawing angular velocity
u	Velocity along x — axis
w	Velocity along z — axis
α	Incidence of control plane
β	Flapping angle
γ	Locks Inertia Number
δ_3	Parameter of feathering due to flapping for skew or effectively skew hinges

ζ	Damping factor Appendix 1
θ_0	Collective Pitch Angle
λ	Inflow ratio
φ	Angle of bank
χ	Inclination of shaft to vert axis
ω_0	Natural frequency
Suffix d	denotes a small disturbance
t	tail rotor

APPENDIX 1

DISTURBANCE FUNCTIONS FOR DETERMINING BLADE FLAPPING

$$F_1 = - \left[\left(\frac{V_i}{V_T} - \lambda \right) (2\bar{B}_1 \mu \theta_0 \sin^2 \psi - \bar{B}_1 \lambda \sin \psi + 2\bar{B}_1 e_0 \theta \sin \psi + 2\bar{E}_1 \theta_0 \sin \psi) - \bar{B}_1 \mu^2 \sin \psi - \bar{B}_1 e \mu - \bar{E}_1 \mu + \frac{W_b}{V_T^2 R} \operatorname{sg} \sin \chi_0 \right] \quad (58)$$

$$F_2 = - \left[\frac{2I_b}{R^4} \sin \psi + \frac{2W_b}{R^2} \operatorname{se} \sin \psi - \bar{A}_1 \cos \psi - \bar{B}_1 e^2 \cos \psi - \bar{B}_1 \frac{\mu e}{2} \sin 2\psi - 2\bar{E}_1 e \cos \psi - \bar{E}_1 \frac{\mu}{2} \sin 2\psi \right] \quad (59)$$

$$F_3 = \left[\frac{I_b}{R_4} \cos \psi + \frac{W_b}{R^2} \operatorname{se} \cos \psi \right] \quad (60)$$

$$F_4 = - \left[2\bar{B}_1 \mu \theta_0 \sin^2 \psi + 2\bar{B}_1 e \theta_0 \sin \psi + 2\bar{E}_1 \theta_0 \sin \psi - \bar{B}_1 \lambda \sin \psi \right] \quad (61)$$

$$F_5 = \left[\bar{B}_1 \mu \sin \psi + \bar{B}_1 e + \bar{E}_1 \right] \quad (62)$$

$$F_6 = \left[W_b s / V_T R^2 \right]$$

$$F_7 = - \left[\bar{A}_1 + \bar{B}_1 e^2 + \bar{B}_1 \mu^2 \sin^2 \psi + 2\bar{B}_1 \mu e \sin \psi + 2\bar{E}_1 e + 2\bar{E}_1 \mu \sin \psi \right] \quad (64)$$

Note $R^4 \bar{A}_1 = \frac{1}{2} \rho a \int_{eR}^{BR} c (r - eR)^3 dr$

$$R^2 \bar{B}_1 = \frac{1}{2} \rho a \int_{eR}^{BR} c (r - eR) dr$$

$$R^3 \bar{E}_1 = \frac{1}{2} \rho a \int_{eR}^{BR} c (r - eR)^2 dr$$

APPENDIX 2

THE DYNAMIC LONGITUDINAL STABILITY CRITERION

Since for crisp control the response is essentially a rapid phenomenon, it may be assumed that the forward velocity is unchanged. Assuming the cyclic pitch which produces the response is B_1 and is instantaneously applied at time $t = 0$, the equations of motion are —

$$\frac{W}{g} (w - Vq) = Z_w w + Z_{B_1} B_1 \quad (65)$$

$$Iq = M_w w + M_q q + M_{B_1} B_1 \quad (66)$$

The characteristic equation is then

$$\frac{W}{g} I \lambda^2 - \lambda \left[\frac{W}{g} M_q + I Z_w \right] + \left[M_q Z_w - \frac{W}{g} V M_w \right] \quad (67)$$

This can be written in the form

$$\lambda^2 + 2 \zeta \omega_0 \lambda + \omega_0^2 = 0 \quad (68)$$

where ω_0 is the natural frequency at zero damping and ζ is the damping factor. By comparison

$$\omega_0^2 = \left(\frac{M_q Z_w g}{W I} - \frac{V M_w}{I} \right) \quad (69)$$

$$2 \zeta \omega_0 = - \left(\frac{M_q}{I} + \frac{Z_w g}{W} \right) \quad (70)$$

The solutions of the quadratic are

$$\lambda_1, \lambda_2 = [-\zeta \pm \sqrt{\zeta^2 - 1}] \omega_0 \quad (71)$$

Applying the Laplace transform

$$w^*, q^* = \int_0^\infty e^{-\lambda t} (w, q) dt \quad (72)$$

to the two equations of motion, the equations yield

$$w^* = \frac{B_1}{\lambda} \begin{vmatrix} Z_{B_1} & -\frac{W}{g} V \\ M_{B_1} & (I\lambda - M_q) \end{vmatrix} = \frac{B_1}{\lambda} \left[\frac{A_2 \lambda + B_2}{\lambda^2 + 2 \zeta \lambda \omega_0 + \omega_0^2} \right] \quad (73)$$

$$\begin{vmatrix} \left(\frac{W}{g} \lambda - Z_w \right) & -\frac{W}{g} V \\ -M_w & (I \lambda - M_q) \end{vmatrix}$$

where $A_2 = g Z_{B_1} / W$ (74)

and $B_2 = \left(\frac{M_{B_1} V}{I} - \frac{Z_{B_1} M_g g}{W I} \right)$ (75)

In the usual way, w^* may be written as

$$\frac{w^*}{B_1} = \frac{B_2}{\omega_o^2 \lambda} + \frac{A_2 \lambda_1 + B_2}{\lambda_1 (\lambda_1 - \lambda_2) (\lambda - \lambda_1)} + \frac{A_2 \lambda_2 + B_2}{\lambda_2 (\lambda_2 - \lambda_1) (\lambda - \lambda_2)} \quad (76)$$

so that inverting the transform

$$\frac{w}{B_1} = \frac{B^2}{\omega_o^2} + \frac{(A_2 \lambda_1 + B_2) e^{\lambda_1 t}}{\lambda_1 (\lambda_1 - \lambda_2)} + \frac{(A_2 \lambda_2 + B_2) e^{\lambda_2 t}}{\lambda_2 (\lambda_2 - \lambda_1)} \quad (77)$$

The only cases of interest are those for which the damping is positive, $i e$, for $\zeta \geq 0$

The solution is then

For $\zeta = 0$,

$$\frac{w}{B_1} = \frac{B_2}{\omega_o^2} + \left[\frac{A_2}{\omega_o} \sin \omega_o t - \frac{B_2}{\omega_o^2} \cos \omega_o t \right] \quad (78)$$

For $0 < \zeta < 1$,

$$\frac{w}{B_1} = \frac{B_2}{\omega_o^2} + \frac{e^{-\omega \zeta t}}{\sqrt{1 - \zeta^2}} \left[\frac{A_2}{\omega_o} \sin \omega_o t \sqrt{1 - \zeta^2} - \frac{B_2}{\omega_o^2} (\zeta \sin \omega_o t \sqrt{1 - \zeta^2} + \sqrt{1 - \zeta^2} \cos \omega_o t \sqrt{1 - \zeta^2}) \right] \quad (79)$$

For $1 < \zeta$,

$$\frac{w}{B_1} = \frac{B^2}{\omega_o^2} + \frac{e^{-\omega \zeta t}}{\sqrt{\zeta^2 - 1}} \left[\frac{A_2}{\omega_o} \sinh \omega_o t \sqrt{\zeta^2 - 1} - \frac{B_2}{\omega_o^2} (\zeta \sinh \omega_o t \sqrt{\zeta^2 - 1} + \sqrt{\zeta^2 - 1} \cosh \omega_o t \sqrt{\zeta^2 - 1}) \right] \quad (80)$$

From these results the growth of the normal acceleration can be evaluated Since the normal acceleration n is given by

$$n = \frac{Z_w w + Z_{B_1} B_1}{W} \quad (81)$$

it follows that there is an initial incremental value of n of magnitude

$\frac{Z_{B_1} B_1}{W}$ and thereafter the variation of n is dependent on w

The condition of $n = 0$ before 2 seconds is the condition that $w = 0$ within two seconds

The values of w and the time τ at which $w = 0$ are given by

For $\zeta = 0$,

$$\frac{w}{B_1} = -A_2 \omega_0 \sin \omega_0 t + B_2 \cos \omega_0 t \quad (82)$$

and $\tan \omega_0 \tau = B^2 / \omega_0 A_2$ (83)

For $0 < \zeta < 1$,

$$\frac{w}{B_1} = e^{-\omega \zeta t} \left\{ \begin{aligned} & \left[\frac{A_2 \omega_0 (2\zeta^2 - 1) - B_2 \zeta}{\sqrt{1 - \zeta^2}} \sin \omega_0 t \sqrt{1 - \zeta^2} \right. \\ & \left. - [2\zeta A_2 \omega_0 - B_2] \cos \omega_0 t \sqrt{1 - \zeta^2} \right] \end{aligned} \right\} \quad (84)$$

and $\tan \omega_0 t \sqrt{1 - \zeta^2} = \frac{(2\zeta A_2 \omega_0 - B_2) \sqrt{1 - \zeta^2}}{\omega_0 A_2 (2\zeta^2 - 1) - B_2 \zeta}$ (85)

For $1 < \zeta$,

$$\frac{w}{B_1} = e^{-\omega \zeta t} \left\{ \begin{aligned} & \left[\frac{A_2 \omega_0 (2\zeta^2 - 1) - B_2 \zeta}{\sqrt{\zeta^2 - 1}} \sinh \omega_0 t \sqrt{\zeta^2 - 1} \right. \\ & \left. - [2\zeta A_2 \omega_0 - B_2] \cosh \omega_0 t \sqrt{\zeta^2 - 1} \right] \end{aligned} \right\} \quad (86)$$

and $\tanh \omega_0 t \sqrt{\zeta^2 - 1} = \frac{(2\zeta A_2 \omega_0 - B_2) \sqrt{\zeta^2 - 1}}{\omega_0 A_2 (2\zeta^2 - 1) - B_2 \zeta}$ (87)

With the aid of these equations, compliance with the two second criterion can be checked from computed derivatives

APPENDIX 3

THE RESPONSE TO A STEP DISTURBANCE LASTING A FINITE TIME

The same methods of analysis can be adopted for this type of input as for a simple step input. It is simpler however to use the previous results and use the fact that the equations are linear in form. It follows from this

last fact that if the response to an input B_1 at $t = 0$ is given by $w(t)$ the response to a cancelling input $-B_1$ at $t = T$ is given for $t > T$ by $-w(t-T)$. Thus for the boundary conditions $t = 0, w = 0, \dot{q} = 0$, and for an input B_1 for $t < T$, and zero input for $t > T$ the velocity disturbance w is given by

For $\zeta = 0$,

$$\frac{w}{B_1} = \left\{ \frac{A_2}{\omega_0} [\sin \omega_0 t - \sin \omega_0 (t - T)] - \frac{B_2}{\omega_0^2} [\cos \omega_0 t - \cos \omega_0 (t - T)] \right\} \quad (88)$$

For $0 < \zeta < 1$,

$$\frac{w}{B_1} = \frac{e^{-\omega \zeta t}}{\sqrt{1 - \zeta^2}} \left\{ \frac{A_2}{\omega_0} [\sin \omega_0 t \sqrt{1 - \zeta^2} - e^{-\omega \zeta \Gamma} \sin \omega_0 (t - \Gamma) \sqrt{1 - \zeta^2}] - \frac{B_2}{\omega_0^2} [\cos \omega_0 t \sqrt{1 - \zeta^2} - e^{-\omega \zeta T} \cos \omega_0 (t - T) \sqrt{1 - \zeta^2}] \sqrt{1 - \zeta^2} - \frac{B_2 \zeta}{\omega_0^2} [\sin \omega_0 t \sqrt{1 - \zeta^2} - e^{-\omega \zeta T} \sin \omega_0 (t - T) \sqrt{1 - \zeta^2}] \right\} \quad (89)$$

For $1 < \zeta$,

$$\frac{w}{B_1} = \frac{e^{-\omega \zeta t}}{\sqrt{\zeta^2 - 1}} \left\{ \frac{A_2}{\omega_0} [\sinh \omega_0 t \sqrt{\zeta^2 - 1} - e^{-\omega \zeta \Gamma} \sinh \omega_0 (t - \Gamma) \sqrt{\zeta^2 - 1}] - \frac{B_2}{\omega_0^2} [\cosh \omega_0 t \sqrt{\zeta^2 - 1} - e^{-\omega \zeta T} \cosh \omega_0 (t - T) \sqrt{\zeta^2 - 1}] \sqrt{\zeta^2 - 1} - \frac{B_2 \zeta}{\omega_0^2} [\sinh \omega_0 t \sqrt{\zeta^2 - 1} - e^{-\omega \zeta T} \sinh \omega_0 (t - T) \sqrt{\zeta^2 - 1}] \right\} \quad (90)$$

the g 's developed at times $t > T$ are given by

$$n = \frac{Z_w w}{W} \quad (91)$$

These relations enable the amplitude of g 's achieved to be evaluated. The period is, of course, the same as for the simple step input, in the absence of damping it is $\frac{2\pi}{\omega_0}$ and with damping it is $2\pi/\omega_0 \sqrt{1 - \zeta^2}$.

Taking the case given by O'Hara where $\frac{\omega_0}{2\pi} = 0.127$ the period with

damping is $\frac{1}{0.127 \sqrt{1-\zeta^2}}$ giving for $\zeta = 0.98$, say, a period of 40 seconds while for $\zeta = 0.5$ say, the period becomes 9 seconds. Unfortunately it is unlikely that the original assumption of forward speed constant will be valid over times of this magnitude. Thus the analysis above is not likely to give exact answers but only a guide to the response behaviour of the helicopter.

The use of the full equations of motion for the analysis of the step type of control input is obviously more complicated, but the method of Appendix 2 can be used to give the corresponding result and this in turn, by using the linearity of the equations of motion can be modified to give more accurate results for the conventional "tooth" input considered here, thereby giving more accurate response characteristics and handling data than the simplified equations above.

Discussion

Mr F O'Hara (*Royal Aircraft Establishment*) (*Member*), said that they must all have been impressed by Dr ROBERTS' account of the problems which have to be faced by a worker in the aerodynamics office of a helicopter firm. As one who had done a certain amount of work on research aerodynamics, in which one dealt with a general theoretical analysis, he sympathised with the need for specific answers to be given and for quantities to be fed into a given design. At one stage of Dr Roberts' Paper, however, he had wondered what was the exact function of the aerodynamics office in the process of the design of the helicopter. Dr Roberts had commented on a method of ground effect analysis by Dr Cheesman and had said that reasonable estimates of ground effect could be obtained by a theory provided by Dr Cheesman. It would appear to him that this was something which an aerodynamicist would elect to have, but Dr Roberts considered that he would prefer to have a fundamentally more satisfactory theory. The requisite object, Mr O'HARA considered, was to supply the design aerodynamicist with a theory which gave the right answer. He did not think Dr Roberts could ask for more.

With regard to the theory of stability, Dr Roberts had given an account of a standard type of classic theory and the more recent developments of that theory in relation to the helicopter, but he had raised questions about the appropriate values of certain concepts in that theory, such as the manoeuvre margin in relation to helicopters. Mr O'HARA said he would like rather to learn from Dr Roberts whether the aerodynamics design office found it possible to get reasonably reliable estimates of manoeuvre margin and whether he felt that on the designs with which he had been concerned he could achieve the value he quoted as desirable. These were points on which information from design aerodynamicists could be particularly interesting.

On the question of evaluating low speed performance, Dr Roberts had referred to Oliver's method by which the ratio of induced power was said to be 1.2. This was not inconsistent with the figure which Dr Roberts had quoted, because in Oliver's analysis separate account was not taken of the fuselage vertical drag. This resulted in an apparent reduction of the efficiency of the rotor, which showed up in a larger ratio between the practical and the theoretical values of the induced velocities.

Commenting on the stability and the handling qualities of a compound type rotor-wing helicopter in which one had a certain amount of control over the distribution of lift between the rotor and the wing, Mr O'HARA suggested this would lead to something in the nature of an additional major control, and was a point on which one had to be careful. It was desirable in flying machines to limit the number of controls. Some of the basic handling difficulties in helicopters arose from the necessity for a collective pitch control in addition to the main control column. The variation of the distribution of lift between the rotor and the wing had to be considered very carefully. He considered that it was probably desirable to work as far as possible with the optimum arrangement for significant flight conditions, removing the factor of lift distribution from the control of the pilot. This was particularly important in