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SHORT NOTES

THE FLOW OF ICE OVER A TILL BED

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Abstract. This paper considers the contribution made to overall glacier flow by deformation of an unlithified bed. The flow pattern in deforming till beneath a glacier is determined for the case of a uniform ice slab resting on a uniform thick till bed. The results obtained show that, once the till is saturated, very little surplus water is needed to produce high velocities in the till near the ice–till interface, the interface velocity varying as the 5/2 power of the surplus water concentration. This sensitivity to water content suggests that this may be a mechanism inducing glacier surging.

Résumé. *L'écoulement de la glace sur un lit morainique.* Cet article examine la contribution apportée à l'écoulement d'ensemble d'un glacier par la déformation d'un lit non rocheux. Le type d'écoulement sur un lit morainique déformable sous un glacier est déterminé pour le cas d'une plaque de glace conforme reposant sur un lit morainique d'épaisseur uniforme. Les résultats obtenus montrent que, une fois la moraine saturée, il suffit de très peu d'eau en supplément pour entrainer de fortes vitesses dans la moraine près de l'interface glace–moraine, la vitesse à cette interface variant comme la puissance 5/2 de la teneur en eau supplémentaire. Cette sensibilité à la teneur en eau fait penser que ce pourrait être un mécanisme produisant les crues glaciaires rapides.

Zusammenfassung. *Das Fliessen von Eis über ein Schuttbett.* Diese Untersuchung gilt dem Beitrag, den die Deformation eines schuttbedeckten Untergrundes zum Gesamtfluss eines Gletschers liefert. Die Fliessvorgänge in verformbarem Schutt unter einem Gletscher werden für den Fall einer einheitlichen Eistafel, die einem gleichmässig dicken Schuttbett aufliegt, bestimmt. Die gewonnenen Ergebnisse zeigen, dass nach der Sättigung des Schuttes mit Wasser sehr wenig zusätzliches Wasser zum Auftreten hoher Geschwindigkeiten im Schutt nahe der Grenzfläche zum Eis genügt, wobei die Geschwindigkeit an der Grenzfläche mit der Potenz 5/2 der Konzentration des überschüssigen Wassers variiert. Diese empfindliche Reaktion auf den Wassergehalt lässt vermuten, dass hier ein Mechanismus zur Auslösung eines Gletscherausbruchs vorliegt.

Previous discussions of glacier flow have assumed that there is a rigid glacier bed, and that the flow results from a combination of deformation of the ice and slipping over the bed. However field work (Boulton, in press) has shown that for some glaciers deformation of the bed contributes most of the forward movement of the ice, and there is evidence indicating that the flow of the Laurentide ice sheet was controlled by the deformable sediments over which it passed.

Consider a glacier resting on a bed of unlithified till. If the glacier is melting at the bottom, the melt water produced penetrates into the till so that the usual mechanisms for glacier sliding are not available. The rheology of the bed is governed by the relative sizes of the particles involved, by the types of material present, and by the amount of water present. If the bed consists of coarse gravel, the melt water percolates freely through the bed, and the bed only becomes unstable if large amounts of water are trapped in it. If, however, the bed consists of fine till containing clay particles, the clay adsorbs the water and tends to immobilize it in the bed, lowering the permeability. If melt water is now produced faster than it can percolate through the bed, the surplus "free" water is dammed near the surface producing a slurry which behaves like a Newtonian fluid with viscosity (Roscoe, 1952)

$$\eta = \eta_0 w^{-5/2},$$

where w is the volume fraction of free water, and η_0 is the viscosity of water. Since most of the free water remains close to the interface, the bed is highly mobile at the top, but at the limit of effective free-water penetration w becomes zero, the viscosity becomes infinite, and the bed is effectively rigid.

In order to study the contribution of bed deformation to the overall flow-rate of a glacier, we consider the simplest situation of an ice sheet of uniform thickness H resting on a bed whose inclination to the horizontal is α (Fig. 1). We take coordinate axes x in the direction of maximum slope, y perpendicular to the bed, and z in the transverse direction, and choose the origin to be at the interface, so that $0 < y < H$ corresponds to the ice layer and $y < 0$ to the till. The shear stress and velocity are assumed to be continuous across the interface $y = 0$.

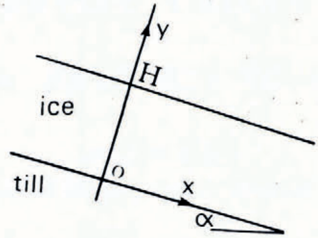

Fig. 1. Ice–till model showing coordinate axes.

If we assume that water penetration into the till is independent of both x and z, the problem reduces to that of determining the velocity u of the ice/till in the x direction as a function of the single independent variable y.

Treating the ice as a Glen fluid, the velocity $u(y)$ for $0 \leqslant y \leqslant H$ is given by (Nye, 1957)

$$u(y) = u_s - (u_s - u_0)(1 - y/H)^{n+1},$$

where u_s is the surface velocity and u_0 is the velocity at the interface. The tangential shear at the interface is $\tau_0 = \rho_i g H \sin \alpha$.

Within the till, we assume that the free water penetrates to a depth l, and that the amount of free water present varies linearly with depth, so that w is given by

$$w(y) = f(y+l)/l, \qquad -l \leqslant y \leqslant 0,$$

where $0 \leqslant f \leqslant 1$ is the volume fraction of free water at the interface. All deformation of the till is confined to the region $-l \leqslant y \leqslant 0$, and in this region the effective viscosity of the till is

$$\eta = \eta_0 (f(y+l)/l)^{-5/2}.$$

The density of the mixture also varies with depth, being $\rho = \rho_w w(y) + \rho_t(1 - w(y))$, where ρ_w and ρ_t are the densities of water and saturated till respectively.

Substituting these values into the Navier–Stokes equations we have, for $-l \leqslant y \leqslant 0$,

$$0 = g \sin \alpha(\rho_t - (\rho_t - \rho_w)\, w(y)) + \frac{\mathrm{d}}{\mathrm{d}y}\left(\eta_0 w(y)^{-5/2} \frac{\mathrm{d}u}{\mathrm{d}y}\right).$$

This equation is solved subject to the boundary conditions that the shear at $y = 0$ is $\rho_i g H \sin \alpha$ and that the velocity at $y = -l$ is zero, giving

$$u(y) = \frac{g \sin \alpha}{\eta_0} (f/l)^{5/2}\{2[\rho_i H + l(\rho_t - \tfrac{1}{2} f(\rho_t - \rho_w))](l+y)^{7/2}/7 - 2\rho_t (l+y)^{9/2}/9 + f(\rho_t - \rho_w)(l+y)^{11/2}/(11 l)).$$

A typical profile for $u(y)$ is shown in Figure 2. Setting $y = 0$, we obtain

$$u_0 = \frac{g \sin \alpha}{\eta_0} f^{5/2} l\{2\rho_i H/7 + [4\rho_t/63 + 8f(\rho_w - \rho_t)]l\}$$

$$\approx \frac{2gHl}{7\eta_0} \rho_i f^{5/2} \sin \alpha,$$

since we expect $l \ll H$. For ice thickness about 30 m and penetration depth about 30 cm, $\rho_i g H l/\eta_0$ is of the order of 10^{15} m/year, so for realistic values of u_0 we must have $f^{5/2} \sin \alpha \approx 10^{-15}$. Hence, for finite slopes, significant deformation of the till occurs when the proportion f of free water at the interface is about 10^{-6}, and when f is 10^{-5} we find that $u_0 \approx 100 \sin \alpha$ m/year. Since doubling f produces a six-fold increase in interface velocity, values of f greater than 10^{-5} will produce velocities in the surge range unless $\sin \alpha$ is very close to zero; i.e. the bed is essentially horizontal. This implies a very sensitive mechanism for triggering glacier surges. If the normal rate of melt-water production is just below the rate at which water can percolate through the bed, any damming of the bed would lead to the trapping of free water in the bed and a rapid rise in the velocity until the obstruction is removed. Alternatively,

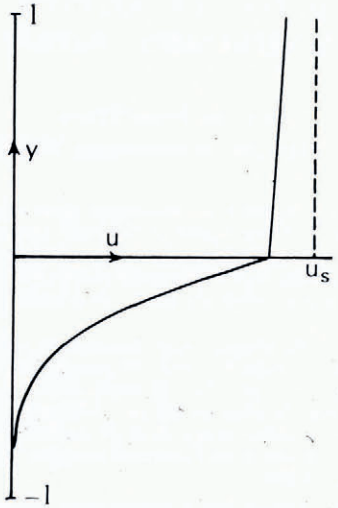

Fig. 2. Typical velocity profile in deforming till and adjacent ice.

a glacier whose rate of melt-water production rose slowly but steadily following a corresponding increase in glacier thickness, would surge when the melting rate passed the critical level. The consequent thinning would lead to a lowering in the temperature at the bed, and hence a reduction in basal melt-water production, and the glacier would return to its quiescent state.

Conclusion

If the bed of a glacier consists of unlithified till, then the stability of the bed depends critically on the rate of melt-water production at the bottom of the ice. Small excess amounts of water liquidize the top of the till, making it very mobile, and resulting in high sliding velocities in the till. If the glacier bed has a finite slope this may result in surging.

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