Resurrecting the Lunar Distance

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Barring electronics and steady timekeepers the Moon's orbit provides the readiest means of finding Greenwich time at sea. But the Moon moves so slowly in her monthly circuit that an error of only 1′ in locating her will cause an error of nearly 2 minutes in GMT. Obviously the navigator needs some idea of how well he has pinpointed her. Even a modest error, unless suspected and allowed for, could be disastrous.

Fortunately, a navigator can learn to estimate the uncertainty in a lunar observation with some confidence. If the altitudes are reasonably high he needs to consider only two things: imprecision of the contact between the Moon's limb and the other body, and unknown errors of the instrument. Other factors are largely filtered out. An error of 2′ or 3′ in one or both of the altitudes (caused by abnormal dip, or by a poorly defined horizon) will seldom affect the outcome at all. The accuracy of a 'lunar' depends almost entirely on the measurement of the distance itself.

For a navigator who has no other way of finding GMT, and is careful, this tight, reliable connection between the sextant measurement and Greenwich time makes the lunar safe. For a navigator who already has GMT, and enjoys using a sextant, it makes the lunar a perfect game of skill. Or it would if there were an acceptable way of doing the calculation.

The method of calculation introduced here was developed to overcome present difficulties. It has been examined for accuracy and universality, and has been used alongside seven of the better-known eighteenth- and nineteenth-century solutions. At least 50 observations have been worked with it.

A book of tables must be published to make the method convenient enough for modern taste, but the book would have to be only slightly larger than the *Nautical Almanac* to be equally legible.

Since clearing a distance of refraction and parallax is generally believed to be a formidable undertaking, the observation itself will be passed over for the moment and the method of clearing it discussed first. The formula does indeed look formidable, and will be given only after the actual procedure has been explained.

The elements needed are: D_a, the apparent distance between centres; m, the apparent altitude of the Moon's centre; and s, the apparent altitude of the Sun's centre, or of the star or planet. Subtracting one altitude from the other produces $m \sim s$, and applying a combined correction for refraction and parallax to this remainder gives $M \sim S$, the difference of true altitudes.

A function called Q is tabulated alongside, and copied out with, the refraction and parallax corrections. But when K is needed it must be looked out of its own table. This much explanation should enable the reader to follow the work form down to the third line from the bottom.

The smaller number on the third line from the bottom is put under the other and subtracted. The remainder is used to enter a critical table to find a Gaussian logarithm (0·032 14) which is put in the adjoining column and subtracted. This produces the K of the cleared lunar distance (25° 57·2′).

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D_a	26° 15·5′			
$m \sim s$	7° 45·2′			
sum	34° 00·7′ K		1.06784	add
difference	18° 30·3′ K		1·58751	
			2·65535	
		half	1·32767,5	add
		Q	179,9	
$M \sim S$ 6° 52·6′ K	2·44407		1·32947	
	−1·32947		−0·03214	
$D =$ 25° 57·2′ ⊠	~~1·11460~~		1·29733	

Because it is easy to blunder by using the wrong number from the last line to enter the K table, that number is struck through as soon as it is no longer needed.

The convenience of the method rests on two features. First, the rules never change. Second, there is no interpolation – or will not be if the tables are published.

The formula, which was derived from two standard formulas of spherical trigonometry, is:

$$\text{hav}\, D = \{\text{hav}\,(D_a + (m \sim s)]\ \text{hav}\,[D_a - (m \sim s)]\}^{\frac{1}{2}} (\cos S/\cos s)(\cos M/\cos m) + hav\,(M \sim S)$$

K is the log haversine left negative. Q is the log of $(\cos S/\cos s)(\cos M/\cos m)$, also left negative. The advantages gained by this, and by using a Gaussian addition log, are central to the method.

In order to find GMT, the distance, once cleared, must be compared with distances taken from the *Nautical Almanac*. These reference distances are computed for the exact hours thought to bracket the time of observation.

The calculation is similar to, but somewhat shorter than, the one for clearing the distance. Since v and d corrections and the yellow pages of the *Almanac* are bypassed, and since both comparing distances can be computed at once (a table opening made for one serves as well for the other) this is not especially burdensome.

In the example the comparing distances, D' and D'', are found to be 25° 52·9′ and 26° 24·5′, respectively. Two small tables of logistical logs (the last tables of the set) proportion for GMT.

$D \sim D'$ 04·3′	(table 10)	1·1447
$D'' \sim D'$ 31·6	(table 10)	−0·2785
8 m 10 s	(table 11)	0·8662

Since D' and D'' were calculated for 05·00 and 06·00, GMT per observation is 5 h 8 m 10 s.

But the interesting part of the lunar problem is the observation itself. A beginner, unless he is on land, probably should attempt only short distances. He will find the longer ones virtually impossible.

The distance must be measured from a body more or less in line with the Moon's orbital motion, which is perpendicular to a line joining her horns. The approximate distance is best found beforehand. One way to do this is to remove the telescope (so

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as to have a wide view), point the sextant toward one object and bring that object's reflected image across the sky to the other, in much the same way that a star is brought to the horizon. With the telescope back in place the best combination of shades can be found and mentally noted.

The observation is begun by measuring and recording the two altitudes, together with their watch times. The preselected shades are then turned into place and the approximate distance set on the arc. As the contact is adjusted the sextant is rocked to find the point on the Moon's limb the other body can just touch. Usually this rocking takes care of itself. Unless the distance is short or the observer is on land it will be impossible to hold the two objects still. The observer judges contact by watching them brush.

After three or more timed distances have been recorded, the observation is brought to a close by again taking the altitudes.

The distances and their watch times are averaged. Three small tables, constructed on the same principle as tables 10 and 11, facilitate the adjustment of the altitudes to what they would have been if measured at the average time of the distances.

Unless he thinks the problem through, a navigator who has no experience with lunars will almost certainly blunder in dealing with corrections, particularly if he uses modern correction tables. At best he will take great pains and get mediocre precision. At worst he will leave the Moon's semidiameter with her refraction and parallax and 'clear' the apparent distance (which is to the Moon's centre, not her upper or lower limb) of a 15′ or 16′ vertical distortion that had not affected it.

A quick, precise, almost blunder-proof procedure was devised for handling corrections. Along with the rest of the system, it follows the logic of the astronomers and navigators who developed the methods of the past.

This compatibility allows old methods of clearing the distance to be substituted for the one provided in the system. The tables, if published, should be useful not only to navigators interested in self-reliance and in developing skill with the sextant, but to those who would like a close, practical understanding of methods used by eighteenth- and nineteenth-century navigators.

Record

Standard Global Reference Datum

This paper was presented by W. Blanchard to a meeting of the Technical Committee on 25 March 1987. It has been forwarded to an IAIN Working Party set up to consider possible contributions to the ICAO Future Air Navigation Systems study.

The common internationally used reference system for navigation is, of course, latitude and longitude. The mathematical parameters laid down for drawing this up are precise and can be found in many different texts, but what has been difficult in the past has been how to reference it to land features so that a system based on one country can be used in another with equal accuracy.

The basic problem has been that, until comparatively recently, in order to perform this referencing operation, observations had to be made of stars or planets to give latitude and longitude for a known point. These observations could not be of very high accuracy

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