e.g., the Poisson parameter is adapted to take into account the trends as well as cycles (by means of an autoregressive process). In addition the problem of forecasting the future flow of business is studied. The method is also adapted for coping with inflation. Investment is a new topic developed. Ruin probabilities for a finite time period come into the picture for discussing the problem of solvency. This chapter ends with the description of the Monte Carlo simulation of risk business.

In Chapter 7 several applications of the risk processes with a time span of several years developed in the previous chapter are given: the evaluation of net retentions, the effect of cycles, the effect of the time span, the effect of inflation, dynamic control rules and a solvency profile.

By means of cohort analysis the results of risk theory are then adapted to the life insurance branch in the following chapter.

In Chapter 9 infinite time ruin probability is studied essentially by means of the adjustment-coefficient. Some practical consequences are deduced.

The final chapter describes the application of risk theory to business planning. In the previous chapters many applications of a risk theory, such as the estimation of a suitable level for the maximum net retention, the evaluation of stability, the safety loading and the magnitude of the funds have been treated as isolated aspects of an insurance business. In this chapter a picture of the management process in its entirety is built up. An integration of the risk theoretical aspects in the context of other management aspects, not of actuarial nature, is carried out.

The book ends with some appendices containing derivations and proofs of some of the mathematical results obtained in the book: derivation of the Poisson and mixed Poisson processes, Edgeworth expansion, Infinite time ruin probability, Computation of limits of finite time ruin probabilities, Random numbers.

In addition the book contains quite a lot of interesting exercises (and their solution), an author index, a bibliography, a subject index as well as a necessary list of symbols.

In conclusion this book on risk theory where formulae are approached from the practical point of view shows to practical actuaries that some of the theoretical results lead to a better understanding of what is going on. To theoretical actuaries (at universities) the book gives a motivation for going on with theoretical research. Although this book has just appeared it is clear from discussions with students that it provides us with insurance models and material which is highly appreciated by people preparing for the actuarial profession.

M. GOOVAERTS

M. GOOVAERTS, F. DE VYLDER and J. HAEZENDONCK (1984). *Insurance Premiums. Theory and Applications.* North-Holland, Amsterdam. xi+406 pages, US $63.75/Dfl.150.00

This book introduces the reader to areas of insurance mathematics which have so far not been published on this scale in the form of a textbook. The individual

themes which the authors have already contributed to in numerous publications are for the first time discussed systematically, starting from the fundamentals.

A brief general introduction is followed by the central chapters on premium calculation principles (Chapter 2) and their properties (Chapter 3). The idea of introducing premium calculation principles goes back to Hans Bühlmann and it is astonishing how many such principles have meanwhile been developed. Based mainly on work by Bühlmann, Gerber and the authors, the following principles are introduced: expected value, maximal loss, variance, standard deviation, semi-variance, mean value, zero utility, Swiss premium, Orlicz, Esscher and mixtures of Esscher principles. Apart from the definition some principles are motivated by e.g., statistical reasoning or utility considerations. In addition properties and characterisations are supplied which at present are not to be found in printed form anywhere else. Because of the mathematically stringent form every expert in this field and every reader interested in mathematics too, who wishes to familiarize himself with the theoretical foundations of premium calculation, will definitely appreciate this book if only for its systematic presentation. On the other hand, it has to be said to every practitioner that it is not the purpose of this book to evaluate the various principles in contrast to each other or to examine the practical feasibility of the principles which use utility concepts. In Chapter 3 *Properties of Premium Calculation Principles* the properties of additivity, translation invariance, iterativity, homogeneity, multiplicativity and some generalisations are investigated as to which principles fulfill them. For example the property of additivity holds that in the case of independent risks X, Y the premium for X and Y should be just the sum of the individual premiums for X and Y. Not all properties are plausible for insurance premiums and not all generalisations important. The question of important and reasonable premium calculation principles and properties, which has not been resolved in the field of theory, should not be dealt with separately for principles and properties, and should not be answered without involving practitioners either. On this point these chapters provide a very successful presentation which will be the basis of every scientific discussion in this very lively field in future.

In the next chapter, entitled *Ordering among Risks*, various possibilities of how to introduce a partial order in the set of all (e.g., bounded) risks are considered. Of course, for a fixed premium principle π a natural order is induced by $\pi(X_1) \leq \pi(X_2)$, but one is also interested in conditions independent of π. The most important term here, the net stop-loss ordering, is introduced and its behaviour as to mixing and convolution investigated. These results allow statements to be made about the influence of these orders of number and size of losses on the orders of the corresponding total losses in the usual risk theoretical model. In this connection the dangerousness of distributions is discussed; after that generalisations of the stop-loss ordering are treated with stochastic dominance, familiar from the theory of finance, and stop-loss dominance.

The chapter *Bounds on Stop-Loss Premiums* takes up the problem which occurs in reinsurance practice of trying to calculate stop-loss premiums with incomplete risk information. Whereas to make an exact calculation of a net stop-loss premium

one requires full knowledge of the distribution function, the situation where one often only has an estimate accepted on trust for the expected value and perhaps the variance too, is looked at very realistically here. Depending on the available information e.g., about the first n moments or the symmetry of the density function, one looks for an upper bound for all stop-loss premiums of those risks which the given information applies to. The step from the noncalculable "true" stop-loss premium to the upper bound is therefore a cautious one. In some cases, e.g., when expected value and variance are regarded as known, lower bounds can also be worked out analogously, i.e., an error estimate. The mathematical tool necessary for this stems from finite dimensional analysis and is presented comprehensively, so that this chapter is self-contained. The most important aid is a duality theorem from convex analysis allowing the original maximisation problem to be transformed into a simple minimisation problem so that analytical or numerical results are gained. In the last chapter on applications the method for estimating stop-loss premiums described above is applied to the case of bounded exposure. Further applications of the ordering of risks and the procedure in case of incomplete information are indicated for questions of determining the optimal critical claim size in a bonus-malus system, for bounds of the ruin probability and bounds for stop-loss premiums for weighted compound distributions.

This book, which only requires little prior mathematical knowledge, is the most comprehensive presentation of the themes dealt with here. Because of the mathematical style the practitioner will miss explanations and a mutual comparison of the results at some points. The book reflects the current level of knowledge in some fundamental partial fields of insurance mathematics in a precise form, thereby providing a base from which theoreticians and practitioners can communicate with each other.

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