Science in Context (2022), **35**, 395–413
doi:10.1017/S0269889724000103

RESEARCH ARTICLE

Brouwer and Hausdorff: On reassessing the foundations crisis

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Abstract
Epistemological issues associated with Cantorian set theory were at the center of the foundational debates from 1900 onward. Hermann Weyl, as a central actor, saw this as a smoldering crisis that burst into flames after World War I. The historian Herbert Mehrtens argued that this "foundations crisis" was part of a larger conflict that pitted moderns, led by David Hilbert, against various counter-moderns, who opposed the promotion of set theory and trends toward abstract theories. Among counter-moderns, L.E.J. Brouwer went a step further by proposing new foundational principles based on his philosophy of intuitionism. Meanwhile, Felix Hausdorff emerged as a leading proponent of the new modern style. In this essay, I offer a reassessment of the foundations crisis that stresses the marginal importance of the various intellectual issues involved. Instead, I offer an interpretation that focuses on tensions within the German mathematical community that led to a dramatic power struggle for control of the journal *Mathematische Annalen*.

Keywords: Cantorian set theory; German mathematics journals; Weimar politics; mathematical power struggles; foundations crisis

Introduction

In 1992, Herbert Mehrtens wrote a short retrospective essay that took up historiographical issues he had addressed some fifteen years earlier in connection with T.S. Kuhn's views on scientific revolutions and their relevance for the history of mathematics (Mehrtens 1992). For those unfamiliar with Mehrtens' work, this essay is well worth reading, partly to gain a sense of his voice as a writer and reflective thinker, with an eye for large issues only rarely addressed in historical studies devoted to mathematics. Some of the contributions to this volume will likely take up some of those larger themes, whereas my aim here is more limited. By drawing on more recent historiography, I will try to throw fresh light on the "foundations crisis" of the 1920s, a famous topic that Mehrtens wrote about in the 1980s (Mehrtens 1984, 1987), but only touched on in his influential *Moderne—Sprache—Mathematik* (*M-S-M*) (Mehrtens 1990). The recently published collection of essays in *The Richness of the History and Philosophy of Mathematics* (Chemla et al. 2023) reveal how the larger issues raised in *M-S-M* have continued to spawn further reflection up to the present day.

My approach owes more to Mehrtens' earlier work, in which he showed how one could reach a better understanding of the foundations conflict during the 1920s by shifting the focus away from the fundamental issues that separated David Hilbert's formalist position from L.E.J. Brouwer's intuitionism. The latter issues were, to be sure, of great interest to the central players who worked on foundational problems, a field that attracted several brilliant young mathematicians. Still, Mehrtens argued that the larger context of circumstances and events had to be taken into account in order to understand how the "foundations crisis" became such a lightning rod at the time, particularly within the German mathematical community.

In *M-S-M*, Herbert Mehrtens put a strong accent on axiomatic set theory as lying at the very heart of modern trends, taking Hilbert and Ernst Zermelo as two key protagonists (on this and related matters, see Rowe 1997). Central to his interpretation was Hilbert's view of Cantorian set theory as representing an ideology of freedom. Those who opposed its principles on foundational grounds (Leopold Kronecker, Brouwer, and Hermann Weyl) belonged to what he called the counter-modern camp. Mathematical knowledge, for them, was rooted in something other than the human capacity to invent formal systems with specified operational rules. Mehrtens traced this tension back to the mid-nineteenth century, when Bernhard Riemann advocated methods in complex function theory that Karl Weierstrass rejected as illegitimate. Thus, the general terms of this conflict were familiar ones within the context of norms for research and mathematical theorizing. For leading practitioners in the German mathematical community, these battle lines hardened over time, but only during the foundations crisis of the 1920s did they spill over into open warfare.

In describing mathematical modernism and its discontents, Mehrtens placed Felix Hausdorff and L.E.J. Brouwer on opposite ends of this ideological spectrum. Hausdorff, as a follower of Friedrich Nietzsche and a foremost champion of Georg Cantor's *Mengenlehre*, fully embraced modernist ideals and methods, including Zermelo's controversial axiom of choice (Moore 1982). Several leading French analysts had objected to the latter principle, including Henri Poincaré (Ferreirós 2007, 311–320). The Dutch topologist Brouwer went beyond them, though, by founding a new brand of intuitionism that became the focus of the famous foundational debates of the 1920s.[1] Hausdorff played no active role in the ensuing controversy, though today his negative views with regard to intuitionism are well known (see his comments to Pavel Alexandrov and Adolf Abraham Fraenkel, cited below).

In Chapter 2 of *M-S-M,* Mehrtens bracketed Hilbert, Cantor, Zermelo, and Hausdorff as the foremost representatives of Cantorian set theory, which he presented as the cutting-edge of modern mathematics. Brouwer surfaced here in counterpoint to Hausdorff, but again in Chapter 3 as the last of four counter-moderns (the other three being Kronecker, Felix Klein, and Poincaré). As I will emphasize below, however, Hausdorff's interests in and contributions to Cantorian set theory differed fundamentally from those of Hilbert and Zermelo. In the present account, I will largely ignore some of the key players, including Hermann Weyl and Paul Bernays (discussed in detail in Mancosu 1998a and 1998b), in order to focus attention on overlooked factors connected with these controversies.[2] In particular, I will argue that the sharp polarization that divided the German mathematical community after the Great War had little to do with the ongoing foundational debates. A far more important factor was the longstanding rivalry between Göttingen and Berlin, which took on strong political overtones during the postwar era (Biermann 1988; Rowe 2004, 2008).

Chapter 5 of *M-S-M* takes up various strands of the modernization process in German mathematics circa 1900. Mehrtens' thumbnail sketch of these developments nicely captures the state of the profession during an era when the German educational system was still adapting to the demands of an industrialized society. His account crystallizes with a portrait of Göttingen mathematics under Klein and Hilbert, featuring the inherent conflict between Klein's praxis-oriented program and Hilbert's liberal appeal to higher mathematical ideals. He points to the important role played by Friedrich Althoff, the autocratic Prussian ministerial official who supported Klein's plan to make Göttingen the dominant center for the mathematical sciences in Germany. Hilbert had numerous opportunities to strike out on his own—he turned down attractive outside offers, including two from Berlin—but chose instead to remain in Göttingen, where he attracted throngs of talented students after 1900. Mehrtens rightly emphasizes that a large part of this success story hinged upon Hilbert's willingness to accommodate himself to

[1]For a thoughtful reassessment of Brouwer's philosophy as it relates to modernity and modernism, see Ferreirós 2023.
[2]For an interesting comparative assessment of the views of Weyl and Hausdorff, see Scholz 2023.

https://doi.org/10.1017/S0269889724000103 Published online by Cambridge University Press

Klein's interests in promoting less highbrow plans for applied mathematics and mathematics education (Rowe 1989).

A major locus of power and influence for Göttingen mathematicians, particularly those who moved in the circles around Klein and Hilbert, was the journal *Mathematische Annalen*. Until the 1920s, when economic problems led the publisher Ferdinand Springer to salvage it, the *Annalen* had long been the leading mathematics journal in Germany, having decades earlier surpassed *Crelle's Journal*, which lay in the hands of mathematicians connected with Berlin (Rowe 2008).[3] The *Annalen*'s dominance ended after World War I, however. In the meantime, the Berlin publisher Springer had launched a new journal, *Mathematische Zeitschrift*, which soon became the preferred venue among younger mathematicians. It featured a multi-layered editorial board, headed by the analyst Leon Lichtenstein[4] together with three other Berliners—Erhard Schmidt, Issai Schur, and Konrad Knopp—and supported by an advisory board of nine distinguished mathematicians. *Mathematische Annalen* also expanded its editorial board when it became a Springer journal, but without fundamentally changing its operational procedures, which continued much as before.

After 1901, when Klein stepped down as editor-in-chief of the *Annalen*, he was succeeded by Hilbert, Klein's handpicked younger colleague. Actually, Klein never ceded the reins of power completely; he would remain one of the *Annalen's* principal editors up until the year before his death in 1925. These curious circumstances reflect an implicit acknowledgement that *Mathematische Annalen* was "Klein's journal," even if no one referred to it by that name. Hilbert's appointment essentially meant that he was destined to inherit the *Annalen* from Klein. Five years later, a second phase in this transition took place when Hilbert's first doctoral student, Otto Blumenthal, became its managing editor, a position he would hold for over three decades. Like Hilbert, Blumenthal was an internationalist who cultivated alliances with mathematicians throughout Europe; he worked especially closely with Brouwer (Rowe 2018b, 241–305; Rowe and Felsch 2019, 107–150, 165–335). In 1914, Blumenthal persuaded Hilbert to expand the *Annalen's* editorial board by inviting Brouwer and Constantin Carathéodory to become associate editors. At the time, no one anticipated the kinds of complications Brouwer's appointment would eventually create.

These new developments in German mathematical publishing played a decisive role in shaping the events of the 1920s (Remmert and Schneider 2010), including the famous power struggle between Brouwer and Hilbert (Rowe and Felsch 2019). Historians have noted that the "foundations crisis" remained simmering on a back burner for many years following an initial stage of controversy during the first decade of the century when the antinomies of set theory first surfaced.[5] While many viewed Cantor's theory with skepticism, very few mathematicians thought that mathematics itself was in a state of disarray. Hermann Weyl famously made that very claim in his polemical essay (Weyl [1921] 1998), when he boldly announced the "new foundations crisis" while further proclaiming that Brouwer's intuitionism was "the revolution" that stood ready to create a new order. Weyl had promised Brouwer to write just such a text, and when he sent it to him in May 1920, he called it a "propaganda pamphlet" designed to "rouse the sleepers" (Van Dalen 2013, 311). Weyl's desertion to the intuitionist camp quickly brought the long-simmering foundational conflict to a full boil, turning what had been a methodological dispute into a full-fledged power struggle. As a member of the advisory board of *Mathematische Zeitschrift*, Weyl took the opportunity to submit his paper to this new journal. He surely never imagined sending it

[3]On its founding and ensuing competition with *Crelle's Journal*, see Tobies and Rowe 1990, 28–46 and Rowe 2018b, 37–46.

[4]Lichtenstein was a Polish Jew who studied in Berlin, where he was employed as an engineer at Siemens during the war. In 1922 he gained a full professorship in Leipzig, where he taught until his death from a heart attack in 1933. Konrad Knopp afterward succeeded him as editor of *Mathematische Zeitschrift*.

[5]Cantor himself had been aware of certain logical problems that arise unless restrictions apply to the formation of sets (Ferreirós 2007, 290–296). Such problems were largely ignored, however, until 1902, when Bertrand Russell informed Gottlob Frege of the antinomy known today as Russell's paradox (see the discussion below).

https://doi.org/10.1017/S0269889724000103 Published online by Cambridge University Press

to Blumenthal, who still held out hopes that a civil exchange of opinions on foundational matters might appear in the pages of *Mathematische Annalen*.

Since the time of Mehrtens' studies a great deal of wide-ranging scholarship, both on Hausdorff's work and on Brouwer's, has appeared. In the case of Hausdorff, his diverse talents are now on full display in the ten volumes of his *Gesammelte Werke*.[6] As part of this massive project, the late Egbert Brieskorn and Walter Purkert have published two detailed accounts of his life and work: the lengthier version (Brieskorn and Purkert 2018) is Volume IB in the Hausdorff edition, whereas their abridged biography (Brieskorn and Purkert 2021) appears in the series *Mathematik im Kontext*. The latter work served as the basis for the recent English translation (Brieskorn and Purkert 2024). Brouwer's collected works appeared long ago in two volumes: the first on his philosophical and foundational writings (Heyting 1975) and the second containing his even more influential papers on topology (Freudenthal 1976). The latter includes Brouwer's important work on dimension theory (for which see also Johnson 1979, 1981). More recently, Dirk van Dalen has documented the broader scope of Brouwer's life and interests in Van Dalen (2013), and Dennis Hesseling undertook a careful study of the reception of intuitionism during the 1920s in Hesseling (2003). Two of Brouwer's important unpublished fragmentary works can be found in Van Dalen and Rowe (2020).

These and other sources now make it possible to form a far more precise picture of the activities and motivations of Hausdorff and Brouwer, two towering figures in the early history of modern topology. Herbert Mehrtens focused almost exclusively on their philosophical views, and quite correctly saw them as holding fundamentally opposed positions. A key question that remains, though, is the significance of foundational issues and debates for understanding the emergence of modern mathematics. Jeremy Gray addressed larger issues connected with modernity in Gray (2008), and more recently Leo Corry has pinpointed strong linkages between modernist trends in art and mathematics in Vienna (Corry 2023). My more limited ambition here is to indicate the respective places of Hausdorff and Brouwer in the context of the controversies that raged within the German mathematical community during the 1920s.

Hilbert and Hausdorff as Exponents of Cantorian Mathematics

In reconsidering Hilbert's place in the history of Cantorian set theory, one should first note that his own research had few direct connections with Cantor's work. Hilbert's main contribution came by way of promoting Cantor's theory in his famous lecture at the Second International Congress (ICM) (Hilbert [1900] 1935). There, he highlighted Cantor's Continuum Problem along with the well-ordering of the real numbers as the first of his 23 problems. At the same time, in the second problem he pointed to the possibility of legitimizing the existence of Cantor's *consistent* infinite sets by placing his theory on firm axiomatic foundations. In fact, Hilbert's brief remarks contained a clear allusion to what we today call Cantor's Paradox, though few took notice of this at the time. What Hilbert left unsaid on that occasion and why are questions well worth pondering, as pointed out in Rowe (2023).

For Hilbert, allying himself so prominently with Cantor's theory was a convenient way to attack their common nemesis: Leopold Kronecker. However, his ensuing efforts to simultaneously establish a foundation for arithmetic as well as for set theory ran aground. In the meantime, though, Ernst Zermelo joined forces with him, and the latter soon emerged as Göttingen's leading expert on Cantorian set theory (Peckhaus 1990). Zermelo and others began to realize that one needed to avoid two types of paradoxes: those involving collections of massive size (like the collection of all sets), and others that arise from semantic ambiguity ("this statement is a lie": true, false, or neither?). Cantor had informed both Hilbert and Dedekind about the former type of

[6]This massive project, involving some seventeen editors, began in the mid-1990s and ended with the publication of Volume 6 in 2021.

https://doi.org/10.1017/S0269889724000103 Published online by Cambridge University Press

antinomy, which played no role in public debates for some years. The latter type of difficulty surfaced in 1902 when Bertrand Russell wrote to Gottlob Frege, who then publicized Russell's Paradox and its devastating implications for his attempt to derive arithmetic from logic. When Frege informed Hilbert of this problem, the latter answered in a letter from November 1903 that Zermelo had found the same paradox three to four years earlier. He then added: "I found other even more convincing contradictions as long as 4–5 years ago; they led me to the conviction that traditional logic is inadequate and that the theory of concept-formation [*Begriffsbildung*] needs to be sharpened" (Kanamori 2004, 490).[7]

One year later, at the Third ICM in Heidelberg, Hilbert sketched a general plan for moving forward with a reformed logic. In his lecture, he began by characterizing various failed efforts to derive arithmetic and to place the real number continuum on a rigorous foundation before describing what he had in mind. Hilbert credited Frege, Dedekind, and Cantor with having made important advances that ultimately floundered in the face of the logical paradoxes. He then remarked that

> In the traditional exposition of the laws of logic certain fundamental arithmetical notions are already used, for example, the notion of set, and to some extent, also that of number. Thus, we find ourselves turning in a circle, and that is why a partly simultaneous development of the laws of logic and of arithmetic is required if paradoxes are to be avoided. (Hilbert 1905, 177)

As pointed out in Dreben and Kanamori (1997, 87), this paper was the only publication on set theory that Hilbert wrote when still in his mathematical prime, though it prefigured his later work in metamathematics and his finitist viewpoint. Between 1905 and 1917, Hilbert turned to entirely new fields of research, including mathematical physics (Corry 2004).

Cantor's theory was again in the limelight at the Heidelberg ICM, but not so much due to Hilbert's presentation. In fact, a previous speaker in the same session, the Hungarian Julius König, managed to upstage him by delivering a sensational mathematical lecture in which he proved (or so it seemed) that the cardinality of the continuum could not be an aleph. Since Cantor's theory demanded that *every* cardinal number had to be an aleph, this result—had it been correct—would have spelled doom for his entire enterprise. Cantor, Hilbert, Zermelo, Hausdorff, and Max Dehn all heard König's lecture, so throughout the congress and even afterward they tried to scrutinize the soundness of his argument. After returning to Leipzig, Hausdorff discovered the source of the error, which arose from an unproven theorem in Felix Bernstein's dissertation (Purkert 2015).

Bernstein had originally studied under Cantor in Halle, but he then left for Göttingen, where he wrote his dissertation under Hilbert. Neither Hilbert nor Zermelo, however, had noticed this error in Bernstein's *Doktorarbeit*. On 29 September 1904, Hausdorff wrote to Hilbert, informing him that in light of Bernstein's oversight he was inclined

> to regard König's proof as faulty and König's proposition as highly improbable. On the other hand, you will scarcely have received the impression that Cantor has found, in the last weeks, what he has sought in vain for 30 years. Thus, your Problem No. 1 appears, after the Heidelberg Congress, to stand precisely where you left it at the Paris Congress. (Translated in Purkert 2015, 16; for the entire letter, see Hausdorff 2012, 330)

Hausdorff then sent a short note to the *Jahresbericht* of the German Mathematical Society, in which he proved his famous recursion formula for aleph exponentiation. At the end of this note, he pointed out that the corresponding formula in Bernstein's dissertation was still unproved. Moreover, "its correctness appears all the more problematic because from it, as J. König has

[7]On Hilbert's interest in these matters, see Peckhaus and Kahle 2002.

shown, would follow the paradoxical result that the power of the continuum is not an aleph and that cardinal numbers exist that exceed every aleph" (Purkert 2015, 14).

Hausdorff was still a young private lecturer in Leipzig at this time. Few in Göttingen knew him, and those who had heard of his work thought of him as an expert on optical astronomy. Hilbert, however, was an exception; in fact, Hausdorff had already pointed out some weaknesses in his axioms for the foundations of geometry in 1900 (Hausdorff 2012, 327–328). Hausdorff's letter, informing Hilbert of the flaw in his student's dissertation, surely made a strong impression on Bernstein's *Doktorvater*. In the meantime, Zermelo had independently discovered the same mistake in König's argument, and with the help of Erhard Schmidt, he found a general method for proving the well-ordering theorem. He wrote up his proof in a letter to Hilbert, sent only a few days before Hausdorff's letter arrived in Göttingen. This new result, in fact, dramatically changed the status of the first Paris problem.

In his Paris lecture, Hilbert spoke about well-ordering in the key case of the real number continuum. At that time, he formulated this by saying that it would be most desirable "to obtain a direct proof of this remarkable statement of Cantor's, for instance by giving an actual arrangement of [all real] numbers such that in every partial system a first number can be identified" (Hilbert [1900] 1935, 299). This formulation suggests that in 1900 Hilbert had little inkling of the real difficulties involved. Four years later, he enthusiastically embraced Zermelo's proof of the general well-ordering theorem by means of the axiom of choice. Since Zermelo's letter took up only three pages, Hilbert immediately inserted it into the next issue of *Mathematische Annalen*. There had been plenty of rumblings of dissatisfaction prior to this, but Zermelo's paper truly opened the floodgates of controversy regarding Cantorian set theory, leading to various efforts to secure, modify, or dismiss its main results (Moore 1982). It also led to the recognition that an actual construction of a well-ordering of the real numbers, as Hilbert originally envisioned this, was in all likelihood impossible.

Foundational issues, in particular the axiomatization of arithmetic and set theory, were the central concerns of Hilbert and Zermelo, whereas Hausdorff had his eyes set on developing Cantor's theory in new directions. This meant accepting the basic premises of so-called naïve set theory, which involved radically new concepts for various orders of infinity. Although sympathetic with axiomatization, Hausdorff probably thought that set theory was not yet ripe for firm foundations. If so, he was unknowingly following Hilbert's own inclinations when the latter spoke of the mathematician as an architect who bore responsibility for both the building and its fundament. In his Paris lecture, he remarked, "only that architect is capable of laying a sure foundation for a building who knows the structure of the building itself thoroughly and in detail" (Hilbert [1900] 1935, 308). Five years later, he noted that the mathematical architect first erects living quarters *before* turning to the foundations. "That," he claimed, "is not a deficiency, but rather the correct and healthy development" (Peckhaus 1990, 51).

Hausdorff had stumbled into Cantor's theory during his philosophical phase, while reading Nietzsche and contemplating his argument for eternal return (Brieskorn and Purkert 2021, 176–180). Already in 1901, he began publishing a series of papers on ordered sets in the *Leipziger Berichte*, described by Arthur Schoenflies in his second report on the theory of point sets for the German Mathematical Society (DMV) (Schoenflies 1908). The first part, (Schoenflies 1900), was initiated when he was still on the Göttingen faculty (he left to take a full professorship in Königsberg in 1899). Mehrtens, following the lead of Hermann Minkowski, identified Hilbert as the "general director" of a modern movement, and noted that Zermelo was certainly one of its prominent members. Yet Schoenflies was not only a leading Cantorian from the older generation, he was also one of Klein's and Hilbert's closest allies.[8] Moreover, he emerges as a central figure for understanding the work of both Hausdorff and Brouwer. As Hans Freudenthal pointed out in his

[8]On Hilbert's interactions with Schoenflies when he was working on his first report on set theory, see Meschkowski and Nilson 1991, 401–404.

https://doi.org/10.1017/S0269889724000103 Published online by Cambridge University Press

commentary for the *Collected Works* (Freudenthal 1976), Brouwer's early work in topology took the form of a sweeping critique of the foundations of what he called "Cantor-Schoenflies topology" (Van Dalen 2013, 143).

Not until 1910 did Felix Hausdorff finally attain his first regular academic appointment. At the age of forty-one, he succeeded Gerhard Kowalewski as associate professor at Bonn University. During the following years, he published very little, as he devoted most of his spare time to writing the work that would assure him a permanent place in the annals of mathematics, his nearly 500-page monograph *Grundzüge der Mengenlehre* (Hausdorff [1914] 2002). By the time he completed it in 1914, Hausdorff had moved on to become full professor of mathematics in Greifswald. Seven years later, Eduard Study succeeded in arranging a renewed appointment in Bonn, where Hausdorff taught up until his retirement in 1935.

Although allied intellectually with Hilbert, Hausdorff was never part of the Göttingen power network. Throughout the 1920s, he published nearly exclusively in *Mathematische Zeitschrift* rather than Hilbert's *Annalen*, and during the 1930s, his preferred outlet was the Polish journal *Fundamenta Mathematicae*.

Hausdorff's early studies built on Cantor's work and were still in the tradition of naïve set theory. His later work, however, which was far more influential, focused on general topology and its applications, but also avoided axiomatic set theory and modern logic. Initially, Hausdorff apparently had no plans to place his work on ordered sets before a larger mathematical public until Cantor approached him directly with this suggestion. He then proceeded cautiously, writing to Hilbert on 15 July 1907:

> Professor Cantor, with whom I spent some time together 14 days ago, suggested that I work out a brief synopsis of my "Investigations on Order Types" and offer this to you for publication in Mathematische Annalen. He thought it desirable that these matters reach a wider audience than the readership of the Leipziger Berichte [a publication of the Scientific Society of Saxony] . . . .
>
> I allow myself thus to ask whether you would be inclined in principle to accept a paper for the Annalen, entitled say "Theory of Order Types" and with a length of between 16 and 24 pages. You may find the suggestion of casting a vote on a paper that has yet to be written somewhat premature. Of course, this would be a statement in principle, which does not obligate you in any way but rather reserves your right to judge the actual work at hand. It is only that I would prefer to save myself the trouble if, for example, the editorial board of the Annalen were inclined to exclude the field of set theory, which is now so widely contested (and with such medieval weapons!). (Hausdorff 2012, 332–333)

Hausdorff was also eager to know whether the backlog of other submissions might lead to a lengthy delay in publication. His last remark with the allusion to medieval warfare—probably with Poincaré in mind—sheds telling light on the uncertain status and controversial position of set theory as a mathematical sub-discipline at this time. Hausdorff ended with a provocative formulation that surely appealed to Hilbert, expressing his hope "that you . . . regard 'Cantorismus' as still somewhat alive, despite Poincaré's declaration of death, and that you will not fail to take interest in a work that adds something new to set theory" (ibid.).

Hilbert gave his assent, which led to "The Fundamentals of a Theory of Ordered Sets" (Hausdorff 1908), his first publication in *Mathematische Annalen.* In the preface, we encounter one of the few instances in which Hausdorff offered a public pronouncement on the controversies surrounding set theory. Here he noted that his study tacitly adopted a clear stance with regard to three different types of criticisms directed against Cantor's theory: 1) those that point to its lack of clarity and axiomatic basis; 2) criticisms affecting mathematics as a whole; and 3) arguments of a scholastic nature that "cling on to words" (Plotkin 2005, 198). In response to the first of these, Hausdorff simply wrote that Cantorians would eventually reach an understanding with regard to axiomatic principles. As for the second type of criticism, he felt assured that one could ignore such sweeping concerns, which were quite unlike the third type; these pseudo-arguments deserved to be harshly condemned.

Jacob M. Plotkin, who translated Hausdorff's papers on ordered sets into English in 2005, distinguished between these contributions and the works of Zermelo. He called Zermelo the "father of abstract set theory," at the same time noting how "Hausdorff eschewed foundations and pursued set theory as part of the mathematical arsenal" (Plotkin 2005, xv) One finds clear confirmation of this in Hausdorff's letter to Adolf Abraham Fraenkel, written on 9 June 1924. This letter responded to the second edition of Fraenkel's monograph on axiomatic set theory (Fraenkel 1923), published in Courant's "yellow series" (Remmert 2008). Hausdorff was then preparing the second edition of his own book (Hausdorff [1927] 2008), and he thanked Fraenkel profusely for having spared him the trouble of writing about matters like axiomatization and the antinomies of set theory, in which he was less interested. His praise extended to Fraenkel's "essential improvement of Zermelo's Axiom of Comprehension," a weak point which Brouwer had attacked in (Brouwer [1919] 1998). Hausdorff then added:

> You have even succeeded in making the oracle pronouncements of Brouwer and Weyl understandable—without making them appear to me any less nonsensical! You and Hilbert both treat intuitionism with too much respect; one must for once bring out heavier weapons against the senseless destructive anger of these mathematical Bolsheviks! (Hausdorff 2012, 293)

Brouwer vs. Schoenflies vs. Hausdorff

Generational conflict animated many cultural spheres in Weimar era Germany, and mathematics was no exception. Arthur Schoenflies, who was eight years younger than Georg Cantor, stood in the vanguard of Cantorian set theory. He was nearly a decade older than Hilbert, who was six years older than Hausdorff. Brouwer, on the other hand, represented a younger generation: he was not yet thirty years old when he burst onto the scene in 1910, full of ambitious plans. Geometric topology[9] was a notoriously difficult field, still very much in its infancy, and Brouwer's contemporaries considered his papers as the ultimate in rigor. His critiques of Schoenflies' publications reinforced that impression.

Throughout the 1920s, Hausdorff's reputation and fame only grew, especially among the younger generation of Polish and Russian topologists, for whom his *Grundzüge der Mengenlehre* became the bible of general topology (Purkert 2002, 55–67). In February 1914, Hausdorff wrote to tell Hilbert he would soon be sending him a copy of his new *magnum opus*. In passing, he noted with regret that Schoenflies had in the meantime published a new version of his original report from 1900 for the DMV (Schoenflies 1913). This comment, though probably not intended ironically, certainly reads as such today, as the *Grundzüge* would very soon render Schoenflies' new *Bericht* obsolete. Hausdorff characterized it as "all the same better than the first edition" (Hausdorff 2012, 338–339). He could hardly have known that Egbertus Brouwer was thickly involved in editing this text. Moreover, Brouwer's involvement led to some very aggravating negotiations with Schoenflies, who enjoyed only limited support from Hilbert (Van Dalen 2013, 225–230).

This situation was, in fact, ironic in a number of ways. Schoenflies had been a longtime protégé of Klein in Göttingen, where he taught Max Dehn during the late 1890s. In fact, Dehn almost

[9]Brouwer studied the topological properties of objects in the Euclidean plane or spaces of higher finite dimension, a field later called geometric topology. These studies were thus more specialized than Hausdorff's *Grundzüge*, which introduced the theory of general topological spaces; however, his second edition (Hausdorff 1927) was restricted to general metric spaces.

https://doi.org/10.1017/S0269889724000103 Published online by Cambridge University Press

surely attended the *Proseminar* Schoenflies taught on set theory during the summer semester of 1898. Yet Dehn, an influential figure in the realm of foundational studies, never spoke fondly about abstract set theory. He even tried to discourage the young Fraenkel from pursuing it! All these names—Hausdorff, Brouwer, Schoenflies, and Dehn—have since found a place in the early history of topology, a discipline that only began to take on clear contours in the 1920s and 1930s. Brouwer's rising reputation in the field was closely connected with insights he gained while critiquing Schoenflies' surveys (Van Dalen 2013, 137–148), during which time Hausdorff quietly wrote the book that would supplant those surveys. How Hilbert saw this situation we do not know, but since Brouwer complained to him incessantly about Schoenflies' inability to recognize his own mistakes, he clearly had a front row seat for viewing these interactions. In this instance, Hilbert was hardly playing the role of a "general director." In fact, Brouwer forced him to act as mediator in his ongoing arguments with Schoenflies, a situation in which Hilbert's former colleague found himself constantly on the defensive (Van Dalen 2013, 204–208, 225–230).

Brouwer explained how this affair began in a letter to Hilbert written on 16 April 1913. During his first trip to Göttingen two years earlier, he reported, some local mathematicians had urged him to write a book on set theory. At the same time, he learned that Schoenflies was preparing a new edition of his report. Since he had neither the time nor the inclination to write his own book, Brouwer thought he could "achieve the desired goal with relatively little loss of time … were [he] given the opportunity to check Schoenflies' work during printing and, if necessary, improve and supplement it" (Brouwer 2011, 155). Schoenflies agreed to this proposal, despite the rough and tumble exchanges he had experienced in his earlier dealings with Brouwer. Predictably, this new effort also went badly. "Schoenflies and I harbor fundamentally different tendencies with regard to the type and intensity of my cooperation," Brouwer wrote, adding that "Schoenflies would like to limit my influence as far as possible to the improvement of false theorems and proofs, whereas I, of course, aim also to supplement and deepen the work" (ibid.). After rattling off a number of specific complaints, Brouwer then indicated how Hilbert could help by gently suggesting to Schoenflies that he might "leave me [Brouwer] as much freedom as possible" (Brouwer 2011, 156).

This case already reflects how Brouwer would later deal with authors as an associate editor of *Mathematische Annalen*. Particularly astonishing in this instance, though, is the review Brouwer wrote of Schoenflies' book (Schoenflies 1913) for the benefit of the German Mathematical Society (Brouwer 1914). Most of his remarks aimed at informing readers about those parts of the text relevant for an intuitionist (i.e., for Brouwer himself), while emphasizing that the other parts of set theory were simply meaningless from this enlightened standpoint. Could this review have been his real motive for taking on the task of editing Schoenflies' text? At any rate, during this stage of his career, Brouwer was hard at work in the very terrain we identify today with Hausdorff's name. Indeed, he owned and carefully studied Hausdorff's *Grundzüge*, and although he often made plans to write a book of his own, he never managed to succeed.[10]

Brouwer as associate editor of *Mathematische Annalen*

Otto Blumenthal, who taught at the Aachen Institute of Technology, carried a heavy burden as managing editor of *Mathematische Annalen* (Rowe 2018b, 187–239). Not only was the workload demanding, so was his boss. Indeed, Blumenthal's situation became complicated owing to his close relationship with David Hilbert, the journal's editor-in-chief, who arranged his appointment in 1906. This type of arrangement, in which a distinguished mathematician persuades a former student to take on the lion's share of the work, was nothing new. In years past, Felix Klein oversaw the *Annalen,* supported by Walther Dyck as his managing editor. The transition to the new regime hardly went smoothly, however, in part because Dyck resented giving way to a Jew (Rowe 2018b,

[10]The closest he came was a plan to publish his Berliner *Gastvorlesungen* from 1927, which appeared posthumously in 1992; reprinted in Van Dalen and Rowe 2020.

https://doi.org/10.1017/S0269889724000103 Published online by Cambridge University Press

28–29). Both he and Klein remained on the board, the latter as its highest authority in disputes over general policy. Felix Klein was a polished diplomat with an extensive network of contacts and alliances, whereas Hilbert was an impulsive personality, famous for his sarcastic remarks. Unlike Klein, who was an avid reader, Hilbert rarely did more than glance through the journal's new submissions, leaving nearly all the real work to Blumenthal.

In 1914, Blumenthal pleaded for assistance, and on his recommendation, Brouwer and Constantin Carathéodory joined the editorial board in July, though Blumenthal had already engaged Brouwer as a referee long before (Rowe 2018b, 291–297). The timing for this decision, coming just before the outbreak of the Great War, turned out to be fortuitous, especially since both appointees were foreigners. Carathéodory was thus able to take over Blumenthal's duties throughout the war years. During the early phase of the war, Blumenthal was stationed on the eastern front, as was his friend, the astronomer Karl Schwarzschild (on their contacts and activities, see Rowe 2018b, 157–185).

Brouwer published nearly all of his most important topological work in *Mathematische Annalen*, beginning in 1909 with a major paper dealing with Hilbert's fifth Paris problem. He met and conversed with Hilbert that same year, around the same time he discovered serious flaws in Arthur Schoenflies' topological studies (Van Dalen 2013, 124–126). Brouwer delivered his inaugural lecture, "Intuitionism and Formalism," in October 1912, and the following year Arnold Dresden published an English translation (Brouwer 1913). Whether Hilbert took notice of it remains unclear, but Blumenthal surely did read it. By this time, in fact, he was in steady contact with Brouwer, owing to a running controversy between the Dutchman and the French analyst Henri Lebesgue (for details, see Rowe 2018b, 241–266). In fact, from 1911 up until the outbreak of the Great War, Brouwer also enjoyed cordial ongoing relations with Hilbert, Klein, and other mathematicians connected with Göttingen.

The invitation Hilbert extended to him in 1914 thus came as no great surprise, especially since Brouwer had been courting the Göttingen mathematicians for years before he gained a full professorship in Amsterdam in 1913. His former mentor and now colleague, Diederik Johannes Korteweg, was by no means happy about this latest turn of events. "I view the work with which you are swamped by the Göttingen people," he admonished Brouwer, "as a very serious and *lasting* hindrance for the continuation of your own work" (Van Dalen 2013, 225). Korteweg also realized that the sirens from over yonder would seriously divert his former student's attention from his duties as a newly appointed professor in Amsterdam. As for the *Annalen*, after the war ended, Blumenthal found himself caught between Brouwer's increasingly insistent demands and Hilbert's highhanded leadership style.

Setting aside Brouwer's personal ambitions, which ultimately proved his undoing, one should not overlook that his general orientation as an editor and referee was decidedly modern. In fact, modern standards for mathematical journals were still a thing of the future; they evolved as a byproduct of disciplinary specialization, which made reliance on a system of referees a necessity. Brouwer's research focused on problems in topology, and he was especially keen to pass judgement on topological papers. Nevertheless, as a board member he handled papers that fell within the general terrain of "geometry." During this era, editors often approached outside experts for their opinions, but only rarely did they expect them to scrutinize a submission carefully. What is more, if the paper came from an established mathematician, editors usually accepted it with little or no scrutiny at all. Blumenthal and Carathéodory both had high respect for Brouwer's industriousness and critical acumen. He often invested countless hours going back and forth with authors in an effort to perfect their submissions, and his insistence on rigorous proofs created real frictions at times. Yet, he also held himself to exceedingly high standards, which deeply impressed his colleagues.[11]

[11]Brouwer's obsession with rigor can be seen at a glance from the constant flow of corrections he submitted to works he had already published (Van Dalen 2013, 834–839).

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Brouwer thus represented a rare exception to standard customs, especially since *Mathematische Annalen*—unlike *Mathematische Zeitschrift*—was a journal steeped in the traditions of the past. To appear on its title page was a high honor, which explains why the names of older colleagues continued to appear even well after they no longer played an active role in the editorial process. By the 1920s, this custom of honoring old-timers exacerbated the generational divide on the board and even hampered the journal's ability to carry out its editorial work efficiently.

As the *Annalen's* managing editor, Otto Blumenthal was quick to sense the danger posed by Springer's new journal *Mathematische Zeitschrift*. He urged Hilbert to strike up negotiations with Springer, who already had plans for a second journal focused on applied mathematics. After the war, Teubner no longer wanted to invest heavily in its mathematics program, thereby setting the stage for Springer to acquire *Mathematische Annalen* (Schneider 2008). These circumstances led to an expansion of the editorial board in 1920, with strong representation in physics. Albert Einstein became one of the four principal editors, though his role was mainly symbolic. As before, Klein and Hilbert oversaw operations, whereas Blumenthal carried out most of the work, supported by Brouwer and Carathéodory. The four older associate editors—Carl Neumann, Max Noether, Walther von Dyck, and Otto Hölder—remained on the board only as figureheads, joined by six new younger members. These were Ludwig Bieberbach, Harald Bohr, Richard Courant, and the physicists Max Born, Theodor von Kármán, and Arnold Sommerfeld (Rowe and Felsch 2019, 15–17). Meanwhile, Courant had already enlisted the support of his father-in-law, the applied mathematician Carl Runge, who helped him in editing Springer's new "yellow series," which began to appear in 1921 (Remmert 2008, 173–185).

By this time, Brouwer had advertised his foundational views in a short essay entitled "Intuitionist Set Theory" (Brouwer [1919] 1998). In it, he attacked Zermelo's Axiom of Comprehension—which aimed to evade the antinomies—as well as Hilbert's assertion that all mathematical problems are solvable (Hilbert [1900] 1935). Blumenthal not only knew these criticisms, he also knew that Hilbert and Bernays had been hard at work on proof theory in an effort to solve the second Paris problem. These clashing ideas were thus in the air, but had not yet been heard in a public forum. Blumenthal thought he saw an ideal opportunity for Hilbert to answer Brouwer's criticisms, namely at the forthcoming DMV meeting in Bad Nauheim in September 1920. One month before it took place, he wrote to Hilbert: "I very much hope that you will come to Nauheim. Aside from everything else, I think dealing with Brouwer's 'intuitionism' is so important that you should not remain silent about it" (Rowe and Felsch 2019, 25). This Bad Nauheim conference turned out to be a massive event, mainly remembered today because of the political agitation that surrounded Einstein's debate with the anti-relativist Philipp Lenard, the founding father of "Deutsche Physik" (Rowe and Felsch 2019, 25–31).

Brouwer had already announced a lecture with the provocative title "Does every real number have a decimal fraction expansion?" (Brouwer 1921). His answer—not surprisingly—was negative, which he interpreted as a refutation of Hilbert's claim that every well-posed mathematical problem is solvable. As an example, Brouwer pointed to the problem of determining whether any given five consecutive digits in the expansion for π are identical or not. Since in nearly all cases, we cannot (even now) answer questions of this sort, Hilbert's resort to the law of the excluded middle offered no help. Brouwer's intuitionism took this to mean that employing this method of argument was illegitimate. Hausdorff commented on Brouwer's claim some years later in a letter to Abraham Fraenkel, calling it "castration mathematics," and mocking Brouwer's whole approach. He likened it with watching a scorpion wasp lay an egg—here formed by some digits from the decimal expansion for π—by inserting this into the definition of a certain number. "It's altogether stupid," Hausdorff wrote, "to then say: we don't know … whether this number has a decimal representation" (Hausdorff 2012, 295).

For whatever reason, Hilbert decided not to attend the Bad Nauheim conference. He afterward learned from Blumenthal, who spent some time visiting with Brouwer in Blaricum, how disappointed his host was over this. Blumenthal also reported to Klein that the *Annalen* should

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remain neutral with regard to the ongoing controversies in set theory. As for the conference in Bad Nauheim, he found it "very interesting and enjoyable":

> The "lion" among mathematicians was definitely Schönflies . . . . His talk on the axioms of set theory was sharp and good. I proposed to him that a reprint of his paper should appear in the *Annalen*, for which Brouwer has given permission. Schönflies' paper should then be placed next to Brouwer's very stimulating lecture . . . . I hope that these two publications will clearly indicate the desire of the editors to make the new crisis in set theory better known and to promote work on it. (Rowe and Felsch 2019, 31)

The papers by Schoenflies and Brouwer appeared exactly as Blumenthal had planned, whereas Hilbert chose to remain offstage working closely with Bernays (see his lecture courses in Hilbert 2013).

Up until 1921, when the "new foundations crisis" broke, Hermann Weyl and Brouwer had maintained close ties with Göttingen. Prior to this, however, they had both published works on foundations in which they strongly distanced themselves from Hilbert's views (Rowe 2018a, 331–341). Despite these differences, Hilbert and the Göttingen faculty were keen to hire either Brouwer or Weyl; both, in fact, had the opportunity in 1920 to assume Klein's former chair (Rowe 2018a, 346–351). After they declined, the way opened for Richard Courant, an analyst with strong interests in mathematical physics, to join forces with Hilbert, his former mentor. Courant took full advantage of Göttingen's glorious legacy, just as Klein had done before, and yet he managed to forge many new networks of power during the Weimar era. Among the more significant were his ties with two Danes, the analyst Harald Bohr and his brother Niels, the famous physicist. Courant and Max Born had studied together in Göttingen during the last years of Hermann Minkowski's short life. Soon after Courant's appointment in 1920, Göttingen gained the services of Born and the experimental physicist James Franck. This marks the beginning of the period when they and their colleagues in Copenhagen produced the ideas that led to the birth of modern quantum physics.

During the immediate postwar years, few people knew about Hilbert's new strategy for attacking the second Paris problem, namely proof theory.[12] Paul Bernays spoke about this approach at the September 1921 DMV conference held in Jena (Bernays [1922] 1998), but by that time Weyl's propaganda paper (Weyl [1921] 1998) was already in print and awaited Hilbert's reply. Weyl's defection to the intuitionist camp was by far the most dramatic event in this conflict, though it only served to harden the positions on both sides. Brouwer perhaps thought that the tide had suddenly turned against Hilbert. He made a concerted effort to bring Weyl to Amsterdam, but to his great disappointment the latter chose to remain in Zurich (Van Dalen 2013, 300–302). He must have felt even more disappointed when Weyl soon withdrew from the arena of conflict, leaving the actual terms of the debate even murkier than before. Brouwer had set forth his intuitionistic principles in the DMV's *Jahresbericht* (Brouwer [1919] 1998) as well as in *Mathematische Annalen* (Brouwer [1921] 1998), but Hilbert responded to neither. He could hardly afford to remain silent, however, in the face of Weyl's frontal attack.

Hilbert chose an interesting venue for his response—the newly founded University of Hamburg, where Erich Hecke, one of his favorite former pupils, now taught. Probably on the latter's invitation, he visited during the summer semester of 1921, at which time he delivered a since famous public lecture, "The New Grounding of Mathematics" (Hilbert [1922] 1998). Unlike Bernays, who a few months later described the efforts of Brouwer and Weyl as legitimate attempts to establish constructive foundations for mathematics, Hilbert employed polemical language, while turning Weyl's political metaphor on its head. He assured his audience that "Brouwer is not, as Weyl believes, the revolution, but only a repetition, with the old tools, of an attempted

[12]For an overview of his work on foundations from this time, see the introductory remarks in Hilbert 2013, 9–15.

https://doi.org/10.1017/S0269889724000103 Published online by Cambridge University Press

coup ...; and now that the power of the state has been armed and strengthened by Frege, Dedekind, and Cantor, this coup is doomed to fail" (Hilbert [1922] 1998, 200).

Brouwer avoided all such polemics, opting instead to promote his cause by publishing new intuitionistic results. Unfortunately, for him, these appeared in nearly unreadable papers devoted to special problems. Three of these studies appeared in *Mathematische Annalen* over the course of the critical years from 1925 to 1927. When Dirk van Dalen described the twelve main topics that dominated Brouwer's correspondence, he noted that the foundations debate was not among them, and that before 1928, it hardly surfaced at all (Brouwer 2011, 6). Thus, the principal protagonists just went their separate ways, at least until the explosive events of 1928. In that year, Brouwer charged Hilbert with coopting his main ideas (Brouwer [1928] 1998).

Climax and aftermath of the power struggle

Once the fireworks from 1921–22 had subsided, the "foundations crisis" afterward drew relatively little attention, whereas the conflict between Hilbert and Brouwer flared up again in an entirely different guise. Beginning in the mid-1920s, this surfaced in the form of a conflict between nationalists and internationalists on the editorial board of *Mathematische Annalen*, and it was in this context that the Berlin mathematician Ludwig Bieberbach entered the picture. Bieberbach's mathematics, unlike Brouwer's, had very little to do with foundations per se. Nevertheless, he gradually aligned himself with Brouwer during a period when the latter's struggle with Hilbert took on deeply political ramifications. By the mid-1920s, Bieberbach was openly siding with Brouwer, as both began to grumble about their marginal status as associate editors of the *Annalen*. They pressured Blumenthal in hopes of playing a larger role in the operations of the journal, thereby indirectly voicing their displeasure with Hilbert's autocratic manner as the journal's editor-in-chief.

Around the time of Klein's resignation in 1924, a meeting of the *Annalen*'s board members took place during the DMV conference held that year in Innsbruck. Bieberbach and Brouwer put forth proposals on this occasion, specifying the rights and responsibilities of the board's associate editors. This produced a provisional agreement, according to which scientific matters would in the future require the consent of the full board, whereas the four main editors reserved the right to decide on broader policy issues affecting the journal's affairs.[13] Blumenthal, too, was patiently pushing for reforms, but Hilbert harshly rebuffed his plan to invite Weyl as a co-editor (Rowe and Felsch 2019, 239–241). Earlier, Hilbert had suggested appointing Hamburg's Erich Hecke to the board, but by late 1925 Hilbert was seriously ill and no longer interested in making any major changes, except to consolidate his own position in Göttingen. Blumenthal had been hoping to step down, but Hilbert dissuaded him from doing so. In his response to Blumenthal's proposal, he informed him that were he to resign, then he, Hilbert, would simply move the *Annalen*'s operational headquarters from Aachen to Göttingen, where Paul Bernays and Erich Bessel-Hagen stood at hand to take over management of the journal.

Less than a year after the *Annalen*'s board members reached an agreement in Innsbruck, Ludwig Bieberbach pressed to set this aside in connection with plans to publish a special issue honoring Bernhard Riemann in 1926, the centenary of his birth (Rowe and Felsch 2019, 205). Bieberbach and Brouwer had learned that the principal editors wished to invite contributions from distinguished foreign authors, including some from France. Since Einstein had excellent relations with mathematicians in Paris, the other three principals asked him to extend an informal invitation to Paul Painlevé, a prominent figure in French politics (he had recently served as President of the *Chambre des députés*). Brouwer objected to this, citing Painlevé's inflammatory

[13]The precise language of the Innsbruck agreements appears not to have survived in extant documents. One can surmise their general import, however, from relevant correspondence, for example in Bieberbach's letter to Blumenthal from 12 January 1925 (Rowe and Felsch 2019, 205).

https://doi.org/10.1017/S0269889724000103 Published online by Cambridge University Press

words about the German war machine, delivered in a speech during the war. Brouwer's correspondence throughout 1925 reveals that he was deeply involved in a high-level propaganda campaign aimed against the Allied boycott of German science (Rowe and Felsch 2019, 217–226).

Einstein's alliance with Hilbert had much to do with their unusually liberal political views (Rowe 2004). Having Einstein's name on the *Annalen*'s title page lent the journal great scientific prestige, of course, though closet anti-Semites surely took this as an affront, perhaps even as a provocation. Einstein's highly publicized trip to Paris in 1922 scandalized many on the right, even though supporters of the Weimar Republic saw this as a major public relations coup (Rowe and Schulmann 2007, 114–120). Mathematicians in both France and Germany were at that time in no mood to restore friendly relations. Hilbert and the other three principal editors hoped that day would come, but the associate editors did not generally share their views. Before the war, Blumenthal had cultivated friendships with Émile Borel and other leading mathematicians during his many visits to Paris. Brouwer, on the other hand, was outspokenly pro-German, a sentiment he mainly expressed through his loathing of French policies, in particular those of the *Conseil international de recherches* aimed at putting a stranglehold on German science (Van Dalen 2013, 327–350).

Brouwer and Bieberbach took umbrage with the way in which Hilbert had engaged Einstein to reach out to French colleagues without first consulting the associate editors on the board. They canvased the other members, some of whom expressed strong misgivings when it came to soliciting French contributions. Hilbert then reacted quickly, informing all members of the board on 14 February 1925 that Einstein would inform the physicist Paul Langevin that the *Annalen*, though welcoming appropriate French contributions, would not issue direct invitations to anyone in France. Bieberbach had spoken to Einstein about this a month earlier, and he afterward wrote Blumenthal to suggest precisely this same procedure. Yet when Hilbert implemented his suggestion, Bieberbach protested that he should have solicited the opinions of the other board members. No doubt he felt confident that most wanted to maintain a hard line against the French. This successful effort to exploit hostilities between French and German mathematicians fit in well with Brouwer's long-term plans for *Mathematische Annalen*, which saw him occupying a seat at the head of its table. Meanwhile, the special issue honoring Riemann only came out in 1927 and without any French or Belgian contributions (Rowe and Felsch 2019, 200–225).

In his portrait of Bieberbach, Herbert Mehrtens published a note from Hilbert's private papers that clearly reveals how he viewed his conflict with Brouwer on the eve of the 1928 ICM in Bologna (Mehrtens 1987, 214–215).

> In Germany there has arisen a [form of] political blackmailing of the worst kind. You are not a German, unworthy of German birth, if you do not speak and act as I prescribe. It is very easy to get rid of these blackmailers. You have only to ask them how long they laid in German trenches [in World War I]. Unfortunately, German mathematicians have fallen victim to this blackmailing, Bieberbach for example. Brouwer has managed to take advantage of this situation, without having himself served in German trenches, in order to stir up the Germans and sow discord among them in order to establish himself as the Master ruling over German mathematics. With complete success. He will not succeed a second time. (Translated from Rowe and Felsch 2019, 260–261)

Hilbert was enraged over Brouwer's behavior, but he also thought his political posturing part of a cynical attempt to gain power through the *Annalen*.

Was this assessment merely paranoia on Hilbert's part? Some of his closest allies thought so, but Otto Blumenthal eventually concluded that the events of 1928 confirmed Hilbert's assessment of Brouwer's character. Einstein took a more generous view of the situation, calling Brouwer "an involuntary proponent of Lombroso's theory of the close affinity between genius and insanity"

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(letter to Hilbert in Rowe and Felsch 2019, 278). Following Hilbert's triumphal appearance at the ICM in Bologna, where he overcame a counter-boycott called by Brouwer and Beieberbach, came a final crushing defeat for Brouwer, who learned from Carathéodory in October that Hilbert planned to expel him from the *Annalen's* board unless he tendered his resignation.

A major factor in Hilbert's determination to nullify Brouwer's influence stemmed from his own precarious health. In November 1925, doctors determined that he was suffering from pernicious anemia, until then an untreatable disease that was normally fatal (Sophus Lie succumbed to it in 1899). Fearing that his death was imminent, Hilbert asked the other three principal editors for authority to dismiss Brouwer from the *Annalen*'s editorial board (Einstein demurred). Brouwer was deeply shocked by this turn of events, which only animated his ferocious instincts to "fight for justice." He appealed to the associate editors, invoking the memory of Felix Klein as *spiritus rector* of the *Annalen*. Though the situation was hopeless—despite the support of Bieberbach and the sympathy of some older board members, Carathéodory, Dyck and Hölder—Brouwer felt betrayed and wanted to fight to the bitter end. He was particularly disgusted that Blumenthal took up Hilbert's side in denouncing him. Einstein remained steadfastly aloof throughout, dismayed by this full display of ferocious backstabbing. Still, he expressed his admiration for Ferdinand Springer, who behaved very coolly while fully accepting that the *Annalen* was "Hilbert's journal" (for full details, see Rowe and Felsch 2019, 276–334).

The legalistic jousting went on for about two months, ending with Springer's decision to dissolve the entire board and appoint Hilbert as its sole responsible editor. This led to the appointment of Hecke and the re-appointment of Blumenthal, but with no new associate editors, an arrangement that soon proved untenable. Nevertheless, it had the decided advantage that outsiders might never suspect the true purpose of this restructuring, namely to purge Brouwer from the editorial board. Courant and Harald Bohr actually hatched this plan. As two loyalists, Hilbert entrusted them with the authority to act legally in his name.

For a brief time afterward, Bieberbach joined the editorial board of *Mathematische Zeitschrift*; but more noteworthy was an action he took in 1934. As secretary of the DMV's *Jahresbericht* he published an "Open Letter to Harald Bohr" that scandalized the international mathematical community (Mehrtens 1987, 220–222). As with Brouwer, the *Annalen* affair left Bieberbach deeply embittered, and after 1933, he felt empowered to strike back at his enemies.

Once Brouwer realized that Hecke and Blumenthal were now running Hilbert's ship, he began casting around for his own journal (Van Dalen and Remmert 2006). In 1930, he wrote to potential allies worldwide about his plan to publish a new mathematical periodical with Noordhoff in Groningen (Rowe and Felsch 2019, 351–352). This new undertaking would stand in sharp contrast with *Mathematische Annalen*—in Brouwer's eyes now a "provincial" journal run by Hilbert loyalists—featuring, by virtue of its international character, a large board with many distinguished names.

As it happened, Brouwer only managed to launch this new periodical, *Compositio Mathematica,* in 1935. He did so, however, with much fanfare. Having won the support of forty-eight mathematicians from sixteen countries for his advisory board, he extolled its international character and commitment to overcoming nationalistic bias. Brouwer also emphasized the need for a broad base of expertise in order to handle the work of mathematicians from various countries who specialized in a variety of research fields. *Compositio Mathematica* thus offered "a guarantee against any one-sidedness with respect to the mathematical character of the published papers" (Van Dalen and Remmert 2006, 1087). By this time, however, Brouwer's enthusiasm for editorial work had passed, and he delegated most tasks to his assistant, Hans Freudenthal. Although it began as a promising venture with many of the hallmarks of a modern scientific periodical, *Compositio* never established itself as one of the leading mathematics journals. Noordhoff published its final volume in 1940 following the German occupation of Holland.

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Brouwer's initial plan called for a structure with five principal editors, one of whom was to have been Ludwig Bieberbach.[14] However, Bieberbach soon found himself in an untenable position, which he described to Brouwer in a letter from 21 June 1934. He had accepted this position with the understanding that others would view his decision favorably. "I assumed," he wrote, "that one would recognize this as an example that the new Germany, notwithstanding its fight with international Jewry, gladly co-operates with other nations, that meet us, if not with sympathy, then at least with loyalty. Instead, people now often see the crucial point in the fact that Jews appear on the cover of *Compositio*" (Van Dalen and Remmert 2006, 1088). There were, in fact, several Jewish members on the board, including three who had already lost or had left their former positions in Hitler's Germany: Reinhold Baer (Halle), Alfred Loewy (Freiburg), and Richard von Mises (Berlin) (Siegmund-Schultze 2009).

An opportunist all his life, Bieberbach now calculated that the time had come for him to establish his credentials as a Jew baiter. He thus informed Brouwer that he would withdraw his name from the editorial board of *Compositio Mathematica,* unless Brouwer agreed to discharge its Jewish members. When the latter refused, Bieberbach submitted his resignation, thereby effectively ending a partnership based on little more than mutual thirst for power and recognition.

After Hitler came to power, Bieberbach soon emerged as the foremost propagandist for "Deutsche Mathematik," founding a new journal under this title in 1936. He and his allies had long harbored resentments against certain Jewish colleagues, but Nazi racial theorists enabled Bieberbach to argue that abstract axiomatics posed a dire danger to German mathematics. The purveyors of such modern theories were promoting, so Bieberbach claimed, a foreign, fundamentally non-Aryan (quintessentially Jewish) style of thought. Few mathematicians openly professed this kind of racially biased opinion prior to January 1933, and probably no one at that time suspected Bieberbach of harboring such views. Yet once the Nazis seized power, German Jews (and others thought disloyal to Germany) came under widespread attack. Antisemitism, once regarded as crude and distasteful, now suddenly seemed reasonable, even insightful. Herbert Mehrtens' portrait of Bieberbach (Mehrtens 1987) remains an impressive study of a mathematician who, more than perhaps any other, embodied the evils of Nazi Germany.

Acknowledgements. My thanks go to Jeremy Gray, Reinhard Siegmund-Schultze, and Lukas Verburgt for their comments on an earlier version of this paper.

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https://doi.org/10.1017/S0269889724000103 Published online by Cambridge University Press

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Cite this article: Rowe, David E. 2022. "Brouwer and Hausdorff: On reassessing the foundations crisis," *Science in Context* 35 (4): 395–413. doi:10.1017/S0269889724000103