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Equatorial Pulsar Winds

Jan Kuijpers

Department of Astrophysics, University of Nijmegen, PO Box 9010,
6500 GL Nijmegen, The Netherlands
Astronomical Institute, Utrecht University, PO Box 80 000,
3508 TA Utrecht, The Netherlands
kuijpers@astro.kun.nl

Abstract: A traditional problem in pulsar wind physics has been the nature of the pulsar wind. On theoretical grounds, the wind is expected to be dominated by Poynting flux associated with the outgoing magnetic field lines anchored on the polar caps of the rotating neutron star, while observations of the Crab Nebula demonstrate that the wind must be dominated by kinetic energy before the termination shock. Here we suggest a new approach to this old problem by studying the *distributed* currents rather than the *singular sheet* currents which have been the object of study in most work. We find that, at a distance well in between the light cylinder and the termination shock, current starvation sets in, and electric fields develop along the magnetic field lines which cause the current to dissipate and convert at least half of the Poynting flux into kinetic energy flux in a relatively thin shell. In the shell, at least half of the current closes across the magnetic field lines, the pitch of the spiralling magnetic field lines jumps downward strongly, and the outer pattern of magnetic field lines slips over the inner pattern.

Keywords: magnetohydrodynamics — stars: neutron — stars: winds — magnetospheres — pulsars: individual (Crab)

1 Introduction

The conversion of the Poynting flux in the equatorial wind of the Crab Pulsar into kinetic energy flux is as yet an unsolved problem. An important role in these discussions is played by the ratio σ of Poynting flux to kinetic energy flux in the wind

$$\sigma = \frac{B^2}{4\pi \Gamma nmc^2}. \qquad (1)$$

Here Γ is the Lorentz factor of the electron–positron plasma with total density n in the frame of the observer. At the base of the wind $\sigma \gg 1$ in pulsar models while observations of the Crab Nebula show that *in the equatorial plane* $\sigma \sim 10^{-3}$ at the termination shock of the wind (Rees & Gunn 1974; Kennel & Coroniti 1984; Hoshino et al. 1992; Shibata 1995, 1996; Tanvir, Thomson, & Tsikarishvili 1997). In an attempt to solve this problem Coroniti (1990) has proposed that the magnetic field energy associated with the sheet current in the striped wind is converted into thermal and kinetic energy by magnetic reconnection. Recently, it has been shown by Lyubarsky & Kirk (2001; see also Kirk & Lyubarsky 2001) that magnetic reconnection of the striped wind is not effective at distances inside the inner shock typical for the Crab Nebula. Another solution to the problem is conversion of the vacuum wave energy by dissipative effects in the large amplitude displacement electric field as has been proposed by Melatos & Melrose (1996a, 1996b). Finally, yet a different attempt to solve the problem is by Begelman (1998) who proposes that a Poynting flux dominated wind is Rayleigh–Taylor unstable so that magnetic turbulence develops and a low sigma environment in the pre-shocked wind is not required anymore for the wind to be compatible with the observations.

Here, in contrast to the papers mentioned above, we focus on *body currents* rather than sheet currents as the former are dynamically important in slowing down the pulsar, and are responsible for accelerating the wind in an MHD picture. We first describe a simplified picture of the MHD wind in an aligned rotator in Section 2, then study the essential modifications occurring in the case of a perpendicular rotator in Section 3, and point out in Section 4 that an important property of the equatorial wind in an oblique rotator, i.e. the existence of *body* currents, has been largely neglected in the, by now traditional, striped wind picture with *sheet* currents only. In Section 5 we then demonstrate that a current starvation problem arises at a radius well within the termination shock. This leads to strong electric fields parallel to the ambient magnetic fields. As a result the current dissipates in this layer and the wind is strongly heated and accelerated. We propose that the main conversion of Poynting flux into kinetic energy of the equatorial wind takes place in this layer. The electric current closes inside this dissipative shell, and the toroidal magnetic field component drops sharply. In Section 6 we describe how the magnetic field structure of the wind outside the dissipative layer rotates much slower and slips over the inside field structure. This process, which allows the external field structure to remain connected to the inner field at each moment, is described by general magnetic reconnection (GMR), a term introduced by Schindler, Hesse, & Birn (1988), and Hesse & Schindler (1988).

2 Aligned Rotator

We consider the wind to be an ideal plasma (infinite conductivity) and investigate where this assumption must break down. The wind energy of a magnetised, rotating star (not necessarily compact) consists of two components: kinetic and (electro)magnetic energy.

Let us have a closer look at the kinetic energy first. Part of the kinetic energy is given to the gas in the lower atmosphere due to such processes as heating and wave damping. Another part is imparted to the wind by the magnetic field which is anchored inside the star and which tends to keep the outflowing gas in corotation. In the MHD limit, this is achieved by a closed current system (see Figure 1) which transports angular momentum flux outward by Maxwell stresses, created by the distorted magnetic field. One can distinguish two domains in the circuit: at and below the stellar surface, the current crosses the magnetic field in such a direction as to brake the stellar rotation; in the atmosphere the current propagates outward and inward, and the magnetic field gets a toroidal (i.e. in the direction of rotation) component. As the distance to the
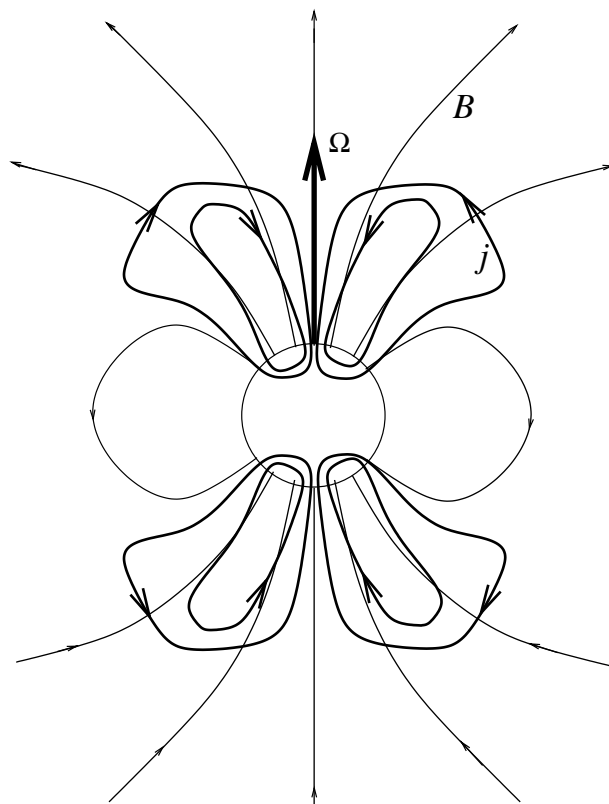

Figure 1 Sketch of the (meridional projection of the) current system and magnetic field lines of an axially symmetric, magnetised stellar wind. Only the part of the circuit is shown which accelerates the wind. In the star cross-field currents brake the stellar rotation. The angular momentum is transported outwards by Maxwell stresses associated with the azimuthal and radial magnetic field components, and gradually imparted to the gas out and above the Alfvén radius where the currents cross the same magnetic flux again, but now in a direction opposite to that in the star. Beyond a number of Alfvén radii the gas coasts practically force-free.

star increases the current direction becomes increasingly oblique with respect to the magnetic field, and the circuit closes. In this layer, the torque created by the Lorentz force increases, and the angular momentum which has been carried outwards by the currents along magnetic field lines is gradually imparted to the wind. Note that an ideal MHD picture suffices to describe the stellar braking and the acceleration of the wind.

The angular momentum loss rate by a wind of mass loss rate $\dot{m} = \rho v_p / B_p$ per flux-tube of unit strength is given by (Mestel 1999)

$$\dot{j} = \dot{m} \Omega_\star R_A^2, \tag{2}$$

where R_A is the Alfvén radius which is defined as the axial distance from the star where the dynamic pressure of the wind in the meridional plane equals the magnetic tension provided by the poloidal magnetic field

$$\rho(R_A) v_p^2(R_A) = \frac{B_p^2(R_A)}{4\pi}. \tag{3}$$

Here ρ is the density, v_p the poloidal (in the meridional plane) wind speed, and B_p is the poloidal component of the magnetic field. The schematic picture of the projected poloidal circuit in Figure 1 is for the simple case of an axially symmetric rotator. Note that for practical purposes the current part that accelerates the wind can be considered to close at an axial distance of a number of Alfvén radii. This current system serves to convert Poynting flux to kinetic energy flow of the wind by electrodynamic means.

Apart from the above described current which acts to accelerate the wind in both toroidal and radial directions, there is a second circuit which extends far out beyond the Alfvén radius but which is, in contrast with the first system, force-free until it closes at an interstellar, 'astrospheric' or 'termination', shock (see Figure 2). The reasons are, firstly, that, in the ideal MHD approximation, the field lines in a stellar wind go out to 'infinity' and both the star and the wind remain frozen into the field, and, secondly, that the gas is coasting unaccelerated beyond a certain distance where it moves along a linear path. The fact that the gas can move only along but not across the field lines which themselves have fixed footpoints at the stellar surface then implies that the geometry is characterised by a spiralling magnetic field. Existence of the latter implies an outward (but force-free) electric current.

As long as the flow is force-free and the field lines do not close, also the current does not close, and the azimuthal magnetic field component falls off along a magnetic field line inversely proportional to the axial distance R

$$B_\phi(R) \propto \frac{1}{R} \tag{4}$$

because of Ampère's law.

Therefore, also far outside the Alfvén radius in the force-free part of the magnetosphere, angular momentum is carried off by the magnetic field in the form of Maxwell stresses, i.e. the component $R B_R B_\phi (4\pi)^{-1}$. Note that in

a force-free field it is not the flow of angular momentum which vanishes but just its divergence, implying that the torque in the region of space considered vanishes.

Associated with the azimuthal field component is a Poynting flux. Choosing a right-handed and orthogonal local coordinate system $\{p, \phi, n\}$ where $\hat{p}$ is taken along the poloidal magnetic field direction (which coincides with the direction of the poloidal velocity), the poloidal part of the Poynting flux can be written as

$$\vec{s}_p = \frac{\hat{p}}{4\pi}(v_p B_\phi^2 - v_\phi B_\phi B_p). \tag{5}$$

Note that both terms are positive as v_ϕ is in the direction of rotation while the magnetic field is always trailing. Here we have used that in an ideal plasma the comoving electric field vanishes so that in the lab system $\vec{E} = -\vec{v} \times \vec{B}/c$. Also, we have used that, in a steady state and force-free wind, the poloidal parts of magnetic field and velocity are aligned. As in the force-free part of the (cold) wind the velocity asymptotically reaches a constant magnitude and direction, the azimuthal magnetic field component varies as $B_\phi \propto 1/R$ (equation (4)) while the poloidal component, because of flux conservation, decreases approximately as the square of the distance r to the star

$$B_p \propto r^{-2}. \tag{6}$$

The part of the Poynting flux which, when integrated over a sphere, does not vanish 'at infinity' is given by

$$s_p = v_p \frac{B_\phi^2}{4\pi}. \tag{7}$$

Note that the second term in equation (5), when divided by c and multiplied by the lever arm R, leads to a non-vanishing flow of angular momentum at infinity.

Whereas the first current system is well understood, the way in which the second current system closes far out in the wind is not well known. The main purpose of the present paper is to suggest a solution to this problem.

3 Oblique Rotator

In case of an oblique rotator, the flux from each polar cap is split into two flux tubes. Part of the open flux from one polar cap connects up to a jet where the magnetic field direction varies smoothly in space while the rest joins up with a similar flux tube from the other polar cap to form a striped (Kennel & Coroniti 1984) equatorial wind which consists essentially of alternating flux tubes wound around each other (Figure 3). The currents in the jets run essentially in a similar manner as in the aligned case (Figure 2). To understand how the currents in the equatorial flux tubes run, let us first consider a magnetised slab of plasma to which a shearing force is applied (Figure 4). In the ideal case a current runs in the directions as sketched.
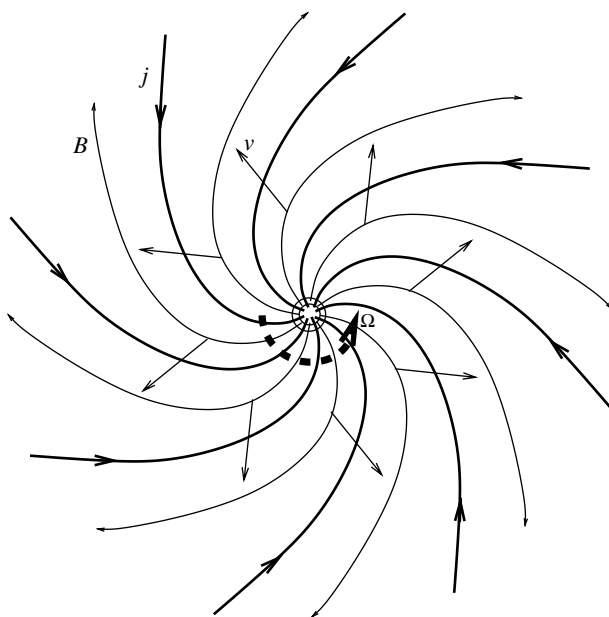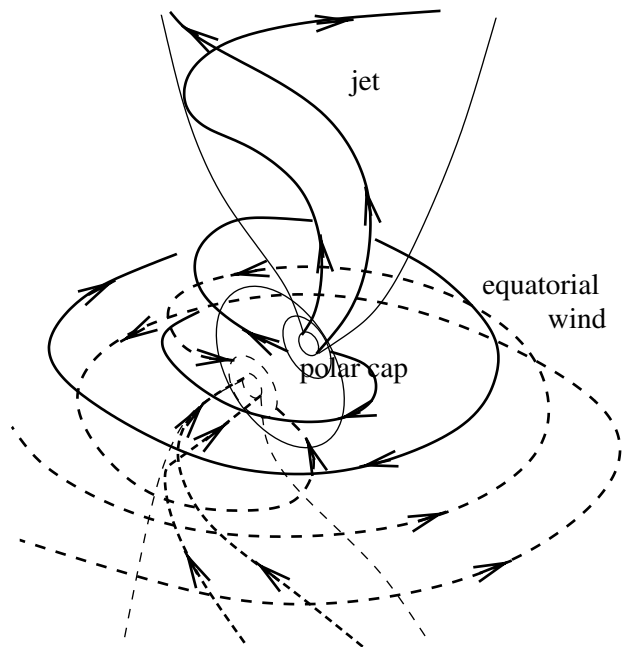

Figure 2 The figure shows the second, largely force-free, part of the current system which continues outside of the Alfvén radius. Sketched is a projection onto the equatorial plane of a few magnetic (B) and current (j) field lines connecting to the northern polar cap. On the polar cap the currents cross the surface of the star equator-ward (not shown) before going out again from the boundary of the polar cap. Also shown are the asymptotic velocities (v) of the gas as observed in the laboratory frame. The sense of stellar rotation is indicated with Ω.

Figure 3 Sketch of the magnetic field of an oblique rotator (solid lines are outgoing, dashed lines are incoming field lines). The central ellipsoid represents the closed part of the magnetosphere extending out to the light cylinder. A bundle of open field lines originates at each polar cap. Each bundle divides into two flux tubes: a central 'polar' flux tube which goes out into a jet of the same polarity, and a cylindrical sheath which joins a similar flux tube with opposite polarity and forms an equatorial 'striped' wind.
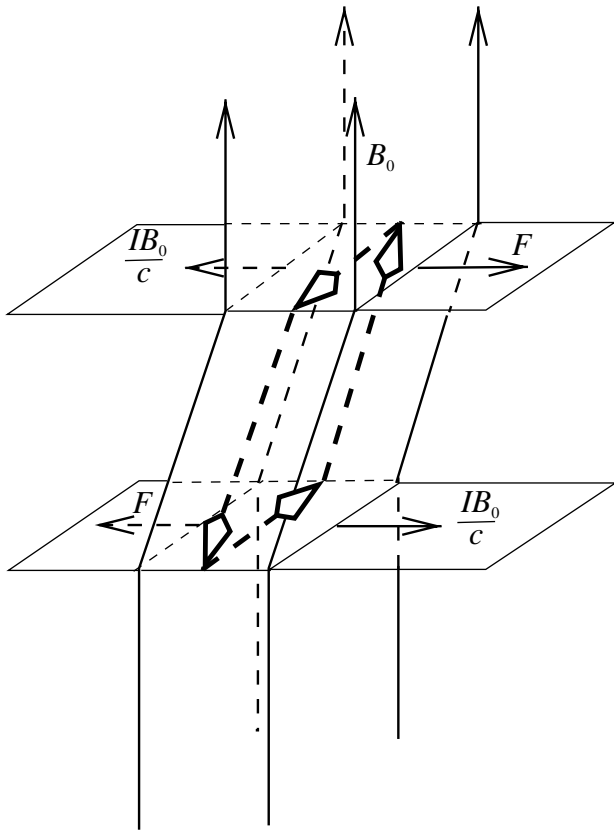
Figure 4 A current system (open arrows) is set up in an ideal magnetised slab which is subjected to a shearing force per unit length $\vec{F}$ so that $IB_0/c = F$.

The situation has some similarity to the Jovian flux tube which passes through Io (Piddington & Drake 1968; Goldreich & Lynden-Bell 1969; Clarke 1996), with the difference that the plasma in Figure 4 is ideal and there is no slip, so that the situation is not a steady state while in Io's case a steady situation requires slip at both ends of the tube.

Let us, for simplicity, consider an exactly perpendicular rotator. We propose that the global shape of the current system can be found from an evolutionary description in which we follow the fate of an initial flux tube which connects north and south stellar magnetic poles and which penetrates the light cylinder (or, more precisely, the Alfvén cylinder). The inertia of the wind will apply a drag on the magnetic field and a current system will be set up in the flux tube as it opens up as sketched in Figure 5.

Such a current system is indeed consistent with electrodynamical considerations of a magnetic rotator which is initially in vacuo and tries to set up an external current. The initial electric field, driving the current, derives from the degree of deviation of the (integrated) local charge density from the (integrated) Goldreich–Julian (GJ) density, and the sign of the electric field component along the magnetic field is the same as that of the integrated GJ density. For an exactly perpendicular rotator, $sgn(\vec{E} \cdot \vec{B}) = -sgn(\vec{\Omega} \cdot \vec{r})$. The electric field points in a direction parallel (antiparallel) to the magnetic field on those parts of the polar
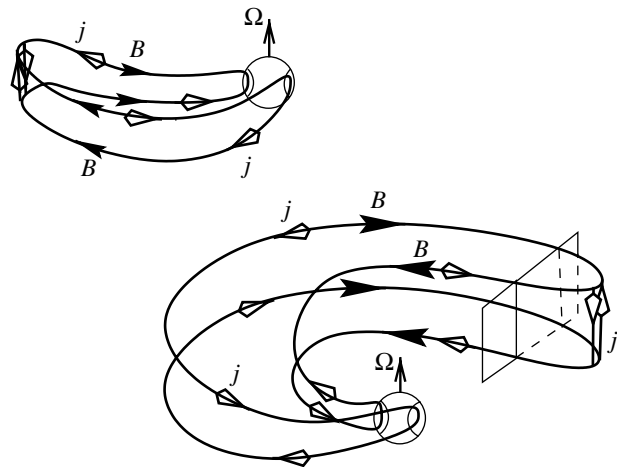

Figure 5 Indicated are the electric currents (open arrows) which will be set up as a flux tube connecting both magnetic poles is drawn out from a rotating star so that it brakes the stellar rotation. For clarity, only two magnetic field lines (black arrows) are shown. The inclined plane on the right is the cut presented in Figure 7 in the asymptotic regime when the top of the loop has vanished to infinity.
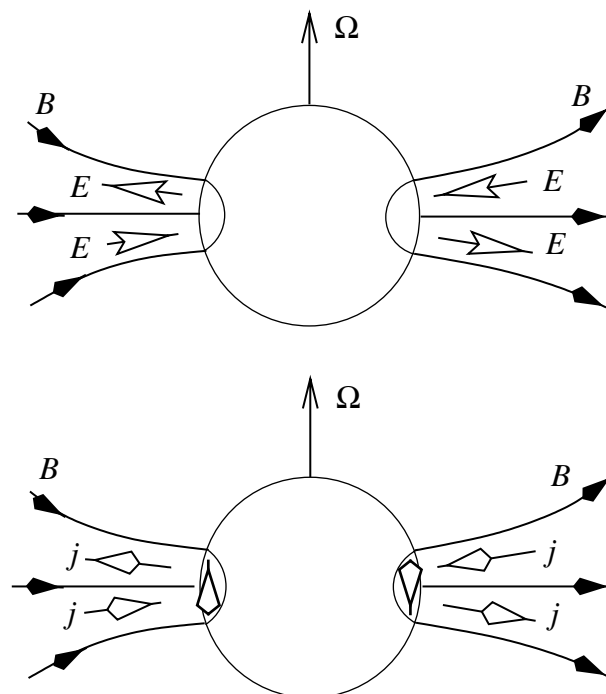

Figure 6 The picture on the top shows the directions of the electric fields which are set up near a perpendicular vacuum rotator. These fields are compatible with the direction of currents required to brake the star as can be seen at the bottom.

caps above (below) the rotational equator. As can be seen from Figure 6 this is indeed the direction required for the electric current to brake the stellar rotation.

We want to find out whether such an ideal current system can be strong enough to brake the pulsar as is observed, and, further, to what distance beyond the light cylinder the currents can be force-free. Whereas the currents in Figure 4 are singular and running on the edges,

in the force-free part of the magnetised wind they will be flowing through the body of the tube of drawn-out open field lines sketched in Figure 5. Similar distributed current systems have been observed in terrestrial flux-tube events, and in the Io–Jupiter system. This does not mean that there would be no sheet currents in the pulsar magnetosphere as well. We just want to investigate the role of the body currents as they have been often neglected, e.g. in the striped wind picture by Kennel & Coroniti (1984), Bogovalov (1999), and Lyubarsky & Kirk (2001).

We can estimate the order of magnitude of the current as follows. If the spindown of the pulsar is entirely due to the action of a current I traversing each of the polar caps the combined torque follows from integrating the Lorentz torques over the polar cap areas. For reference purposes, we first derive the current for an aligned rotator. Assuming that a uniformly distributed current I travels down along the open field lines onto each of the polar caps, then diverts in meridional planes, and travels upwards at the footpoints of the last open field lines, the total torque acting on the star is found to be

$$K_0 = \frac{I\Psi}{c\pi}. \qquad (8)$$

Here Ψ is the magnetic flux integrated over a polar cap area

$$\Psi = \pi r_{pc}^2 B_0 = \frac{\pi r_*^3 \Omega_* B_0}{c}, \qquad (9)$$

with B_0 the magnetic field strength at the magnetic poles. The torque inferred from the observed 'dipole' power loss rate U is

$$K_{obs} = \frac{U}{\Omega_*} = \frac{\Omega_*^3 r_*^6 B_0^2}{6c^3}. \qquad (10)$$

Therefore, for the current to spin down the star its magnitude should be

$$I = \frac{B_0 \Omega_*^2 r_*^3}{6c} = \frac{|\tau_{GJ0}|}{3} c\pi r_{pc}^2, \qquad (11)$$

which is comparable to the maximum current that can be drawn from the polar cap ($\tau_{GJ0} = \Omega_\star B_0 (2\pi c)^{-1}$ is the GJ density at the pole of an aligned rotator).

Surprisingly, for a perpendicular rotator a similar result obtains. The required current is, apart from a factor of order unity, again the GJ current, now from one half of a polar cap (see Figure 6). Although the lever arm for the torque is larger than in the aligned case by a factor $r_*/r_{pc} = (c/\Omega_* r_*)^{1/2}$ the GJ density $-\vec{\Omega} \cdot \vec{B}(2\pi c)^{-1}$, and, therefore, the maximum current in a perpendicular rotator is smaller by the same factor. Neglecting the minor difference in area between aligned and perpendicular polar caps, we find for the perpendicular case

$$K = 2 \int \frac{R j_\theta B_r}{c} dS \approx \frac{I\Psi}{c} \left(\frac{c}{\Omega_* r_*}\right)^{1/2}, \qquad (12)$$

where we have approximated $R \approx r_*$, $j_\theta \approx I/(2r_{pc})$, $|\tau_{GJ}| \approx \Omega\langle B_z\rangle/(2\pi c)$, and $\langle B_z\rangle \approx 1.5 B_0(\Omega_* r_*/c)^{1/2}$.

Finally, one finds that the required current is, as announced,

$$I \approx I_{GJ0} \frac{3}{4} \left(\frac{\Omega_* r_*}{c}\right)^{1/2}. \qquad (13)$$

Here we have introduced the GJ current for an aligned rotator by the maximum current that can be drawn from one polar cap $I_{GJ0} = |\tau_{GJ}|c\pi r_{pc}^2 = \Omega\Psi/2\pi$.

We come to the conclusion that sufficiently large body currents can, in principle, be set up to brake the pulsar. In the next section we will consider the current path in the equatorial wind.

4 Equatorial Wind

For an oblique rotator we expect a wind to be established as drawn in Figure 3 which consists essentially of two jets aligned with the rotation axis and an equatorial wind perpendicular to it. As the jets and the equatorial wind each have a closed current system they can be considered as separate electrodynamic 'motors'. Here we consider the equatorial wind. In Figure 5 it can be seen that the currents which brake the pulsar are arranged in a series of adjacent flux tubes of different magnetic field orientation, arranged in the radial direction. Note that the direction of the body currents changes also in the radial direction (see Figure 7).

Surprisingly, this picture is very different from the conventional picture of currents in the so-called striped equatorial wind (Kennel & Coroniti 1984; Melatos & Melrose 1996a, 1996b; Bogovalov 1999) which is shown in Figure 8. These authors investigate an asymptotic solution which is characterised by singular sheet currents, and small body currents which can be ignored. They investigate if the current sheets can dissipate to create a kinetic energy dominated flow. In our description, however, these currents are not dynamically important as they are not
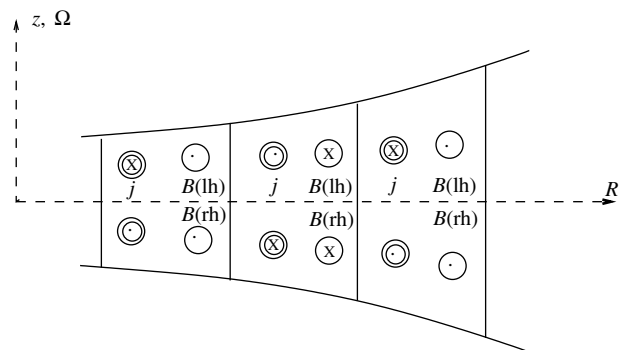

Figure 7 Sketch of the magnetic fields and the electric currents over a radial cut of the equatorial wind. The cut is the same plane as drawn in Figure 5 but now after a large number of windings of the original flux tube. The field in each segment connects to the corresponding polar cap on the star. The horizontal width of a flux tube is roughly constant in the force-free cold flow, and equal to πr_{lc} where r_{lc} is the light cylinder radius. The force-free nature of the current implies that the magnetic field in each flux tube is helical as indicated in brackets (lh for left-handed; rh for right-handed). The pitch of the field on the northern hemisphere is left-handed.
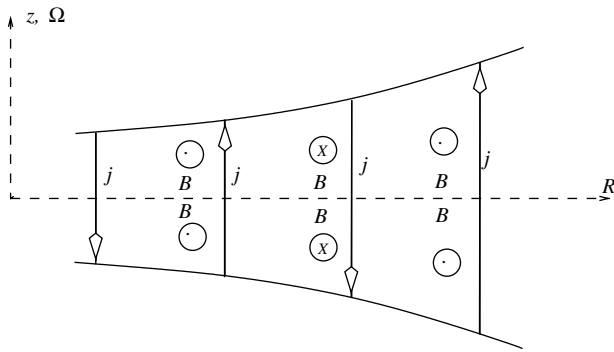
Figure 8 The conventional current picture differs greatly from Figure 7 in that the body currents are unimportant while one focuses on the singular current sheets associated with the contact continuity.

the ones braking the star but just arise from contact discontinuities between alternating flux tubes. This view is supported by the prescription of Bogovalov (1999) which allows the construction of an oblique rotator from an axial one by simply reversing the field direction in suitable sections of the wind without having any consequence at all to the shape or dynamics of the wind. Note that we dispute neither the *existence* of singular currents nor the existence of those asymptotic solutions. The question is whether such a singular current solution can be reached in practice. As to the problem at hand, we have argued above that distributed currents evolve naturally when setting up a pulsar wind. In the next section we will argue that such body currents do in fact lead to the conversion of Poynting flux into kinetic energy flux.

Michel (1971) has proposed that body currents in the equatorial zone of alternating flux tubes are pushed into sheet currents at the boundaries between flux tubes to satisfy the requirements of magnetostatic equilibrium. The argument is that a distributed current profile would give rise to magnetic pressure gradients in the radial direction which are not balanced by tension in the relatively thin flux tubes as is required in a force-free situation. We think that the argument is erroneous. First, the magnetic pressure can and will be balanced in a force-free flux tube, not by the tension due to azimuthal winding but due to winding of magnetic flux lines around the axis of each flux tube. The solar corona offers plenty of examples of such connected and twining flux tubes. This, of course, implies that the field lines as drawn in Figure 7 are only accurate to first order. Superimposed is a winding of magnetic field lines in an individual flux tube around each other. Secondly, in an ideal, force-free plasma it is difficult to move an electric current from one field line to another, and, while it is true that shock waves and dissipative effects may be set up in a pulsar wind it remains unclear how they can lead to the singular current sheets.

5 Parallel Field Acceleration

In a force-free situation the total body current I running through the flux tube is independent of distance R to the

star. As the cold flow eventually reaches its asymptotic speed this creates the following problem. The maximum current density decreases as the particle density

$$j_{\max} = nec \propto \frac{1}{R^2}. \tag{14}$$

However, the current density, required to maintain the equatorial wind, varies inversely with the tube cross-section

$$j \approx \frac{I_{GJ\text{perp}}}{R\pi r_{\text{lc}}\delta}, \tag{15}$$

where δ is the opening angle of the equatorial wind, and we have introduced the abbreviation $I_{GJ\text{perp}} = \int \Omega_\star B_z(r_\star, \theta)\, dS/2\pi$ for the maximum current that can be drawn from (one half of) the polar cap at the equator of a perpendicular rotator. The plasma in the wind, therefore, cannot provide the required number of current carriers beyond an axial distance

$$\frac{R_{\max}}{r_{\text{lc}}} \approx \frac{M}{2\Gamma}, \tag{16}$$

a few tens to hundreds of light cylinder radii. Here M is the multiplicity of the pair plasma (the ratio of secondary pairs to primary particles), and n is measured in the observer's frame. The Lorentz factor Γ of the asymptotic cold wind enters as equation (14) for the maximum current density is valid in the comoving frame. The current density is essentially transverse to the boost direction and remains invariant when transforming to the observer's frame, in contrast to the particle density (Melatos & Melrose 1996a). Note that our result (16) remains correct if the poloidal trajectories of the field lines are curved rather than straight as long as the particles are tied to the field lines and follow the same curved paths.

6 General Magnetic Reconnection

The current carriers in the outgoing wind consist mainly of electrons and positrons, created at the base of the wind by the primary particles. Along those field lines where the current goes outwards, the current is carried by an outward drift of positrons relative to electrons while the electrons drift outwards with respect to the positrons in the region where the electric currents return to the star. Beyond a typical distance given by equation (16), the number of particles is insufficient to carry the required current density, and electric fields develop aligned with the ambient magnetic field. These electric fields accelerate the particles in opposite directions depending on their charge. However, as the drift speed of the current carriers is already relativistic the current starts to dissipate, and with it its transverse magnetic field which is responsible for the Poynting flux. We can obtain an idea about the thickness of the shell inside which at least half of the Poynting flux is converted into kinetic energy flux — and in which, therefore, σ falls from a large value to a value of order unity — as follows. Equations (14)–(15) imply that, between a distance $R_{\max}$ and $\sqrt{(2)}R_{\max}$, the total current
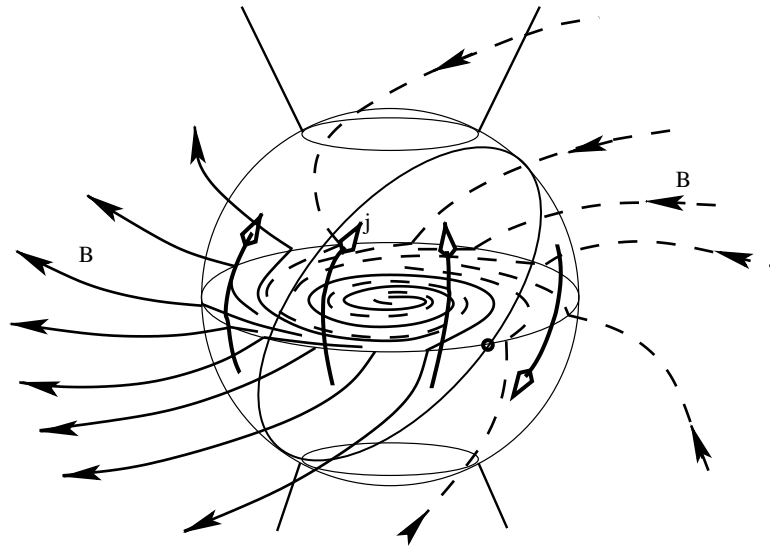
Figure 9 Sketch of the shell where the ideal MHD approximation breaks down because of current starvation. For simplicity, the shell is presented as thin while in reality it has a thickness of the same order as the distance of its lower boundary to the star. Also only a few field lines in the equatorial plane have been drawn, and then only with a number of windings much less than in reality. Inside the shell, the toroidal magnetic field component changes as $1/R$; outside the shell the same is true but at the shell there is a strong decrease in pitch of the field lines. At the shell the current crosses the field and closes across the equator as indicated by open arrows. The field topology changes at the poles where the jets cross. The shell consists of two regions of different magnetic polarity separated by the oblique circle. The outer field drifts with respect to the inner field lines and stays of course on one side of the circle which separates both polarities.

falls below a fraction $1/\sqrt{(2)}$ of what is needed to maintain the magnetic field as prescribed by equation (4), even if no current closure across the field would be assumed in that shell. This means that the Poynting flux decreases by a factor $1/2$ over the same distance, and σ drops to a value of order unity. Therefore, half of the entire wind energy is dissipated in a relatively narrow shell between $M/2\Gamma$ and M/Γ light cylinder radii, deep inside the termination shock! But of course, this energy conversion itself implies that an electrodynamic torque is applied inside the same shell to the extent $K_{\text{electro}} = U/\Omega$, which then shows that half of the existing current closes across the field.

This current dissipation in a relatively thin shell has a number of important aspects:

- The Poynting flux carried by the wind into the shell is locally converted into kinetic wind energy. Above the shell, the wind energy is not dominated any more by electromagnetic energy.
- Both above and below the shell the plasma can be considered to be largely ideal. However, in the shell the strong electric fields destroy the frozen-in condition and the outward magnetic fields reconnect continuously with the inward field structure. This form of reconnection has been termed general magnetic reconnection (GMR; Schindler et al. 1988; Hesse & Schindler 1988; Priest & Forbes 2000), and has, at least in principle, been known for a long time (Dungey 1958).
- The parallel electric voltage drops allow the inner and outer field patterns to slip past each other. The inner magnetic field pattern rotates substantially faster

than the outer pattern, and the amount of slippage is determined by the magnitude of the potential drop.
- A large part of the current closes in the shell as is required by the decrease in Poynting flux, and therefore also in toroidal field strength. Such current closure is in agreement with the existence of a torque that necessarily accompanies the energy release. The cross-field particle drifts come along with the inertial force on the wind. As a result of the partial current closure the pitch of the spiralling field lines strongly decreases in the shell.

The corresponding electrodynamic structure is sketched in Figure 9.

Finally, we note that our prediction of the locus of conversion of the wind magnetic into kinetic energy is compatible with the measured γ-ray flux of the Crab Nebula. According to Bogovalov & Aharonian (2000) inverse Compton γ-ray emission would be too high if the dissipation radius were less than about 30 r_{lc}.

7 Conclusion

We suggest that the solution to the traditional Poynting flux problem in the equatorial part of a pulsar wind such as in the Crab Nebula can be found in the effects of *distributed* or *body* currents of the pulsar wind. These currents are usually not considered, but we have shown that they show an interesting property. At lower heights the pulsar is able to set up a wind with an ideal MHD current system which is largely force-free — note that we neglect localised effects of non-force-free neutral sheets which play a central role

in Mestel (2001) — and is caused by relative drifts of electrons and positrons in the secondary pair plasma. As the distance to the star increases the number density of particles decreases more ($\propto 1/R^2$) than the current density required to sustain the toroidal magnetic field component ($\propto 1/R$), and a current starvation problem occurs. As a result, strong voltage drops develop along the magnetic field lines, the current dissipates and closes across the field lines, the pitch of the spiralling field lines decreases sharply, the rotation rate of the outer magnetic field pattern drops strongly, and a substantial part of the Poynting flux is converted into kinetic energy. We predict that this region of conversion of a Poynting flux dominated wind into a kinetically dominated one occurs in a relatively narrow shell at an axial distance relative to the light cylinder radius of order $R_{\max}/r_{\mathrm{lc}} \approx M/2\Gamma$, where Γ is the Lorentz factor of the incoming wind and M is the multiplicity of the pair plasma at the base.

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