

## MICRO-METEOROLOGICAL CONDITIONS FOR SNOW MELT

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**ABSTRACT.** The energy budget of a snow or ice surface is determined by atmospheric variables like solar and atmospheric long-wave radiation, air temperature, and humidity; the transfer of energy from the free atmosphere to the surface depends on the stability of the atmospheric boundary layer, where vertical profiles of wind speed and temperature determine stability, and on surface conditions like surface temperature (and thus surface humidity), roughness, and albedo.

This paper investigates the conditions exactly at the onset or the end of melting using air temperature, humidity, and as the radiation term the sum of global and reflected short-wave plus downward long-wave radiation. For the turbulent exchange in the boundary layer, examples are computed with a transfer coefficient of  $18.5 \text{ W m}^{-2} \text{ K}^{-1}$  which corresponds to the average over the ablation period on an Alpine glacier. Ways to estimate the transfer coefficient for various degrees of stability are indicated in the Appendix.

It appears from such calculations that snow may melt at air temperatures as low as  $-10^\circ\text{C}$  and may stay frozen at  $+10^\circ\text{C}$ .

### I. INTRODUCTION

The fact that snow melts at a temperature of  $T_s = 0^\circ\text{C}$  has often lead to the simplified assumption that it also melts at air temperatures of  $T_a = 0^\circ\text{C}$ . This statement disregards the intricacies of the energy budget of the snow surface in which radiation fluxes and turbulent exchange of latent heat operate independently of air temperature. In the following, a formulation is derived that permits the assessment of the meteorological conditions for snow melt in terms of absorbed radiation, humidity, and temperature of the atmospheric boundary layer above the snow surface.

The treatment of the problem is essentially an exercise in micro-meteorology. The partition of the energy budget obeys the conservation of energy law and is formally straightforward. Its application to real snow, however, is complicated by three facts:

- (i) The separate variables chosen to describe a particular situation are not entirely independent, for instance, the atmospheric vapor density  $\rho_{va}$  is limited by air temperature  $T_a$ , and the long-wave downward radiation flux  $L\downarrow$  is to a certain degree coupled to  $T_a$  and  $\rho_{va}$ . Such interconnections need to be considered when specifying a particular set of variables and will be explicitly stated in two examples in the text.
- (ii) Because of the diurnal variation of the energy fluxes, stationarity is only approximated.
- (iii) The transfer of heat through a turbulent boundary layer depends on external parameters such as wind speed and air temperature as well as internal ones like surface temperature and surface roughness.

In view of these complications, an *a priori* determination of the transfer coefficient is not attempted here. A value of  $18.5 \text{ W m}^{-2} \text{ K}^{-1}$ , as found from long-term observations on Alpine firn and ice (Kuhn, 1979), was used

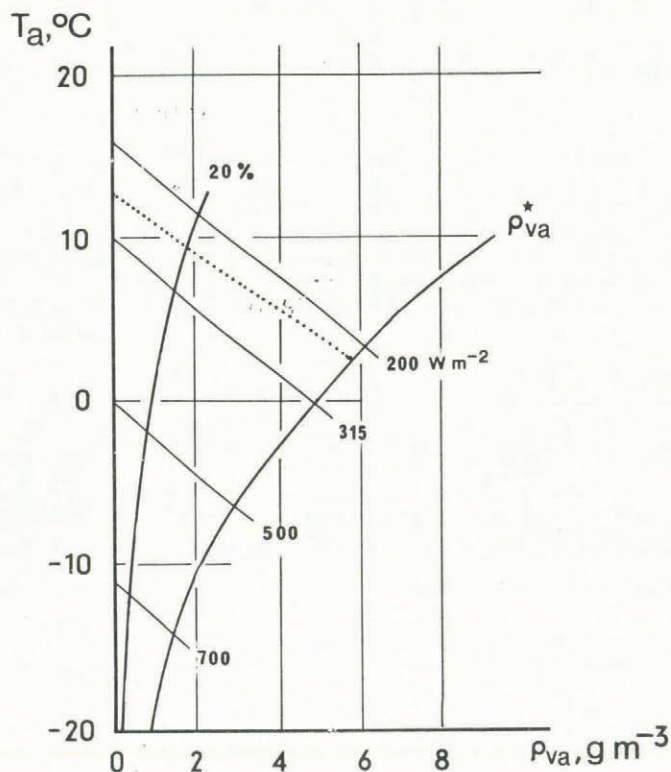


Fig. 1. Air temperature  $T_a$  at the onset of melting for various values of absorbed radiation  $R = S\downarrow - S\uparrow + L\downarrow$  and water-vapor density  $\rho_{va}$ . Curves show saturation and 20% relative humidity. The dotted line is explained in the text. This diagram is valid for a thermal resistance  $r_H = 65 \text{ s m}^{-1}$  (equivalent to a transfer coefficient  $\alpha_H = 18.5 \text{ W m}^{-2} \text{ K}^{-1}$  at  $\rho_a = 1.2 \text{ kg m}^{-3}$ ).

in the quantitative examples in section IV and in Figure 1. Various ways of expressing turbulent-heat transfer and its dependence on stability are briefly summarized in the Appendix.

### II. PARAMETERIZATION OF THE ENERGY BUDGET

Energy fluxes at the surface balance such that the sum of short-wave ( $S\downarrow, S\uparrow$ ) and long-wave ( $L\downarrow, L\uparrow$ ) radiation fluxes, sensible ( $H$ ), and latent ( $V$ ) turbulent-heat fluxes are available for either changing the temperature or the phase of the snow at a rate  $Q$ .

$$S\downarrow - S\uparrow + L\downarrow - L\uparrow + H + V = Q. \quad (1)$$

Surface temperature  $T_s$ , air temperature  $T_a$ , and vapor density of air  $\rho_{va}$  and surface  $\rho_{vs}$  are obvious choices of meteorological parameters in specifying this budget. As downward long-wave fluxes  $L\downarrow$  are only weakly related to



$T_a$  (while  $L\uparrow$  is unambiguously tied to  $T_s$ ), the first three of the four radiation terms are taken as one independent variable

$$R = S\downarrow - S\uparrow + L\downarrow \tag{2}$$

The turbulent fluxes are taken to be proportional to the differences ( $T_s - T_a$ ) and ( $\rho_{vs} - \rho_{va}$ ), where the factor of proportionality can be expressed as a transfer coefficient, or resistance, which depends on stability as explained in the Appendix. The budget can then be written as

$$R - \epsilon_s \sigma T_s^4 - \frac{\rho_a c_p}{r_H} (T_s - T_a) - \frac{L_v}{r_v} (\rho_{vs} - \rho_{va}) = Q \tag{3}$$

where  $\epsilon_s$  is the surface emissivity and is approximately equal to unity,  $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$  is the Stefan-Boltzmann constant,  $\rho_a$  is the air density,  $c_p$  is the specific heat of air,  $r_H$  and  $r_v$  are the resistance to heat and vapor transfer, respectively,  $L_v = 2.5 \text{ MJ kg}^{-1}$  the latent heat of evaporation, and the other terms have been explained before.

Considering that surface vapor density is determined by surface temperature

$$\rho_{vs} = \rho_{vs}^*(T_s), \tag{4}$$

there are four independent variables ( $R$ ,  $T_s$ ,  $T_a$ , and  $\rho_{va}$ ) and one dependent variable  $Q$  for given values of  $\rho_a$ ,  $r_H$ , and  $r_v$  in Equation (3).

### III. MELTING CONDITIONS

At the onset (or cessation) of melting, however,  $Q = 0$ ,  $T_s = 0^\circ\text{C}$ ,  $\epsilon_s \sigma T_s^4 = 315 \text{ W m}^{-2}$ , and  $\rho_{vs}^* = 4.8 \times 10^{-3} \text{ kg m}^{-3}$ . This situation facilitates the analysis of Equation (3) by reducing the number of variables to three ( $R$ ,  $T_a$ , and  $\rho_{va}$ ), each one of which can be studied as it reacts to the other two. That means, for given values of  $R$  and  $\rho_{va}$ , there is only one value of  $T_a$  that fulfils the condition of incipient melting.

By taking partial derivatives in Equation (3), one can determine the values of  $(\partial T_a / \partial \rho_{va})_R$ ,  $(\partial T_a / \partial R)_{\rho_{va}}$ ,  $(\partial R / \partial \rho_{va})_{T_a}$ , which may be considered as sensitivity coefficients that indicate how much one variable has to change in order to compensate for an imbalance caused by one of the others.

### VI. NUMERICAL EXAMPLES

Choosing, for example, values for  $\rho_a c_p = 1200 \text{ J kg}^{-1} \text{ K}^{-1}$  and  $r_H = r_v = 65 \text{ s m}^{-1}$ , Equation (3) becomes

$$R - 315 + 18.5 T_a - 185 + 3.85 \times 10^4 \rho_{va} = 0 \tag{5}$$

$$\text{or } T_a = 27.0 - R/18.5 - 2.08 \rho_{va}$$

From this equation, the sensitivity coefficients can be derived

$$\partial T_a / \partial \rho_{va} = -2.1 \times 10^3 \text{ K (kg m}^{-3}\text{)}^{-1} \tag{6}$$

which means that a decrease in atmospheric vapor density by  $1 \text{ g m}^{-3}$  must be counterbalanced by an increase in air temperature of  $2.1 \text{ K}$  or, in other words, less condensation (stronger evaporation) is compensated by an increased flux of sensible heat towards the surface.

Similarly,

$$\partial T_a / \partial R = -0.054 \text{ K (W m}^{-2}\text{)}^{-1} \tag{7}$$

A decrease in absorbed radiation by  $1 \text{ W m}^{-2}$  needs to be compensated by an increase of  $T_a$  by  $0.054 \text{ K}$ . Inspecting the reciprocal of this value, one finds that it is equal to

$\rho_a c_p r_H^{-1}$  and thus to the transfer coefficient for sensible heat.

Finally,

$$\partial R / \partial \rho_{va} = -3.85 \times 10^4 \text{ W m}^{-2} \text{ (kg m}^{-3}\text{)}^{-1} \tag{8}$$

A decrease in atmospheric vapor density by  $1 \text{ g m}^{-3}$  requires an increase in absorbed radiation by  $38.5 \text{ W m}^{-2}$  for compensation.

It is clear from Equations (3) and (5) that the coefficients just described are independent of the value of the respective third variable ( $R$ ,  $\rho_{va}$ ,  $T_a$ ) but that they change with the assumptions made for  $\rho_a$  and  $r_H$  or  $r_v$ .

Let us now use the example in Equation (5) in order to estimate a reasonable range of air temperatures under which melting might be expected.

For a given value of absorbed radiation  $R$ , the maximum conceivable air temperature will occur in an absolutely dry atmosphere. A somewhat less stringent lower limit for  $T_a$  is set at  $\rho_{va} = \rho_{va}^*(T_a)$ .

In Figure 1, which shows a graphical solution of Equation (5), the two limits  $\rho_{va}^*$  and  $\rho_{va} = 0$  are entered as well as the more likely lower limit of 20% relative humidity.

The extreme values of  $T_a$  for melting conditions are further determined by extremes of  $R$ . Although these cannot be calculated exactly, some likely situations will be discussed in the following examples.

#### 1. Minima of $R$

During darkness, when  $S\downarrow$  and  $S\uparrow$  vanish,  $R = L\downarrow$ , minima of which are to be expected under a clear sky.  $L\downarrow$  is a function of the respective vertical profiles of  $T$ ,  $\rho_v$ , and of other emitting trace gases, but it can be sufficiently approximated by  $L\downarrow = 0.7 \sigma T_a^4$  for Alpine, clear-sky conditions.

An approximate expression for the minimum value of  $R$  can then be derived by inserting  $R = 0.7 \sigma T_a^4$  into Equation (5). The solution is the dotted line on Figure 1, which shows that at 20% relative humidity,  $10^\circ\text{C}$  is a likely maximum value of  $T_a$  above non-melting snow.

#### 2. Intermediate values, $R = L\uparrow$

A particular situation arises when the absorbed radiation equals that emitted ( $L\uparrow = 315 \text{ W m}^{-2}$  for  $T_s = 0^\circ\text{C}$ ). Equation (5) then describes  $T_a$  as the dry-bulb temperature of a psychrometer for a fixed wet-bulb temperature of  $0^\circ\text{C}$  and  $\partial T_a / \partial \rho_{va}$  can be recognized as the psychrometric constant.

#### 3. Maxima of $R$

From Equation (2), maxima of  $R$  are to be expected for snow exposed to a normally incident solar beam under conditions of high atmospheric transparency (maximum  $S\downarrow$ ), low albedo (minimum  $S\uparrow$ ) with warm, humid air (maximum  $L\downarrow$ ). Extremes of  $S\downarrow$  and  $L\downarrow$  are unlikely to occur simultaneously; as a matter of fact, the vertical changes  $\partial S\downarrow / \partial z$  and  $\partial L\downarrow / \partial z$  are nearly equal and opposite under Alpine conditions. In Figure 1, high values of  $R$  are associated with low values of  $T_a$  which is found in extra-polar mountains.

In order to avoid excessive speculation, let us use mid-summer, Alpine measurements (Wagner, 1979, 1980) over an almost horizontal snow surface ( $S\downarrow = 1020 \text{ W m}^{-2}$ ,  $L\downarrow = 280 \text{ W m}^{-2}$ ) and an extreme albedo of 0.2 as applicable to dirty glacier ice or a thin, translucent ice cover on a dark rock. This means  $S\uparrow = 204$  and  $R = 1096 \text{ W m}^{-2}$ , yielding a debatable  $T_a = -32^\circ\text{C}$ . This result is questionable, as it implies a strong lapse-rate above the snow, and turbulent exchange should be significantly higher than initially assumed. If we take only half the resistance as before in Equation (5), (now  $r_H = 32 \text{ s m}^{-1}$ ) the resulting  $t_a$  becomes  $-16^\circ\text{C}$ .

### V. CONCLUSIONS

Since a change in resistance by a factor of 2 is not at all unreasonable and may occur in small space and time intervals, it is impossible to make a quantitative but



generally true statement about the upper and lower limits of  $T_a$ , except, maybe, that the range  $-10^\circ < T_a < +10^\circ\text{C}$  is likely to be encountered above snow at the beginning or end of melting ( $Q = 0$ ).

Figure 1 treats both stable ( $T_a > 0^\circ\text{C}$ ) and unstable situations with the same heat-transfer resistance but it is not intended to suggest that  $r_H$  might be a constant for snow. To predict a value of  $r_H$ , a number of micro-meteorological hypotheses need to be resolved and values of  $T_a$ ,  $u$ ,  $u_*$ , and  $z_0$  need to be known, as described in the Appendix. However, for climatological applications of the present problem, the range of  $r_H$  is small and likely to be centered around the value of  $65\text{ s m}^{-1}$  found in the ablation period of an Alpine glacier.

Considering that the determination of  $r_H$  is a crucial problem in calculating the energy budget, we may even venture to use a form of Equation (3) to determine  $r_H = r_v$  from measurements of  $R$ ,  $T_a$ , and  $\rho_{va}$  at a time when  $Q = 0$

$$r_H = r_v = \frac{\rho_a c_p \Delta T + L_v \Delta \rho_v}{R - 315} \quad (9)$$

taking care that  $R$  is sufficiently different from  $315\text{ W m}^{-2}$ .

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APPENDIX

The turbulent fluxes of sensible heat  $H$  and of latent heat  $V$  are defined here as positive when energy flows to the surface. According to the gradient hypothesis, they can be expressed

$$H = \rho_a c_p K_H \left[ \frac{\partial T}{\partial z} + \frac{g}{c_p} \right] \text{ and } V = L_v K_v \frac{\partial \rho_{va}}{\partial z} \quad (A.1)$$

With

$$K_H = \kappa u_* z \phi_H^{-1} \text{ and } K_v = \kappa u_* z \phi_v^{-1}, \quad (A.2)$$

and neglecting  $g/c_p \ll \partial T/\partial z$ ,

$$H = \rho_a c_p \kappa u_* \phi_H^{-1} \frac{\partial T}{\partial \ln z} \text{ and } V = L_v \kappa u_* \phi_v^{-1} \frac{\partial \rho_{va}}{\partial \ln z} \quad (A.3)$$

where  $\kappa$  is the von Kármán constant,  $u_*$  is the friction velocity, and  $\phi$  is a function of stability.

The desired form with finite differences and a bulk transfer number

$$F = \alpha_H (T(z) - T_s) \text{ and } V = \alpha_v (\rho_{va}(z) - \rho_{vs}) \quad (A.4)$$

is obtained by integration of Equations (A.3) from the surface ( $z_0$ ) to the level of measurements.

$$F \int_{z_0}^z \phi_H d(\ln z) = \rho_a c_p \kappa u_* (T(z) - T_s) \quad (A.5)$$

and

$$V \int_{z_0}^z \phi_v d(\ln z) = L_v \kappa u_* (\rho_{va}(z) - \rho_{vs}).$$

Comparison of Equations (A.4) with Equations (A.5) yields

$$\alpha_H = \frac{\rho_a c_p \kappa u_*}{\int_{z_0}^z \phi_H d(\ln z)} \quad (A.6)$$

and with  $\phi_v = \phi_H$

$$\alpha_v = L_v \rho_a^{-1} c_p^{-1} \alpha_H \quad (A.7)$$

which can be expressed as a function of wind speed by inserting for  $u_*$  from the logarithmic wind profile

$$u(z) = \kappa^{-1} u_* \int_{z_0}^z \phi_M d(\ln z) \quad (A.8)$$

where  $\phi_M$  is the stability function for momentum flux.

The function  $\phi$  was determined from experiments as reviewed by Brutsaert (1982), Bush (1973) or Carson and Richards (1978), among others. The simplest expression holds for stable layering where

$$\phi_M = \phi_H = \phi_v = 1 + azL^{*-1} \quad (A.9)$$

and  $a = 5$ , approximately.

The Monin-Obukhov length is

$$L^* = \frac{u_*^3 T \rho_a c_p}{\kappa g H (1 + 0.07H/V)} \quad (A.10)$$

using the notation and sign convention of this paper.

The stability functions may also be expressed in terms of the Gradient-Richardson number  $Ri$

$$Ri = \frac{g}{T} \frac{(\partial T/\partial z + g/c_p)(1 + 0.007H/V)}{(\partial u/\partial z)^2} \approx \frac{g}{T} \frac{\partial T/\partial z}{(\partial u/\partial z)^2} \quad (A.11)$$

Under stable conditions ( $\phi_M = \phi_H$ )

$$Ri = \frac{z}{L^*} \frac{\phi_H}{\phi_M^2} = \frac{z}{L^*} \frac{1}{\phi_M}, \quad (A.12)$$

from which a useful approximation can be derived with  $a = 5$

$$\phi_M = (1 - 5Ri)^{-1} \approx 1 + 5Ri. \quad (A.13)$$