1

Introduction

1.1 Motivation

Under ordinary circumstances, matter on Earth occurs in the three phases of solid, liquid and gas. Here, 'ordinary' refers to the circumstances relevant for human life on this planet. This state of affairs does not extrapolate beyond earthly scales: astronomers agree that, ignoring the more speculative nature of dark matter, matter in the Universe consists of more than 90% plasma. Hence, *plasma is the ordinary state of matter in the Universe*. The consequences of this fact for our view of nature are not generally recognized yet (see Section 1.3.4). The reason may be that, since plasma is an exceptional state on Earth, the subject of plasma physics is a relative latecomer in physics.

For the time being, the following crude definition of plasma suffices. *Plasma is a completely ionized gas, consisting of freely moving positively charged ions, or nuclei, and negatively charged electrons*.[1] In the laboratory, this state of matter is obtained at high temperatures, in particular in thermonuclear fusion experiments ($T \sim 10^8$ K). In those experiments, the mobility of the plasma particles facilitates the induction of electric currents which, together with the internally or externally created magnetic fields, permits magnetic confinement of the hot plasma. In the Universe, plasmas and the associated large-scale interactions of currents and magnetic fields prevail under much wider conditions.

Hence, we will concentrate our analysis on the two mentioned broad areas of application of plasma physics, viz.

(a) *Magnetic plasma confinement for the purpose of future energy production by controlled thermonuclear reactions* (CTR); this includes the pinch experiments of the 1960s and early 1970s, and the tokamaks and alternatives (stellarator, spheromak, etc.) developed in the 1980s and 1990s and, at present, sufficiently matured to start designing prototypes of the fusion reactors themselves.

(b) *The dynamics of magnetized astrophysical plasmas*; this includes the ever growing research field of solar magnetic activity, planetary magnetospheres, stellar winds, interstellar medium, accretion discs of compact objects, pulsar magnetospheres, etc.

The common ground of these two areas is the subject of *plasma interacting with a magnetic field*. To appreciate the power of this viewpoint, we first discuss the conditions for laboratory fusion in Section 1.2, then switch to the emergence of the subject of plasma-astrophysics in Section 1.3, and

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[1] In plasma physics, one can hardly avoid mentioning exceptions: in pulsar electron–positron magnetospheres, the role of positively charged particles is taken by positrons. In considerations of fusion reactions with exotic fuels like muonium, the role of negatively charged particles is taken by muons.

finally refine our definition(s) of plasma in Section 1.4. In the latter section, we also provisionally formulate the approach to plasmas by means of magnetohydrodynamics.

The theoretical models exploited lead to *nonlinear partial differential equations*, expressing *conservation laws*. The boundary conditions are imposed on an extended spatial domain, associated with the *complex magnetic plasma confinement geometry*, whereas the temporal dependence leads to *intricate nonlinear dynamics*. This gives theoretical plasma physics its particular, mathematical, flavour.

1.2 Thermonuclear fusion and plasma confinement

1.2.1 Fusion reactions

Both fission and fusion energy are due to nuclear processes and, ultimately, described by Einstein's celebrated formula $E = mc^2$. Hence, in nuclear reactions $A + B \rightarrow C + D$, net energy is released if there is a mass defect, i.e. if

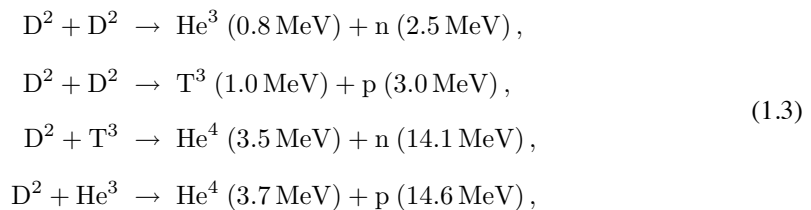$$(m_A + m_B) c^2 > (m_C + m_D) c^2 . \tag{1.1}$$

In laboratory fusion, reactions of hydrogen isotopes are considered, where the deuterium-tritium reaction (Fig. 1.1) is the most promising one for future reactors:
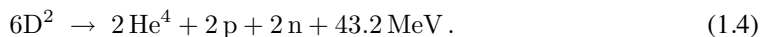
$$D^2 + T^3 \;\rightarrow\; He^4 \,(3.5\,\text{MeV}) + n\,(14.1\,\text{MeV}) . \tag{1.2}$$
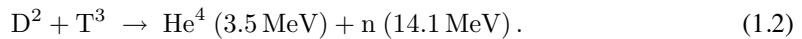
This yields two kinds of products, viz. α particles (He^4), which are *charged* so that they can be captured by a confining magnetic field, and neutrons, which are *electrically neutral* so that they escape from the magnetic configuration. The former contribute to the heating of the plasma (so-called α particle heating) and the latter have to be captured in a surrounding Li^6/Li^7 blanket, which recovers the fusion energy and also breeds new T^3.

Deuterium abounds in the oceans: out of 6500 molecules of water one molecule contains a deuteron and a proton instead of two protons. Thus, in principle, 1 litre of sea water contains 10^{10} J of deuterium fusion energy. This is a factor of about 300 more than the combustion energy of 1 litre of gasoline, which yields 3×10^7 J.

A number of other reactions also occur, in particular reactions producing T^3 and He^3 which may be burned again. Complete burn of all available D^2 would involve the following reactions:
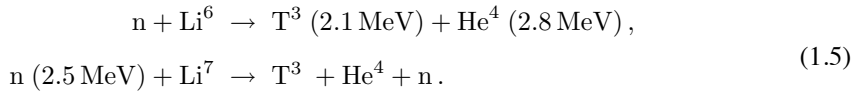
$$\begin{aligned}
D^2 + D^2 \;&\rightarrow\; He^3 \,(0.8\,\text{MeV}) + n\,(2.5\,\text{MeV}) , \\
D^2 + D^2 \;&\rightarrow\; T^3 \,(1.0\,\text{MeV}) + p\,(3.0\,\text{MeV}) , \\
D^2 + T^3 \;&\rightarrow\; He^4 \,(3.5\,\text{MeV}) + n\,(14.1\,\text{MeV}) , \\
D^2 + He^3 \;&\rightarrow\; He^4 \,(3.7\,\text{MeV}) + p\,(14.6\,\text{MeV}) ,
\end{aligned} \tag{1.3}$$

so that in effect

$$6D^2 \;\rightarrow\; 2\,He^4 + 2\,p + 2\,n + 43.2\,\text{MeV} . \tag{1.4}$$

In the liquid Li blanket, fast neutrons are moderated, so that their kinetic energy is converted into

heat, and the following reactions occur:

$$n + Li^6 \rightarrow T^3 \, (2.1\,\text{MeV}) + He^4 \, (2.8\,\text{MeV}) \, ,$$

$$n \, (2.5\,\text{MeV}) + Li^7 \rightarrow T^3 + He^4 + n \, . \tag{1.5}$$

This provides the necessary tritium fuel for the main fusion reaction, described by the third item of Eq. (1.3) [442].
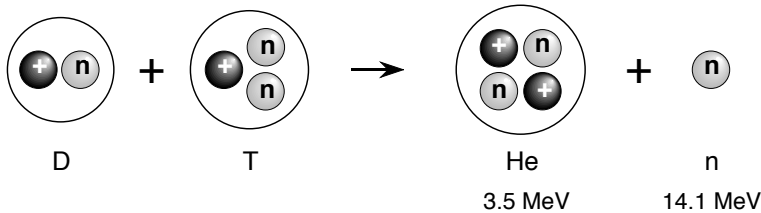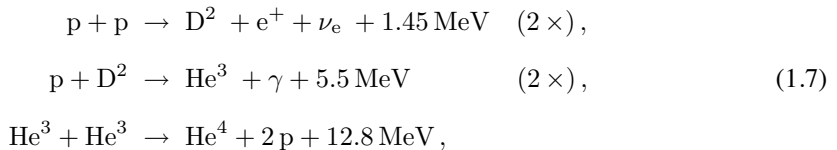

Fig. 1.1 Nuclear fusion reaction of the hydrogen isotopes deuterium and tritium.

Typical numbers associated with thermonuclear fusion reactors, as presently envisaged, are:

$$\text{temperature} \quad T \sim 10^8 \, \text{K} \, (10\,\text{keV}) \, , \quad \text{power density} \sim 10\,\text{MW}\,\text{m}^{-3} \, ,$$

$$\text{particle density} \quad n \sim 10^{21}\,\text{m}^{-3} \, , \qquad \text{time scale} \quad \tau \sim 100\,\text{s} \, . \tag{1.6}$$

It is often said that controlled thermonuclear fusion in the laboratory is an attempt to harness the power of the stars. This is actually a quite misleading statement since the fusion reactions which take place in, e.g., *the core of the Sun* are different reactions of hydrogen isotopes, viz.

$$p + p \rightarrow D^2 + e^+ + \nu_e + 1.45\,\text{MeV} \quad (2\times) \, ,$$

$$p + D^2 \rightarrow He^3 + \gamma + 5.5\,\text{MeV} \qquad (2\times) \, , \tag{1.7}$$

$$He^3 + He^3 \rightarrow He^4 + 2\,p + 12.8\,\text{MeV} \, ,$$

so that complete burn of all available hydrogen amounts to

$$4\,p \rightarrow He^4 + 2\,e^+ + 2\,\nu_e \, (0.5\,\text{MeV}) + 2\,\gamma \, (26.2\,\text{MeV}) \, . \tag{1.8}$$

The positrons annihilate with electrons, the neutrinos escape, and the gammas (carrying the bulk of the thermonuclear energy) start on a long journey to the solar surface, where they arrive millions of years later (the mean free path of a photon in the interior of the Sun is only a few centimeters) [510]. In the many processes of absorption and re-emission the wavelength of the photons gradually shifts from that of gamma radiation to that of the visible and UV light escaping from the photosphere of the Sun, and producing one of the basic conditions for life on a planet situated at the safe distance of one astronomical unit (1.5×10^{11} m) from the Sun.

At higher temperatures another chain of reactions is effective, where carbon acts as a kind of catalyst. This so-called CNO cycle involves a chain of fusion reactions where C^{12} is successively converted into $N^{13}, C^{13}, N^{14}, O^{15}, N^{15}$, and back into C^{12} again. However, the net result of incoming and outgoing products is the same as that of the proton–proton chain, viz. Eq. (1.8).

Typical numbers associated with thermonuclear reactions in the stars, in particular the core of the Sun, are the following ones:

$$\text{temperature} \quad T \sim 1.5 \times 10^7 \, \text{K} \, , \quad \text{power density} \, \sim 3.5 \, \text{W m}^{-3} \, ,$$

$$\text{particle density} \quad n \sim 10^{32} \, \text{m}^{-3} \, , \qquad \text{time scale} \quad \tau \sim 10^7 \, \text{years} \, . \tag{1.9}$$

Very different from the numbers (1.6) for a prospective fusion reactor on Earth!

1.2.2 Conditions for fusion

Thermonuclear fusion happens when a gas of, e.g., deuterium and tritium atoms is sufficiently heated for the thermal motion of the nuclei to become so fast that they may overcome the repulsive Coulomb barrier (Fig. 1.2) and come close enough for the attractive nuclear forces to bring about the fusion reactions discussed above. This requires particle energies of $\sim 10 \, \text{keV}$, i.e. temperatures of about $10^8 \, \text{K}$. At these temperatures the electrons are completely stripped from the atoms (the ionization energy of hydrogen is $\sim 14 \, \text{eV}$) so that a plasma rather than a gas is obtained (cf. our crude definition of Section 1.1).
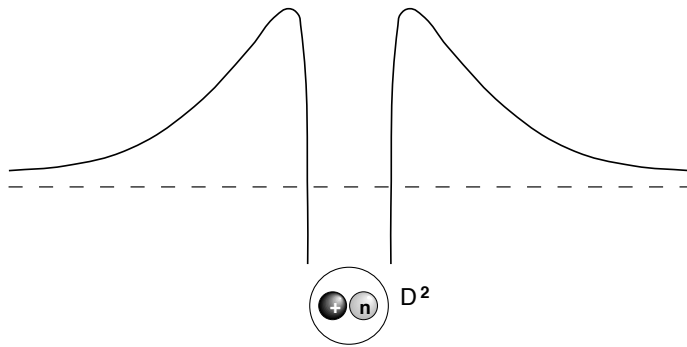

Fig. 1.2 Nuclear attraction and Coulomb barrier of a deuteron.

Because the charged particles (occurring in about equal numbers of opposite charge) are freely moving and rarely collide at these high temperatures, a plasma may be considered as a *perfectly conducting fluid* for many purposes. In such fluids, electric currents are easily induced and the associated magnetic fields in turn interact with the plasma to confine or to accelerate it. The appropriate theoretical description of this state of matter is called *magnetohydrodynamics* (MHD), i.e. the dynamics of magneto-fluids (Section 1.4.2).

Why are magnetic fields necessary? To understand this, we need to discuss the power requirements for fusion reactors (following Miyamoto [442] and Wesson [647]). This involves three contributions, viz.

(a) the thermonuclear output power per unit volume:

$$P_{\text{T}} = n^2 f(\widetilde{T}) \, , \qquad f(\widetilde{T}) \equiv \tfrac{1}{4} \langle \sigma v \rangle E_{\text{T}} \, , \quad E_{\text{T}} \approx 22.4 \, \text{MeV} \, , \tag{1.10}$$

where n is the particle density, σ is the cross-section of the D-T fusion reactions, v is the relative speed of the nuclei, $\langle \sigma v \rangle$ is the average nuclear reaction rate, which is a well-known function

of temperature, and E_T is the average energy released in the fusion reactions (i.e. more than the $17.6\,\text{MeV}$ of the D-T reaction (1.3)(c) but, of course, less than the $43.2\,\text{MeV}$ released for the complete burn (1.4));

(b) the power loss by Bremsstrahlung, i.e. the radiation due to electron–ion collisions:

$$P_B = \alpha n^2 \widetilde{T}^{1/2}, \quad \alpha \approx 3.8 \times 10^{-29}\,\text{J}^{1/2}\,\text{m}^3\,\text{s}^{-1}; \tag{1.11}$$

(c) the losses by heat transport through the plasma:

$$P_L = \frac{3n\widetilde{T}}{\tau_E}, \tag{1.12}$$

where $3n\widetilde{T}$ is the total plasma kinetic energy density (ions + electrons), and τ_E is the energy confinement time (an empirical quantity). The latter estimates the usually anomalous (i.e. deviating from classical transport by Coulomb collisions between the charged particles) heat transport processes.

Here, we have put a tilde on the temperature to indicate that energy units of keV are exploited:

$$\widetilde{T}\,(\text{keV}) = 8.62 \times 10^{-8} T\,(\text{K}),$$

since $\widetilde{T} = 1\,\text{keV} = 1.60 \times 10^{-16}\,\text{J}$ corresponds with $T = 1.16 \times 10^7\,\text{K}$ (using Boltzmann's constant, see Appendix Table B.1).

If the three power contributions are considered to become externally available for conversion into electricity and back again into plasma heating, with efficiency η, *the Lawson criterion* [400],

$$P_B + P_L = \eta\,(P_T + P_B + P_L), \tag{1.13}$$

tells us that there should be power balance between the losses from the plasma (LHS) and what is obtained from plasma heating (RHS). Typically, $\eta \approx 1/3$. Inserting the explicit expressions (1.10), (1.11) and (1.12) into Eq. (1.13) leads to a condition to be imposed on the product of the plasma density and the energy confinement time:

$$n\tau_E = \frac{3\widetilde{T}}{[\eta/(1-\eta)]\,f(\widetilde{T}) - \alpha\widetilde{T}^{1/2}}. \tag{1.14}$$

This relationship is represented by the lower curve in Fig. 1.3. Since Bremsstrahlung losses dominate at low temperatures and transport losses dominate at high temperatures, there is a minimum in the curve at about

$$n\tau_E = 0.6 \times 10^{20}\,\text{m}^{-3}\,\text{s}, \quad \text{for } \widetilde{T} = 25\,\text{keV}. \tag{1.15}$$

This should be considered to be the threshold for a fusion reactor under the given conditions.

By a rather different, more recent, approach of fusion conditions, *ignition* occurs when the total amount of power losses is balanced by the total amount of heating power. The latter consists of α-particle heating P_α and additional heating power P_H, e.g. by radio-frequency waves or neutral beam injection. The latter heating sources are only required to bring the plasma to the ignition point, when α-particle heating may take over. Hence, at ignition we may put $P_H = 0$ and the power balance becomes

$$P_B + P_L = P_\alpha = \tfrac{1}{4}\langle\sigma v\rangle n^2 E_\alpha, \quad E_\alpha \approx 3.5\,\text{MeV}. \tag{1.16}$$
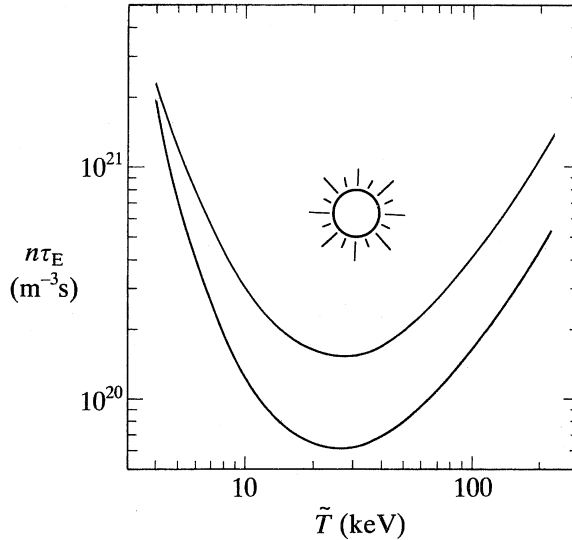
Fig. 1.3 Conditions for net fusion energy production according to the Lawson criterion (lower curve) and according to the view that power losses should be completely balanced by α-particle heating (upper curve). (Adapted from Wesson [647].)

Formally, this may be described by the same equation (1.14) taking now $\eta \approx 0.135$ so that a 2.5 times higher threshold for fusion is obtained:

$$n\tau_{\mathrm{E}} = 1.5 \times 10^{20}\,\mathrm{m}^{-3}\,\mathrm{s}\,, \quad \text{for } \widetilde{T} = 30\,\mathrm{keV}\,. \tag{1.17}$$

This relationship is represented by the upper curve of Fig. 1.3.

Roughly speaking then, products of density and energy confinement time $n\tau_{\mathrm{E}} \sim 10^{20}\,\mathrm{m}^{-3}\,\mathrm{s}$ and temperatures $\widetilde{T} \sim 25\,\mathrm{keV}$, or $T \sim 3 \times 10^8\,\mathrm{K}$, are required for controlled fusion reactions. As a figure of merit for fusion experiments one frequently constructs the product of these two quantities, which should approach

$$n\tau_{\mathrm{E}}\widetilde{T} \sim 3 \times 10^{21}\,\mathrm{m}^{-3}\,\mathrm{s}\,\mathrm{keV} \tag{1.18}$$

for a fusion reactor. To get rid of the radioactive tritium component, one might consider pure D-D reactions in a more distant future. This would require yet another increase of the temperature by a factor of 10. Considering the kind of progress obtained over the past 40 years though (see Fig. 1.1.1 of Wesson [647]: a steady increase of the product $n\tau_{\mathrm{E}}T$ with a factor of 100 every decade!), one may hope that this difficulty eventually will turn out to be surmountable.

Returning to our question on the magnetic fields: no material containers can hold plasmas with densities of $10^{20}\,\mathrm{m}^{-3}$ and temperatures of 100–300 million K during times of the order of minutes, or at least seconds, without immediately extinguishing the 'fire'. One way to solve this problem is to make use of the *confining properties of magnetic fields*, which may be viewed from quite different angles:

(a) the charged particles of the plasma rapidly and tightly gyrate around the magnetic field lines (they 'stick' to the field lines, see Section 2.2);
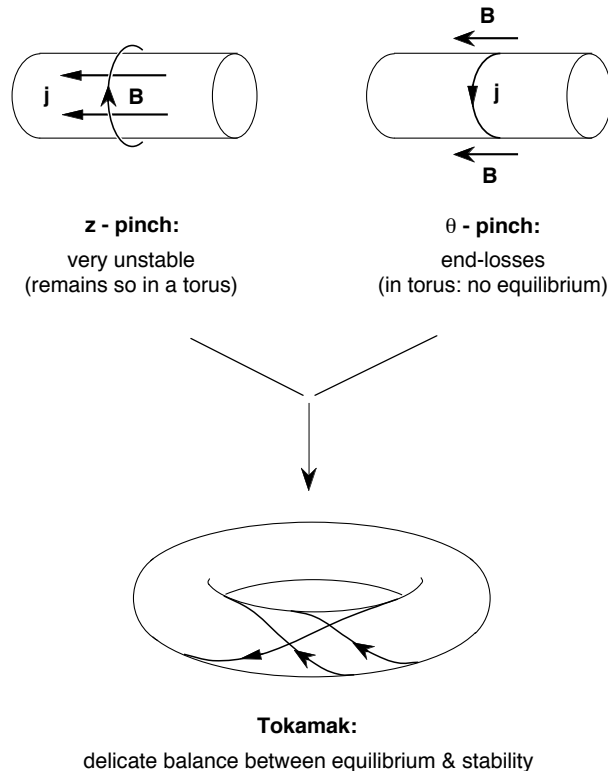
Fig. 1.4 Interaction of currents and magnetic fields: the history of plasma confinement experiments.

(b) fluid and magnetic field move together ('the magnetic field is frozen into the plasma', see Section 2.4), so that engineering of the geometry of the magnetic field configuration also establishes the geometry of the plasma;

(c) the thermal conductivity of plasmas is extremely anisotropic with respect to the magnetic field, $\kappa_\perp \ll \kappa_\parallel$ (Sections 2.3.1 and 3.3.2), so that heat is easily conducted along the field lines and the magnetic surfaces they map out, but not across.

Consequently, what one needs foremost is a *closed magnetic geometry* facilitating stable, static plasma equilibrium with roughly bell-shaped pressure and density profiles and nested magnetic surfaces. This is the subject of the next section.

1.2.3 Magnetic confinement and tokamaks

Controlled thermonuclear fusion research started in the 1950s in the weapons laboratories after the 'successful' development of the hydrogen bomb: fusion energy had been unleashed on our planet! The development of the peaceful, controlled, counterpart appeared to be a matter of a few years, as may become clear by considering the simplicity of early pinch experiments. The history of the subject is schematically illustrated in Fig. 1.4. In the upper part the two early attempts with the
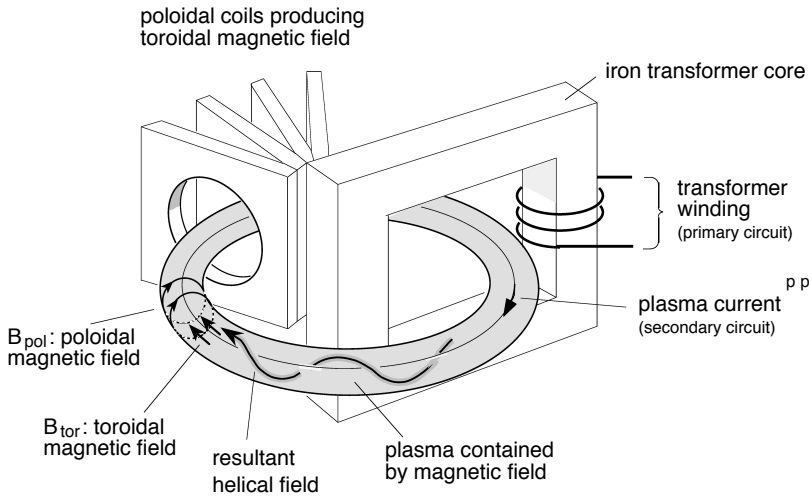
poloidal coils producing
toroidal magnetic field

iron transformer core

transformer
winding
(primary circuit)

plasma current
(secondary circuit)

B$_{pol}$: poloidal
magnetic field

B$_{tor}$: toroidal
magnetic field

resultant
helical field

plasma contained
by magnetic field

Fig. 1.5 Schematic presentation of magnetic confinement in a tokamak.

simple schemes of θ- and z-pinch are shown. Here, θ and z refer to the direction of the plasma current in terms of a cylindrical r, θ, z coordinate system. Since it is relatively straightforward to produce plasma by ionizing hydrogen gas in a tube, a very conductive fluid is obtained in which a strong current may be induced by discharging a capacitor bank over an external coil surrounding the gas tube. In a *z-pinch experiment*, this current is induced in the z-direction and it creates a transverse magnetic field B_θ, so that the resulting Lorentz force $(\mathbf{j} \times \mathbf{B})_r = -j_z B_\theta$ is pointing radially inward. In this manner, the confining force as well as near thermonuclear temperatures ($\sim 10^7$ K) are easily produced. There is only one problem: the curvature of the magnetic field B_θ causes the plasma to be extremely unstable, with growth rates in the order of microseconds. To avoid these instabilities, the orthogonal counterpart, the *θ-pinch experiment*, suggested itself. Here, current is induced in the θ-direction, it causes a radial decrease of the externally applied magnetic field B_z, so that the net Lorentz force $j_\theta \, \Delta B_z$ is again directed inward. In the θ-pinch, thermonuclear temperatures are also obtained, and the plasma is now macroscopically stable. However, pinching of the plasma column produces unbalanced longitudinal forces so that the plasma is squirted out of the ends, again terminating plasma confinement on the microsecond time scale. In conclusion, in pinch experiments the densities and temperatures needed for thermonuclear ignition are easily produced but the confinement times fall short by a factor of a million to a billion.

With these obstacles ahead, the nations involved with thermonuclear research decided it to be opportune to declassify the subject. This fortunate decision was landmarked by the Second International UN Conference on Peaceful Uses of Atomic Energy in Geneva in 1958, where all scientific results obtained so far were presented. Prospects then gradually became much brighter with the emergence of *the tokamak* alternative line (bottom part of Fig. 1.4 and Fig. 1.5) developed in the 1960s in the Soviet Union, and internationally accepted in the 1970s as the most promising scheme towards fusion. Crudely speaking, the tokamak configuration cures the main problems of the z-pinch (its instability due to the curvature of the transverse magnetic field) and of the θ-pinch (its end losses), both destroying the configuration on the microsecond time scale, by combining them

into a single scheme. The vessel is now a torus rather than a straight tube and the magnetic field is helical, with a poloidal and a larger toroidal component. The latter component of the magnetic field provides the crucial longitudinal 'backbone' for stability. Whereas the toroidal geometry simply eliminates the end-loss problem of the θ-pinch, it is not quite true that the kink instability problem of the z-pinch is eliminated as well. Instead, the basic MHD problem of tokamak confinement turns out to be a delicate balance between equilibrium considerations, favouring a large toroidal current, and stability considerations, which favour a minimum current so as to eliminate the driving force of the kink instabilities. Thus, tokamak performance is an *intricate optimization problem* which makes it both interesting and impressive. Concerning the latter: to have improved upon a technological parameter by a factor of 10^6 in thirty years (from confinement times of microseconds in the sixties to seconds in the nineties) is a kind of progress which is only paralleled by developments in computer technology.

For more on the history of fusion research: see Braams and Stott [86].

1.3 Astrophysical plasmas

We have sketched the efforts in controlled thermonuclear confinement experiments, where the prospect of abundant energy has driven scientists to ever deeper exploration of the plasma state. At this point an entirely different line of research should enter the presentation. This is the rapidly growing field of plasma-astrophysics, which has much older credentials than laboratory plasma research. We will introduce this topic by means of the example of the solar system, where the usual gravitational picture completely masks the dynamics of the plasmas that are present. To understand how this picture has changed in recent times, we introduce some basic astrophysical notions and recall events in space research. We will also use the opportunity to introduce numerical values of certain quantities that may not be familiar to some readers.

1.3.1 Celestial mechanics

To set the stage, recall *the traditional picture of the solar system*: the Sun is the central massive object (a thousand times more massive than Jupiter) which keeps the nine planets orbiting around it by its gravitational attraction. (See Fig. 1.6 and the numerical values summarized in Table B.6.)

Recall that the planets move according to *Kepler's laws* (1610):

(a) *The planetary orbits are ellipses* lying in or close to the ecliptic (the orbital plane of the Earth) *with the Sun in one of the focal points*. The inclination of the orbit with respect to the ecliptic is modest ($< 4°$) for most of the planets, whereas the largest values occur for the innermost planet (Mercury: $7°$) and for the outermost 'planet' (Pluto: $17°$). The ellipses are characterized by the eccentricity parameter $e \equiv c/a = (1 - b^2/a^2)^{1/2}$, where c is the distance of the focal points to the origin and a and b are the lengths of the semi-axes of the ellipse. Again, the highest eccentricities occur for Mercury ($e = 0.206$) and Pluto ($e = 0.250$), whereas they are small for the other planets ($e < 0.1$). Incidentally, it is to be noted that the ellipticity as measured by the ratio of the semi-axes, $b/a = \sqrt{1 - e^2}$, is $\sqrt{0.96} \approx 0.98$ for Mercury and 0.97 for Pluto, i.e. just deviations of 2% and 3% from a circle, and much less for the other planets. The original approximation of circular motion by the ancients appears not all that stupid. The big effect is not the deviation from a circle though, but
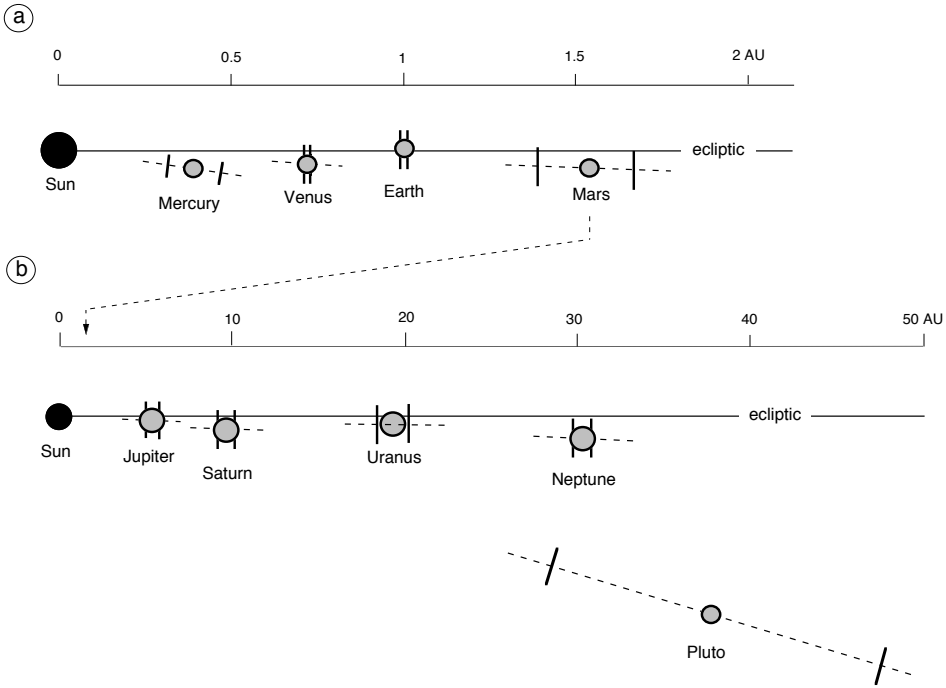
Fig. 1.6 Schematic representation of the size and inclination of the planetary orbits: (a) terrestrial planets, (b) giant planets and Pluto. The planets (not to scale) are drawn at a distance a from the Sun, where the vertical dashes indicate the extrema $a - c$ (perihelion) and $a + c$ (aphelion) of the orbital motion.

the eccentricity, i.e. *the shift c of the near-circular orbit*. This gives rise to significant variations in the distance to the Sun, as measured by the ratio $(a - c)/(a + c) = (1 - e)/(1 + e)$ which is 0.66 for Mercury and 0.60 for Pluto, as shown in Fig. 1.6.

(b) *The radius vector of the Sun to the planet sweeps out equal areas in equal times*. Hence, the orbital velocity is highest in the perihelion (the orbital point closest to the Sun) and smallest in the aphelion (the point farthest from the Sun). This *law of areas* is a consequence of conservation of angular momentum.

(c) *The harmonic law: The cubes of the semi major axis a of the orbits of the planets are proportional to the squares of the orbital period τ*,

$$a^3/\tau^2 = \text{const} \approx GM_\odot/4\pi^2 = 1\,(\text{AU})^3/\,\text{y}^2 = 3.38 \times 10^{18}\,\text{m}^3\,\text{s}^{-2}. \tag{1.19}$$

Here, G is the gravitational constant, $M_\odot$ is the mass of the Sun, $1\,\text{AU} = 1.5 \times 10^{11}\,\text{m}$ is the distance from the Earth to the Sun (the astronomical unit) and $1\,\text{y} = 3.16 \times 10^7\,\text{s}$ is, of course, the orbital period of the Earth.

▷ **Exercise** Use Table B.6 to check this number for the different planetary orbits.   ◁

Next, Kepler's laws were then founded on the laws of mechanics, in particular *Newton's law of*

gravitational attraction (1666):

$$F_{gr} = G \frac{M_1 M_2}{r^2} = -\frac{dV_{gr}}{dr}, \qquad (1.20)$$

where $V_{gr} = -G M_1 M_2/r$ is the gravitational potential energy. This law implies that the planets move as point particles in the gravitational field of the Sun whereas the whole solar system is kept together in dynamical equilibrium by gravity. All this belongs to the subject of *celestial mechanics* which is at the root of classical mechanics, which in turn constitutes the basis of physics. Thus, progress in understanding may schematically be depicted by the sequence Kepler (1609) → Newton (1687) → Lagrange (1782), Laplace (1799) → Hamilton (1845). After the work of these giants, the subject of classical mechanics (as, e.g., summarized by Goldstein [243]) has long been considered a closed subject. However, the sequence continues with the more recent names of Kolmogorov, Arnold and Moser (1964) associated with fundamental work on the stability of dynamical systems. At the present time, there is a resurgence of the subject of Hamiltonian mechanics through the development of the science of nonlinear dynamics.

So far, plasmas did not appear on the stage. Obviously, the gravitational attraction dominates everything. Gravitational and centrifugal acceleration balance perfectly in the leading order picture where the celestial bodies are treated as massive point particles. Since this is so, next order effects should be quite important (just like astronauts in an orbiting spacecraft may be accelerated by forces that are totally negligible as compared to gravity). Hence, when the internal structure of the stars (in this case, the Sun) and the planets is taken into account, the whole picture changes dramatically.

1.3.2 Astrophysics

In the nineteenth and twentieth century, there is a gradual shift away from exclusive interest in celestial mechanics towards the study of *the structure and evolution of stars and stellar systems*: the subject of astrophysics is born. Here, a basic postulate provides the guiding principle, viz. that *the laws of physics are valid throughout the Universe*. In historical perspective, the revolutionary character of this point of view can hardly be overestimated: Quintessence (according to *Webster's* Dictionary, 'the fifth and highest essence in ancient and medieval philosophy that permeates all nature and is the substance composing the heavenly bodies') is no longer essential, and 'heavenly' or 'celestial' are no longer descriptive adjectives for astronomical objects. A particularly relevant example is provided by the work of Kirchhoff and Bunsen (1859) who interpreted the observed dark lines in the spectrum of solar light, discovered by Fraunhofer (1814), as due to absorption by chemical elements in exactly the same way as spectra obtained in the laboratory. Consequently, most of our knowledge of the stars comes from *spectroscopy*, i.e. atomic physics applied to the photospheres of the stars where the spectra are determined by the temperature T of the surface and the different abundances of the chemical elements.

A measure for the relative brightness of a star is *the apparent magnitude m*,

$$m \equiv m_0 - 2.5 \times {}^{10}\log(l/l_0), \qquad (1.21)$$

where l is the flux, i.e. the amount of electromagnetic radiation energy passing per unit time through a unit area (taken at the position of our eye, or any other observing apparatus on Earth), and the subscript 0 refers to a reference star. The value of m_0 for the reference star is fixed by convention.

This definition bas been chosen to conform with the ancient classification based on what the human eye can distinguish, viz. five steps in a brightness scale ranging from $m = 0$ for the brightest star to $m = 5$ for the faintest one, corresponding to a decrease by a factor of $1/100$ in the flux.

Obviously, two stars of equal apparent magnitude may have a completely different value of the luminosity L, which is the total radiation energy output per unit time, since the flux l depends on the distance d from the star according to

$$l = L/(4\pi d^2). \tag{1.22}$$

Hence, a quantity of more intrinsic physical interest is *the absolute magnitude M*, which is based on the flux $\hat{l}$ that would be produced at the position of the Earth (ignoring atmospheric extinction) if the star were moved from its actual distance d to a fictitious distance $\hat{d} = 10\,\mathrm{pc}$ (parsec) from the Earth.[2] In other words, the absolute magnitude is defined as the apparent magnitude the star would have if positioned at $\hat{d}$, so that we obtain from Eqs. (1.21) and (1.22):

$$M \equiv m_0 - 2.5 \times {}^{10}\!\log(\hat{l}/l_0) = m - 2.5 \times {}^{10}\!\log(\hat{l}/l)$$

$$= m - 2.5 \times {}^{10}\!\log(d^2/100) = m + 5 - 5 \times {}^{10}\!\log d, \tag{1.23}$$

where d is measured in pc. For the Sun, with $d_\odot = 1\,\mathrm{AU} = 1.5 \times 10^8\,\mathrm{km} = 5 \times 10^{-6}\,\mathrm{pc}$, we get a huge difference between the absolute and the apparent magnitude: $M_\odot = m_\odot + 31.5$. (Note that we have used the same symbol $M_\odot$ already in Section 1.3.1 to indicate the solar mass; every now and then, we will not be able to avoid context-dependent notation.) The reason is clear: the apparent magnitude is based on night-time observation and, hence, totally out of range for the Sun. On the other hand, for the absolute magnitude of the Sun the very ordinary value $M_\odot = 4.7$ is obtained: apart from its proximity, the Sun is just an ordinary star.

A particularly effective way of representing the absolute magnitudes of a large number of stars is the celebrated *Hertzsprung–Russell diagram*, where the absolute magnitude M is plotted versus the effective surface temperature T_{eff}, or the associated spectral class indicated by the letters O, B, etc. (Fig. 1.7). A crude estimate of the curve for the main sequence stars may be obtained by using the Stefan–Boltzmann black-body radiation law for the luminosity,

$$L = 4\pi R^2 \sigma\, T_{\mathrm{eff}}^4. \tag{1.24}$$

Here, R is the radius of the star, $\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$ is the constant of Stefan–Boltzmann and T_{eff} is the effective surface temperature of the star. For stars of equal size, we obtain from the first line of Eq. (1.23) the following difference in their absolute magnitudes:

$$\Delta M = -2.5 \times \Delta({}^{10}\!\log\hat{l}) = -2.5 \times \Delta({}^{10}\!\log L) = -10 \times \Delta({}^{10}\!\log T_{\mathrm{eff}}). \tag{1.25}$$

This roughly checks with the overall slope of the Hertzsprung–Russell diagram.

For the Sun, $R_\odot = 700\,000\,\mathrm{km}$ and $T_{\mathrm{eff},\odot} = 5777\,\mathrm{K}$ (i.e., spectral class G) so that $L_\odot =$

---

[2] Note on *distance scales*: a star at a distance $d = 1\,\mathrm{pc}$ produces a parallax of $1'' = 4.85 \times 10^{-6}\,\mathrm{rad}$, so that $1\,\mathrm{pc} = (4.85 \times 10^{-6})^{-1}\,\mathrm{AU} = 2.06 \times 10^5\,\mathrm{AU} = 3.26\,\text{light-years} = 3.09 \times 10^{16}\,\mathrm{m}$. The distance to the next nearest star, Alpha Centauri, is $1.3\,\mathrm{pc}$. The size of our Galaxy (the Milky Way) is $50\,\mathrm{kpc} = 1.6 \times 10^5\,\text{light-years} = 1.3 \times 10^8 \times$ the size of the solar system (the diameter of the orbit of Pluto, i.e. $2 \times 40\,\mathrm{AU}$).

Light also provides useful estimates for *time scales*: a photon would take $2\,\mathrm{s}$ to travel from the centre of the Sun to the surface if the Sun were optically thin. (In reality, because of the innumerable absorptions and re-emissions it takes about 10^7 years, as we have already noted in Section 1.2.1.) It then takes $8.3\,\mathrm{min}$ to reach Earth, 5.6 hours to reach Pluto, and 4.2 years to reach Alpha Centauri.
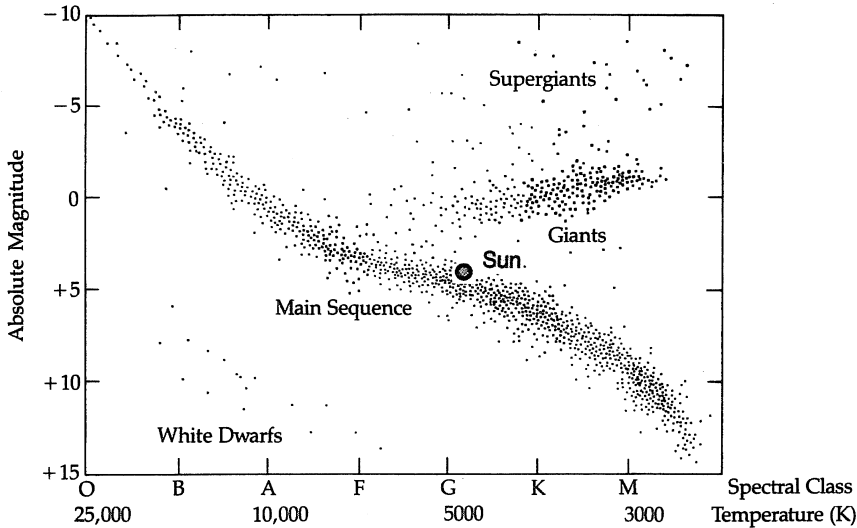
Fig. 1.7 Hertzsprung–Russell diagram: the Sun is an ordinary, main sequence, star. (Adapted from Zeilik and Smith [666].)

3.89×10^{26} W. Incidentally, the flux $l = L_\odot/(4\pi d^2)$ at the position of the Earth is called the solar constant. Since $d = 1$ AU, its value turns out be $l = 1.38$ kW/ m^2: just the right value for human and other life. However, at this point of our exposition, we have turned away from Earth-centred considerations to the intrinsic properties of the stars. The central position of the point representing the Sun in the Hertzsprung–Russell diagram is then just another way of expressing that the Sun is but an ordinary main sequence star. Yet, as far as distance is concerned, we should consider ourselves lucky to have a typical star close enough to permit *spatially resolved* observations! This is crucial for our understanding of plasma dynamics in the Universe as a whole.

Not only is the solar system kept together by gravity, but the individual celestial bodies of the Sun and the planets are also kept together by gravity and, as a result, they *contract*. Stars with masses like that of the Sun (Jupiter is just too small to qualify as a star) contract so much that in the centre *densities* and *temperatures* are reached that are *high enough* for *thermonuclear burn by fusion* reactions of hydrogen, viz. $T_c = 1.5 \times 10^7$ K, and $\rho_c = 1.5 \times 10^5$ kg m^{-3}. We have encountered these fusion reactions and their conditions in Eqs. (1.8) and (1.9). Recall that, under these conditions, matter is ionized so that we encounter the plasma state again in the core of the Sun and the other stars. It appears that we have closed the circle and that the announcement of the theme for this book will simply be: laboratory fusion and astrophysical fusion reactions require the study of plasma physics. This is not the case. Reality is much more interesting (and subtle) than this.

1.3.3 Plasmas enter the stage

With the discovery of Bethe and von Weiszäcker in 1939 that thermonuclear fusion reactions take place in the centre of the Sun and the other stars, we know the ultimate source of the enormous amounts of power emitted in the form of visible and ultraviolet light. However, there is quite some

distance in space and time between this source and the starlight it eventually produces. In the intermediate stages, this huge thermonuclear energy source indirectly excites a wide variety of additional, plasma dynamical, phenomena. Could we 'see' that? If only the blinding brightness of the solar disc were blocked for a few minutes we would be able to tell. Fortunately, provision has been made for that: the relative sizes of the Moon and the Sun, and their distances to the Earth, are precisely of the right magnitude to permit occultation of the Sun every now and then to exhibit an extremely beautiful phenomenon. At the moment of the eclipse, even the birds hold their breath, and a human being lucky enough to be at the right spot at the right moment can see a hot (millions of degrees) plasma with his own eyes: a diffuse light due to scattering of sunlight by the coronal plasma and stretching out over several solar diameters. Even the magnetic structures supporting it are visible to the (admittedly prejudiced) physicist in the form of streamers of plasma tracing out magnetic field lines and helmet structures associated with magnetic cusps (see Chapter 8). Hence, at a solar eclipse, one catches a wonderful glimpse of a huge magnetized plasma structure which engulfs the whole solar system.

This structure is the solar corona expanding into the *solar wind*, which forms magnetospheres when encountering the magnetic fields of the planets and which is a giant magnetosphere by itself, called the heliosphere, terminating only at distances beyond the solar system. The solar wind carries the wave-like signals of its creation, but it also carries the intermittent radiation and high-energy particle signatures of violent outbursts of magnetic energy releases by *flares* and *coronal mass ejections* (CMEs) at the solar surface (Chapter 8). This highly unsteady plasma dynamical state creates the critical conditions for magnetic storms in the magnetosphere. (*The* magnetosphere is always shorthand for the magnetosphere of the Earth, or *our* magnetosphere, like *the* Galaxy always stands for the Milky Way, or *our* Galaxy.) It forms a threat for safety of personnel and proper functioning of spacecrafts. This aspect of solar wind dynamics is called *space weather*.

Receding now to the interior of the star, closer to the thermonuclear energy source, we encounter the phenomena responsible for all this: radiation transport and convection which, together with the differential rotation of the star, create the conditions required for a dynamo. This dynamo produces magnetic fields that do not stay inside the star but are expelled, with the plasma, to form the extremely hot coronae and stellar winds that are the characteristics of X-ray emitting stars. Incidentally, the creation of magnetic fields in the interior of stars and the high temperatures of coronal plasmas are two plasma physical problems that are far from being solved at present. While we do not pretend to solve them here, we do believe that for progress one needs to delve deeply into basic magnetohydrodynamics, which is the subject of this book. Hence, the connection between laboratory and astrophysical plasmas is not the thermonuclear fusion reactions but their indirect result far away: *magnetized plasmas are present everywhere in the Universe!*

How do we know? High-resolution astronomical observations over the whole range of the electromagnetic frequency spectrum by means of 'telescopes', ground-based or from space vehicles, have produced irrefutable evidence for that. Whereas Sputnik (1957) and the Apollo flight to the Moon (1969) have spoken to the imagination of a large public, the less-known observations of the Sun and stars by means of X-ray telescopes on board rockets and the risky (manned) Skylab missions of 1973 and 1974 may have produced a more lasting revision of our scientific picture of the cosmos. It revealed the tremendously dynamic magnetic structure of the solar atmosphere and corona with myriads of closed magnetic flux tubes, containing hot plasma, bordering open magnetic regions, so-called coronal holes where the cooler plasma is associated with reduced X-ray emission. These early

observations were finally superseded by the higher resolution images obtained from the Japanese satellite Yohkoh and the NASA-ESA Solar and Heliospheric Observatory SOHO, launched in 1992 and 1995, respectively. Meanwhile, the plasma physics picture of the solar system has been augmented considerably by planetary missions like Voyager 2 (launched 1989) travelling to the outer edges of the solar system and also measuring the magnetic fields of the giant planets (see Table B.6), or the flight of Ulysses (launched 1990), whereas Cluster II (launched in 2000) provides many more details of the three-dimensional structures of the magnetospheres. In the same period, the picture of the structure of the Galaxy and the Universe, essentially including galactic and cosmic magnetic fields, has evolved explosively due to the ever improved resolution of the traditional telescopes, the radio telescopes in large and very large arrays, and the numerous space missions, culminating in the launch of the Hubble Space Telescope in 1990. At the time of this writing, the Parker Solar Probe and the Solar Orbiter are about to be launched to reach positions extremely close to the Sun to finally, hopefully, disclose the mysteries of the coronal temperature and of the solar wind acceleration.

We summarize by making a few sweeping statements, obviously not meant to present final scientific truths:

– By means of X-ray observations, the few minutes of a solar eclipse have been extended almost indefinitely to provide a picture of the corona as a high-temperature plasma with *extremely complex dynamical magnetic structures* (Priest [510]).

– The interaction of the solar wind with the planetary magnetospheres is one of the most interesting plasma laboratories in space, offering the possibility of studying *spatially resolved plasma dynamics*.

– Finally, since the Sun is an ordinary star, what has been learned there may be extrapolated to other stars, of course with due modifications (Schrijver and Zwaan [544]). Going one step further, including neutron stars and pulsars (Mestel [438]), accretion disks about compact objects, etc.: what has been learned from magnetic plasma structures in the solar system may be extrapolated, again with due modifications, to the more exotic astrophysical objects that cannot be observed with spatial resolution but that do provide *intricate temporal signatures*.

Thus, a secondary layer (considering gravity and nuclear fusion as the primary layer) of phenomena has been revealed in the solar system that is present everywhere in the Universe. This brings us to our next subject.

1.3.4 The standard view of nature

Consider the standard view of nature, as developed in the twentieth century and widely held to provide the correct scientific representation of the Universe (Fig. 1.8). The four fundamental forces govern phenomena at immensely separate length scales, at least at times beyond 'The First Three Minutes' (Steven Weinberg, 1978) after the big bang. At the risk of caricaturing the wonderful achievements of elementary particle physics, on a scale of increasing dimensions, the weak and strong nuclear forces in the end just produce the different kinds of nuclei and electrons which constitute the main building blocks of matter. In a sense, these forces are exhausted beyond the length scale of 10^{-15} m. Since nuclei are positively charged and electrons negatively, the much longer

$$\boxed{\textbf{Nuclear forces}}$$

$$\Downarrow$$

quarks / leptons

nuclei $(+)$ / electrons $(-)$ $10^{-15}\,\mathrm{m}$

$$\boxed{\textbf{Electrostatic forces}}$$

$$\Downarrow$$

atoms / molecules $10^{-9}\,\mathrm{m}$

(ordinary matter: electrically neutral)

$$\vdots$$

$$\boxed{\textbf{Gravity}}$$

$$\Downarrow$$

stars / solar system $10^{9}/10^{13}\,\mathrm{m}$

galaxies / clusters $10^{20}/10^{23}\,\mathrm{m}$
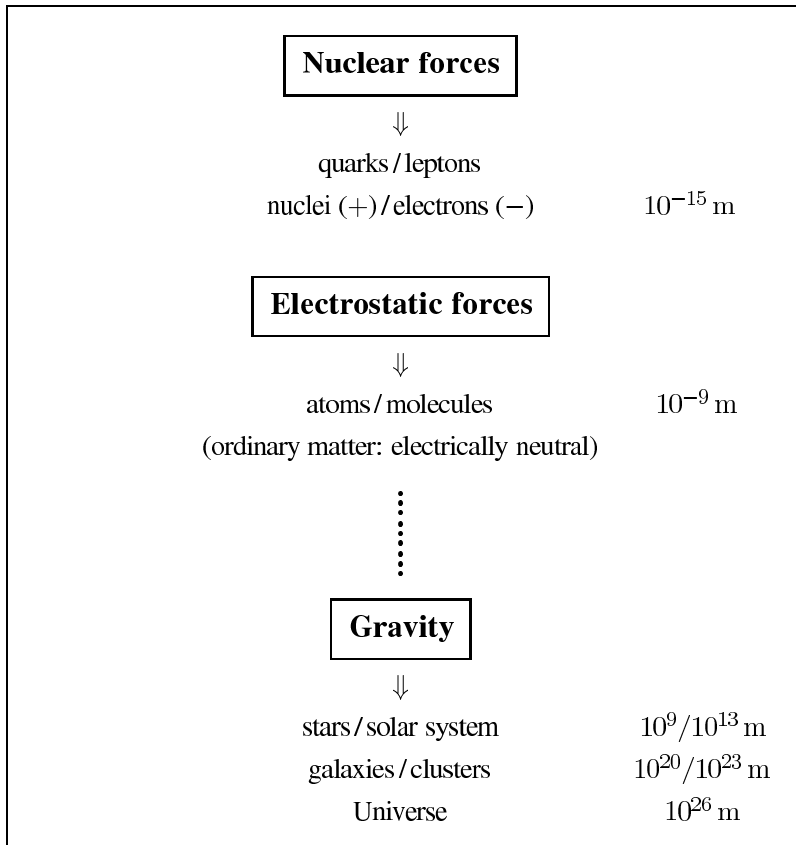
Universe $10^{26}\,\mathrm{m}$

Fig. 1.8 The standard view of nature: not accounting for the abundance of plasma in the Universe.

range electric forces then take over, giving rise to the next stage of the hierarchy: 'ordinary' matter consisting of atoms and molecules with sizes of the order of 10^{-9} m. Since these particles are electrically neutral, all there appears to remain is the gravitational force which requires the collective effect of huge amounts of matter over length scales beyond 10^{9} m in order to become sizeable. This gives rise to the different astronomical systems of stars, galaxies, clusters of galaxies, etc. Since the gravitational force is a long-range force which is solely attractive (there is no screening by repulsive negative mass particles), this force is only 'exhausted' at the scale of the Universe itself.

It will be noticed that the 'picture' of Fig. 1.8 jumps the eighteen orders of magnitude from atoms to stars (indicated by the dots) under the assumption that nothing of fundamental interest happens there. One could remark that we just happen to live on the least interesting level of the physical Universe, or one could dwell on the disproportion of man between the infinities of the small and the large (Pascal), or one could join the recent chorus of holistic criticism on the reductionism of physics. So much appears to be correct in the latter viewpoint that the given picture does not have any place for the complexities of solid state physics, fluid dynamics or biological systems, to name just a few. It should come as a big disappointment that nature would hang together from elementary particles to cosmology without really involving the intermediate stages.

For our subject, however, another misrepresentation is implicit in Fig. 1.8. We have started our discussion in Section 1.1 by noting that more than 90% of matter in the Universe is plasma so that *the Universe does not consist of ordinary matter (in the usual sense) but most of it is plasma*! It is true that plasma is usually also almost electrically neutral, like atoms and molecules, but the important difference is that the ions and electrons are not tied together in atoms but *move about freely as fluids*. The large-scale result of this dynamics is the formation of *magnetic fields* which in turn determine the plasma dynamics: a highly nonlinear situation. These magnetic fields not only bridge the gap between microscopic and macroscopic physics, but they also reach far into the astrophysical realm at all scales. Hence, the subject of plasma-astrophysics is of basic importance for understanding phenomena occurring everywhere in the Universe.

It will have been noted that we have ignored the unification of electric and magnetic forces brought about by Maxwell's theory of electromagnetism. There is a good reason for this since, in the domain of plasma dynamics, electric and magnetic forces are associated with quite different effects operating on immensely different length scales with the magnetic forces dominating on the longer length scales. Consequently, most of plasma dynamics is well described by exploiting the so-called pre-Maxwell equations, i.e. Maxwell's equations without the displacement term. We will see in later chapters that the dynamics of magnetic fields is so interwoven with the dynamics of the plasma itself that its proper description takes precedence over the one where electric and magnetic fields are treated on an equal footing.

The most important law for magnetic fields is $\nabla \cdot \mathbf{B} = 0$, which implies that there are no sources or sinks. This law is incompatible with spherical symmetry so that the simplest basic geometries of magnetised plasmas are completely different from the ones prevailing on the atomic and gravitational scales. In particular, large-scale tubular magnetic structures occur which move with the plasma so that magnetic forces are transmitted with the fluid. One could hardly imagine a bigger contrast with central electrostatic and gravitational forces decaying in vacuum with distance as r^{-2}! Striking examples are solar flares, the X-ray emitting corona of the Sun, and coronal mass ejections (plasma expelled from the main body of the Sun against the gravitational pull), the interaction of the solar wind with the planetary magnetic fields, waves and flows in neutron star magnetospheres, extragalactic jets, spiral arm instabilities, etc.

In conclusion: the standard view of nature fails over a wide range of scales because it does not recognize the presence of magnetized plasmas all over the Universe. Magnetic fields are an important aspect of modern astrophysics. Hence, *the nonlinear interaction of plasma and complex magnetic structures* presents itself as an important common theme of laboratory and astrophysical plasma research.

1.4 Definitions of the plasma state

1.4.1 Microscopic definition of plasma

Turning now to the subject of plasmas proper, we need to refine the crude definition given in Section 1.1. This involves a closer study of the microscopic properties required for the plasma state. To that end, we follow the exposition given by F. F. Chen in the first chapter of his book on Plasma Physics [117].

First, we need to relax the condition of complete ionization given in our crude definition since

plasma behaviour is already encountered when the ionization is only partial. A simpler definition of plasma would then be: *a plasma is an ionized gas*. However, how much ionization is required? An estimate may be obtained from the *Saha equation* which gives an expression for the amount of ionization of a gas in thermal equilibrium:

$$\frac{n_i}{n_n} = \left(\frac{2\pi m_e k}{h^2}\right)^{3/2} \frac{T^{3/2}}{n_i} \, e^{-U_i/kT} \approx 2.4 \times 10^{21} \frac{T^{3/2}}{n_i} \, e^{-15.8 \times 10^4/T} \,. \tag{1.26}$$

Here, n_i and n_n are the particle densities of ions and neutrals (in m^{-3}), U_i is the ionization energy (in J), T is the temperature (in K) and the other symbols have their usual meaning. The numbers on the RHS are obtained by exploiting Table B.1, $(2\pi m_e k/h^2)^{3/2} = 2.4 \times 10^{21} \, K^{-3/2} \, m^{-3}$, and inserting the ionization energy of hydrogen, $U_i = 13.6 \, eV$. (Ionization energies are usually given in eV, where $1 \, eV = 1.6 \times 10^{-19} \, J$, which corresponds with $1.16 \times 10^4 \, K$ if one divides by the Boltzmann factor k.)

For *air* at room temperature, where $n_n = 3 \times 10^{25} \, m^{-3}$, $T = 300 \, K$, $U_i = 14.5 \, eV$ (ionization potential for nitrogen), one finds a huge negative factor (-560) in the exponent of Eq. (1.26) so that the final ratio of the densities of ions and neutrals is extremely small: $n_i/n_n \approx 2 \times 10^{-122} \ll 1$. As expected, the degree of ionization is totally negligible: air is not a plasma. For hydrogen in *a tokamak machine* with $T = 10^8 \, K$ and $n \equiv n_e = n_i = 10^{20} \, m^{-3}$, one finds that the expression in the exponent $U_i \ll kT$ so that $\exp(U_i/kT) \approx 1$ and $n_i/n_n \approx 2.4 \times 10^{13} \gg 1$: in such machines genuine plasmas are obtained. However, for *the core of the Sun* with $T = 1.6 \times 10^7 \, K$ and $n = 10^{32} \, m^{-3}$, one finds that $n_i/n_n \approx 1.5$. Surprisingly, although thermonuclear reactions take place, ionization is not complete in the core of the Sun and plasma behaviour is not completely dominant! This is due to the extremely high densities there. On the other hand, *in the corona of the Sun*, with typical values of $T = 10^6 \, K$ (not thermonuclear, but anomalously high: a subject of concern in later chapters) and $n = 10^{12} \, m^{-3}$, we have $n_i/n_n = 2.4 \times 10^{18}$: matter in the corona is an excellent plasma!

Even though we now have a measure for the degree of ionization required for plasmas, we still do not have a criterion for plasma behaviour. A much more precise definition, as given by Chen, reads: *a plasma is a quasi-neutral gas of charged and neutral particles which exhibits collective behaviour*.

In an ordinary gas neutral molecules move about freely (there are no net electromagnetic forces) until a collision occurs. This is a *short-range binary* event in which two particles hit each other. In a hard-sphere model of the molecules, the cross-section for such a collision is just the cross-section of the particles. In a plasma, on the other hand, the charged particles are subject to *long-range collective* Coulomb interactions with many distant encounters (so-called pitch angle scattering). Although the electrostatic force between two charged particles decays with the mutual distance ($\sim 1/r^2$), the combined effect of all charged particles may not even decay, since the interacting volume increases ($\sim r^3$). This is a typical collective effect, the result of the statistics of many particles, each moving in the average electrostatic field created by all the other particles.

We now discuss this electrostatic collective aspect quantitatively. For collective plasma behaviour, according to Chen [117], three conditions should be satisfied.

(a) The long-range Coulomb interaction between charged particles should dominate over the short-range binary collisions with neutrals. Indicating typical *time scales* of collective oscillatory motion

by τ ($\sim 1/\omega$ when ω is the angular frequency of the oscillations), this implies that

$$\tau \ll \tau_{\mathrm{n}} \equiv \frac{1}{n_{\mathrm{n}}\sigma v_{\mathrm{th}}} \approx \frac{10^{17}}{n_{\mathrm{n}}\sqrt{T}}\,, \tag{1.27}$$

where τ_{n} is the mean time between collisions of charged plasma particles with neutrals. The estimate on the RHS is obtained by writing $\tau_{\mathrm{n}} \approx \lambda_{\mathrm{mfp}}/v_{\mathrm{th}}$, where λ_{mfp} is the mean free path and v_{th} is the thermal speed of the particles. With $\lambda_{\mathrm{mfp}} = (n_{\mathrm{n}}\sigma)^{-1}$, where the cross-section $\sigma = \pi a^2 \approx 10^{-19}\,\mathrm{m}^2$ is obtained by taking the radius $a \approx 2 \times 10^{-10}\,\mathrm{m}$ of a neutral H atom, and $v_{\mathrm{th}} \approx (kT/m_p)^{1/2} \approx 100\sqrt{T}$, we obtain expression (1.27) for τ_{n}. Since we are interested in plasma conditions, we should convert this expression from neutral density n_{n} to ion density n_{i} by means of the Saha equation (1.26). For solar coronal plasma with $T = 10^6$ K and $n_{\mathrm{i}} = 10^{12}\,\mathrm{m}^{-3}$, so that $n_{\mathrm{n}} = 4 \times 10^{-7}\,\mathrm{m}^{-3}$, this implies $\tau \ll \tau_{\mathrm{n}} \approx 2 \times 10^{20}\,\mathrm{s}$ (!). For tokamaks with $T = 10^8$ K and $n_{\mathrm{i}} = 10^{20}\,\mathrm{m}^{-3}$, the condition becomes $\tau \ll \tau_{\mathrm{n}} \approx 2.4 \times 10^6\,\mathrm{s}$. Clearly, the condition (1.27) represents very mild restrictions on the time scales for plasma behaviour.

(b) The *length scale* of plasma dynamics should be much larger than the minimum size over which the condition of *quasi-neutrality* holds. Production of overall charge imbalance creates huge electric fields which in turn produce huge accelerations, so that such an imbalance is neutralized almost instantaneously and the plasma maintains charge neutrality to a high degree of accuracy. However, local charge imbalances may be produced by thermal fluctuations. To estimate their size, one should compare the thermal energy kT of the particles with their electrostatic energy $e\Phi$. The latter can be estimated through Poisson's law, $dE/dx = -d^2\Phi/dx^2 = -(1/\epsilon_0)\,en$, so that $kT \approx e\Phi \approx (1/\epsilon_0)\,e^2n\lambda_{\mathrm{D}}^2$. Here, the gradient length has been equated to the *Debye length*, which is the typical size of a region over which charge imbalance due to thermal fluctuations may occur. Hence, length scales for a quasi-neutral plasma should satisfy

$$\lambda \gg \lambda_{\mathrm{D}} \equiv \sqrt{\frac{\epsilon_0 kT}{e^2 n}} \approx 70\sqrt{\frac{T}{n}}\,, \tag{1.28}$$

where $n \equiv n_{\mathrm{e}} \approx Zn_{\mathrm{i}}$ (with Z the ion charge number). Inserting the numbers for coronal plasma again, we find $\lambda_{\mathrm{D}} = 0.07\,\mathrm{m}$. For typical transverse length scales of coronal loops, $\lambda \sim 10000\,\mathrm{km} = 10^7\,\mathrm{m}$, the condition is easily satisfied.

▷ **Exercise** Exploit the tables of Appendix B to also find out what this condition means for other cases, like tokamak plasmas. ◁

Note that the concept of Debye length alleviates our original statement about long-range electrostatic forces considerably: sizeable regions with charge accumulation do not form through thermal fluctuations alone. A free charge, which in vacuum would have a potential $\Phi = q/r$, in a plasma is surrounded by a cloud of particles of opposite charge, which effectively shields the Coulomb potential for distances much larger than the Debye length: $\Phi_{\mathrm{eff}} = (q/r)\exp(-r/\lambda_{\mathrm{D}})$ (called *Debye shielding*). This just implies that $Z\,n_{\mathrm{i}} \approx n_{\mathrm{e}}$, i.e. *quasi* charge-neutrality holds. It does not mean that electric fields do not arise in plasmas. Actually, quite the opposite: electric fields arise almost automatically when plasmas move in a magnetic field. However, charge imbalances are extremely small when measured in terms of the total charge of the separate species:

$$|Zn_{\mathrm{i}} - n_{\mathrm{e}}|/n_{\mathrm{e}} \ll 1\,. \tag{1.29}$$

Hence $Zn_{\mathrm{i}} \approx n_{\mathrm{e}}$ holds to a high degree of accuracy in plasmas.
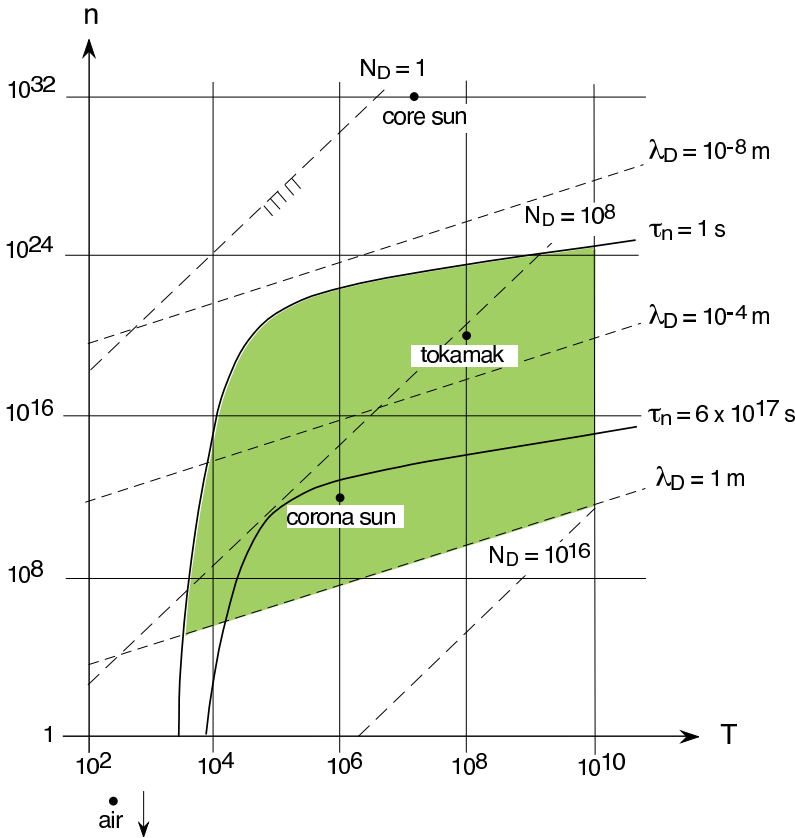
Fig. 1.9 Conditions for collective plasma behaviour, in terms of the density $n \equiv n_e \approx Z n_i$ and temperature $T \sim T_e \sim T_i$, are satisfied in the green area for time scales $\tau < \tau_n = 1\,\text{s}$ and length scales $\lambda > \lambda_D = 1\,\text{m}$, where $N_D \gg 1$. The restrictions on the upper time limit of low-density astrophysical plasmas quickly approach the age of the Universe, whereas the restrictions on the lower length limit for high density laboratory fusion experiments approach microscopic dimensions.

(c) Finally, in order for statistical considerations to be valid, sufficiently many particles should be present in a *Debye sphere*, i.e. a sphere of radius λ_D:

$$N_D \equiv \tfrac{4}{3}\pi \lambda_D^3\, n \approx 1.4 \times 10^6\, \sqrt{T^3/n} \gg 1\,. \tag{1.30}$$

For our example of a coronal plasma, this yields $N_D = 1.4 \times 10^9 \gg 1$, which is again easily satisfied. Note that both $\lambda_D \sim n^{-1/2}$ and $N_D \sim n^{-1/2}$ so that very high density plasmas are OK with respect to condition (1.28), but not with respect to condition (1.30). For example, for the core of the Sun, $\lambda_D = 3 \times 10^{-11}\,\text{m}\,(!)$, but $N_D \approx 9$: not so good for the application of statistical mechanics.

In conclusion: collective plasma behaviour is encountered when the time scales are sufficiently short with respect to collision times with neutrals, $\tau \ll \tau_n$, the length scales are much larger than the Debye length, $\lambda \gg \lambda_D$ and there are many particles in a Debye sphere, $N_D \gg 1$. These

conditions can be translated in terms of conditions on the density and the temperature, which are satisfied under a wide variety of conditions, as shown in Fig. 1.9. This picture confirms our statement of Section 1.1: plasma is a very normal state of matter in the Universe.

1.4.2 Macroscopic approach to plasma

So far, the most important physical variable in laboratory and astrophysical plasmas, viz. the magnetic field, has been conspicuously absent from our definition of the plasma state. The reason is that we have followed the traditional exposition of basic plasma theory, which starts with the microscopic point of view and stresses the collective phenomena involving electric fields. Whereas the length and time scales appropriate for these phenomena may be discussed in terms of the *local* values of the plasma density n and the temperature T, the magnetic field $\mathbf{B}$ brings in entirely different, *global*, considerations. (Incidentally, here one may detect one of the ways in which reductionism fails to recognize the emergence of new levels in the description of nature.) We have already observed the central importance of magnetic fields in confinement of fusion plasmas (Section 1.2) and in the dynamics of an enormous variety of astrophysical objects (Section 1.3), where we have stressed their basic non-locality. We now have to quantify these observations.

The macroscopic point of view does not set aside the microscopic conditions derived in Section 1.4.1 but it incorporates them as follows. A macroscopic description requires (1) frequent enough collisions between electrons and ions to establish fluid behaviour, (2) in addition to the microscopic conditions of length and time scales involving the density and temperature, global conditions on length and time scales involving the magnetic field. The latter quantities have to be large in order to permit *averaging over the microscopic dynamics*. To quantify this step requires the consideration of the cyclotron (or gyro) motion of the electrons and ions, which will only be discussed in the next chapter. Anticipating that discussion, the cyclotron radii $R_{e,i}$ and the inverse cyclotron frequencies $\Omega_{e,i}^{-1}$ of the electrons and ions will be shown to be inversely proportional to the magnetic field strength, $R_{e,i} \sim B^{-1}$ and $\Omega_{e,i}^{-1} \sim B^{-1}$, where the ion expressions provide the most limiting conditions on macroscopic length and time scales. Consequently, 'large enough' means that macroscopic length and time scales should be much larger than R_i and Ω_i^{-1}, respectively. This is possible when the magnetic field is large enough for the plasma volume under consideration to contain many ion gyro radii and when the dynamic phenomena last many ion gyro periods.

Summarizing: *for a valid macroscopic model of a particular magnetized plasma dynamical configuration, size, duration, density and magnetic field strength should be large enough to establish fluid behaviour and to average out the microscopic phenomena (i.e. collective plasma oscillations and cyclotron motions).*

The distinguishing feature for macroscopic plasma dynamics is the interaction of plasma motion and magnetic field geometry. This fluid aspect of plasmas concerns the motion of the plasma as a whole, without considering the separate electrons and ions, under the influence of magnetic fields. These fields are, in turn, generated by the plasma motion itself: a highly nonlinear situation. *The theoretical tool to describe this global interplay of plasma and magnetic field is called MHD ≡ magnetohydrodynamics.* The objective of this book is to demonstrate how this theory provides the common basis for the description of laboratory and astrophysical plasma dynamics.

The (surprisingly many) different aspects of the given definition of a macroscopic plasma model

will be discussed one by one in the following chapters. In particular, Chapters 2 and 3 will provide the missing quantitative elements of microscopic plasma physics needed for the foundation of macroscopic plasma dynamics. The basic Chapter 2 may be skipped by readers that are already familiar with basic plasma physics. The advanced Chapter 3 may be skipped as well by readers that wish to start with magnetohydrodynamics proper as soon as possible.

One question, answered in detail in the next chapters, must be addressed at least provisionally here, viz.: why is the electric field not even mentioned in the above discussion of macroscopic plasma dynamics? The reason is that the electric field becomes, in fact, a secondary quantity in MHD. Large electrostatic fields due to charge imbalances only occur over Debye length scales, which are averaged out, and electromagnetic waves are absent in non-relativistic MHD since the displacement current is negligible. The electric field is then determined from the primary variables of the velocity $\mathbf{v}$ and the magnetic field $\mathbf{B}$ by means of 'Ohm's law' for a nearly perfectly conducting plasma: $\mathbf{E} + \mathbf{v} \times \mathbf{B} \approx 0$, i.e. the electric field in a frame moving with the plasma vanishes.

1.5 Literature and exercises

Notes on literature

Some general references for the whole book are given under the different headings below. The complete information on the references is given at the end of the book.

Introductory plasma physics

– Boyd and Sanderson, *Plasma Dynamics*, one of the older textbooks on plasma physics, has been revised completely in *The Physics of Plasmas* [84]. It may be recommended for complementary reading since it contains a thorough discussion of the various models used to describe plasma physics.
– Chen, *Introduction to Plasma Physics and Controlled Fusion* [117] is the most readable, and probably most widely used, basic textbook on plasma physics.
– Bittencourt, *Fundamentals of Plasma Physics* [70] is a basic theoretical course on plasma physics with detailed calculations.
– Sturrock, *Plasma Physics* [580] is a basic text on plasma physics written for graduate students from astrophysics, space science, physics and engineering departments.
– Goldston and Rutherford, *Introduction to Plasma Physics* [244] is a basic text on plasma physics based on teaching by two experts in tokamak physics.
– Hazeltine and Waelbroeck, *The Framework of Plasma Physics* [298] develops a basic understanding of plasma and the wide ranging applications of plasma behaviour in a thorough and systematic manner.
– Gurnett and Bhattacharjee, *Introduction to Plasma Physics* [272] is a comprehensive text on basic plasma theory with applications to space and laboratory plasmas.
– Fitzpatrick, *Plasma Physics, An Introduction* [178] provides the theoretical framework describing the most common plasmas of nature in a clear and concise way.

Topics in advanced plasma physics

– Leontovich (ed.), *Reviews of Plasma Physics*, Vols. 1–5 [402] contain the unsurpassed Russian expositions of the basics of plasma theory after the declassification of the 1958 Geneva Conference.
– Akhiezer, Akhiezer, Polovin, Sitenko and Stepanov, *Plasma Electrodynamics* [8] is another classic from one of the Soviet theory schools, systematically building up plasma physics by kinetic and hydrodynamic methods and progressing to the diverse linear and nonlinear manifestations of the plasma state.
– Dendy (ed.), *Plasma Physics: An Introductory Course* [150] contains the material taught at the yearly Culham summer schools on plasma physics.

– Donné, Rogister, Koch and Soltwisch (eds.), *Proc. Second Carolus Magnus Summer School on Plasma Physics* [162] contains the material taught at that summer /school held every other year.

Magnetohydrodynamics

– Freidberg, *Ideal MHD* [186] is a textbook on ideal MHD, based on lectures at MIT for graduate students and researchers, which puts perfect conductivity and the applications to fusion research centre stage.

– Lifschitz, *Magnetohydrodynamics and Spectral Theory* [410] is an advanced text on MHD stressing the unity of physics and mathematics through spectral theory. This kind of complex, yet faultless, calculations is a rare commodity in plasma physics.

– Davidson, *An Introduction to Magnetohydrodynamics* [140] is an introductory textbook on MHD for students in physics, applied mathematics and engineering, with stress on the fluid dynamics foundations and a wide variety of applications.

– Polovin and Demutskii, *Fundamentals of Magnetohydrodynamics* [502] is an introduction to MHD, written in the lucid style of the great Russian theoreticians.

Tokamaks

– Wesson, *Tokamaks* [647] is a veritable encyclopedia of the plasma physics involved in nuclear fusion research in tokamaks.

– White, *Theory of Toroidally Confined Plasmas* [649] contains the material of a graduate course at Princeton University on fundamental plasma theory of tokamaks.

– Hazeltine and Meiss, *Plasma Confinement* [296] provides the advanced theory of magnetic plasma confinement with stress on derivations from first principles.

– Braams and Stott, *Nuclear Fusion: Half a Century of Magnetic Confinement Fusion research* [86] gives the history of nuclear fusion research up to the present, leading up to the famous citation of Artsimovich "Fusion will be there when society needs it".

The Sun

– Priest, *Magnetohydrodynamics of the Sun* [510] is the classical introduction of magnetohydrodynamics of the Sun, in particular the solar corona.

– Stix, *The Sun* [569] is a textbook on the physics of the Sun with innumerable observational facts.

– Foukal, *Solar Astrophysics* [181] aims at making the advances in understanding of the Sun accessible to students and non-specialists by means of simple physical concepts and observations.

Space physics

– Hasegawa and Sato, *Space Plasma Physics* [293] is a monograph on the physics of stationary plasmas, small amplitude waves and the stationary magnetosphere.

– Kivelson and Russell (eds.), *Introduction to Space Physics* [368] is an introduction of all aspects of space and solar plasmas for senior undergraduate and graduate students, written by experts in the various fields.

– Baumjohann and Treumann, *Basic Space Plasma Physics*, and (same authors in reverse order) *Advanced Space Plasma Physics* [42] are the basic material presented in a space plasma physics course at the University of Munich, and the advanced nonlinear aspects of the various waves and instabilities.

Plasma astrophysics

– Battaner, *Astrophysical Fluid Dynamics* [40] is a systematic theoretical treatise of the dynamics of classical, relativistic, photon and plasma fluids, progressing from stars to the Universe at large.

– Choudhuri, *The Physics of Fluids and Plasmas* [122] is an introduction to fluid dynamics, plasma physics and stellar dynamics for graduate students of astrophysics.

– Mestel, *Stellar Magnetism* [438] is a monograph on MHD applied to the magnetism of stars, including stellar dynamos, star formation and pulsar electrodynamics.

– Kulsrud, *Plasma Physics for Astrophysics* [385] introduces plasma physics as a comprehensible field that can be grasped largely on the basis of physical intuition and qualitative reasoning.

Exercises

The exercises are meant to increase understanding of the principles of plasma dynamics. Estimating orders of magnitude is an essential part. Frequent use of the numerical appendices is recommended. Difficult problems are marked with a star.

[1.1] *Fusion reactions*

We know two methods of energy production by nuclear processes, namely nuclear fission and nuclear fusion. For both, the net energy released is described by the same formula.

– What is expressed by that formula? What is the major difference between fission and fusion? What is actually expressed by the following formula for the most likely reaction in future fusion reactors: $D^2 + T^3 \rightarrow He^4 \,(3.5\,\text{MeV}) + n\,(14.1\,\text{MeV})$?

[1.2] *Fusion power*

If we want a tokamak reactor to produce energy and sustain itself, we need balance between thermonuclear power output and power losses. This leads to a condition on required particle density n, energy confinement time τ_E, and temperature $\widetilde{T}$.

– If the power output is given by $P_T = n^2 f(\widetilde{T})$, where f is a known function of temperature, and the power losses consist of Bremsstrahlung, $P_B = \alpha n^2 \widetilde{T}^{1/2}$, and heat transport, $P_L = 3n\widetilde{T}/\tau_E$, derive the criterion for fusion energy production. Assume that all power contributions can be converted in plasma heating with efficiency η.

– A more recent approach states that for ignition of a fusion reactor, α-particle heating of the plasma, P_α, should make up for the power losses. Express this criterion by a similar equation to that above. In that case, the product of the three mentioned quantities will have to be $n\tau_E \widetilde{T} \approx 3 \times 10^{21} \,\text{m}^{-3}\,\text{s}\,\text{keV}$. Give some estimates for n, τ_E and $\widetilde{T}$. Using Table B.1, convert the temperature $\widetilde{T}$ in keV to T in degrees K.

– Why do magnetic fields play such an important role in thermonuclear fusion?

[1.3] *Solar plasmas*

All the light we receive is the result of specific nuclear reactions which release energy. For instance, nuclear reactions in the centre of the Sun are the ultimate cause of light escaping at positions where the Sun becomes optically thin for this radiation.

– Which nuclear reactions take place in the centre of the Sun? What kind of radiation is produced? Why doesn't this extremely energetic light escape right away? Why is the light we collect on Earth mostly in the visible range? Why is it the less energetic light that escapes from the Sun? Explain what this says about the density profile.

– Normally we cannot see the solar atmosphere further outwards since we are blinded by the escaping light. A beautiful exception occurs during a solar eclipse. Explain why we can observe it at all on Earth.

– The coronal structures visible during a solar eclipse are the footprints of a giant engine that produces the solar activity. What is the mechanism and how is that connected to the structures observed?

[1.4] *Plasma definitions and applications*

Putting plasmas in a wide perspective, discuss the following aspects.

– What is a plasma? How is it different from ordinary gases and fluids?

– Name some of the numerous applications of plasma physics.

– How can plasmas be confined?

[1.5] *Forces in nature*

Explain the major forces present in nature, together with their relative strength and decay distance and, thus, the scale at which they are dominant. Explain why gravity is such a special force. What forces dominate the plasma regime?