A STATISTICAL TREATMENT OF LOW-N SYSTEMS

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The long-term evolution of N-body systems with high N ($>10^4$) cannot yet be studied by direct integration of the equations of motion. Instead, the study of systems with smaller N can give insight into the higher N limit in two ways. First, we can measure the dependence on N of all quantities of interest for a range of N-values, and extrapolate the results to higher N. Second, even a very high-N system will eventually undergo core collapse, at which point the central dynamics will be dominated by a subset of stars with small N (10-100).

For both reasons it is interesting to explore the extent to which we can meaningfully define statistical quantities, even for very low N. As a first step in this direction, we report here some results obtained with $N = 8$, using a regularized code written by S. Aarseth. The initial conditions have been drawn from a Plummer model, a polytrope of index $n = 5$. We have now 35 realizations of this system. We are currently extending these explorations to higher N, and we hope to study the different characteristics of the evolution when approaching the high-N (Fokker-Planck) limit.

Our results indicate that multi-body encounters, as opposed to two-body scattering, play an important role in the dynamical evolution of systems with small N, especially in their early stages (cf. Aarseth & Lecar 1975). Multi-body encounters favor the formation of binaries, which - during their formation and subsequent hardening - can release enough energy to unbind the cluster (see Fig. 3).

After the formation of a hard binary the evolution of the system can be sketched as follows. The binary will gain binding energy in discrete steps, corresponding to encounters with field stars. As its binding energy increases, encounters will become less frequent - because of the smaller geometrical cross-section - but more energetic (Hut 1984). If, after one of these encounters, the binary acquires enough energy to unbind the cluster, the system will be disrupted. If, on the other hand, the separation becomes too small, the binary is "frozen" and will no longer play a role in the dynamics of the system.

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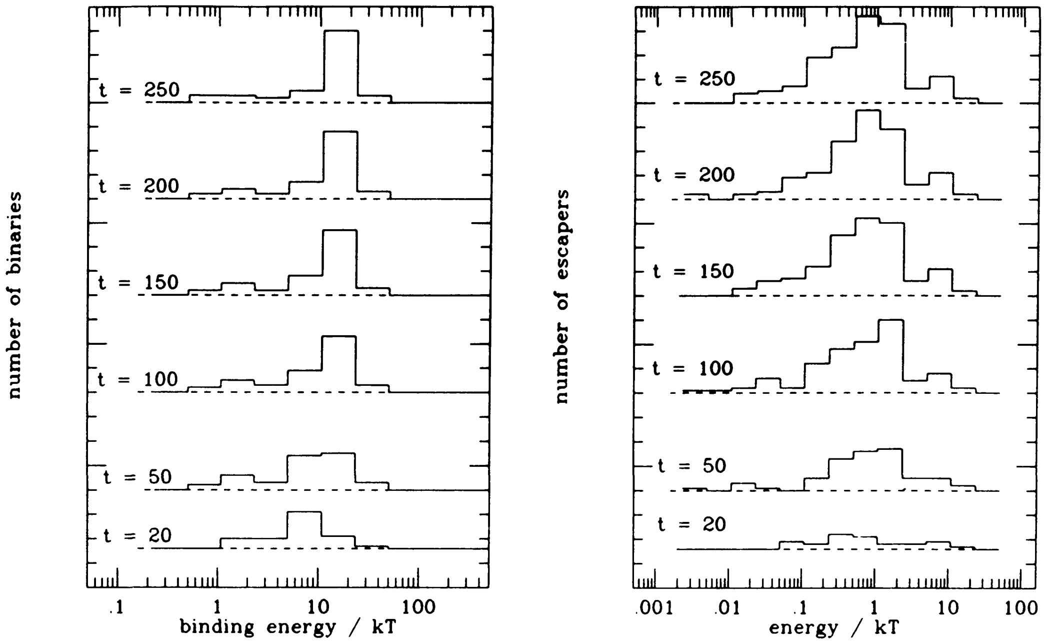

Figure 1: Spectrum of (a) binding energy of binaries and (b) energy of escapers at different times. Tickmarks are in units of ten. In all figures, a time unit is used in which the crossing time $t_{cr} = 2\sqrt{2}$.

This schematic description is illustrated in Fig. 1a, where we show the variation with time of the cumulative spectrum of binaries, summed over all 35 runs. The distribution of binaries evolves in time towards higher energies, until - at the end of the time interval used (approximately 90 crossing times) - almost every system has a binary in the range 10 - 25 kT, where T is defined in terms of the initial velocity dispersion. Fig. 1b shows the corresponding evolution in the spectrum of escapers.

In Fig. 2 we show the number of surviving systems as a function of time (the system is disrupted when four particles of the initial eight escape). The probability of disruption seems to decrease after the initial period. This agrees with the fact that binaries become less effective in disrupting the system as they harden. Note that, even for N=8, the average survival time is orders of magnitude larger than the crossing time. For our sample, the harmonic mean of individual lifetimes is about 100 (35 t_{cr}).

A more detailed diagnostic of the evolution is in Fig. 3a, where we give the energy balance of the system as a function of time. There

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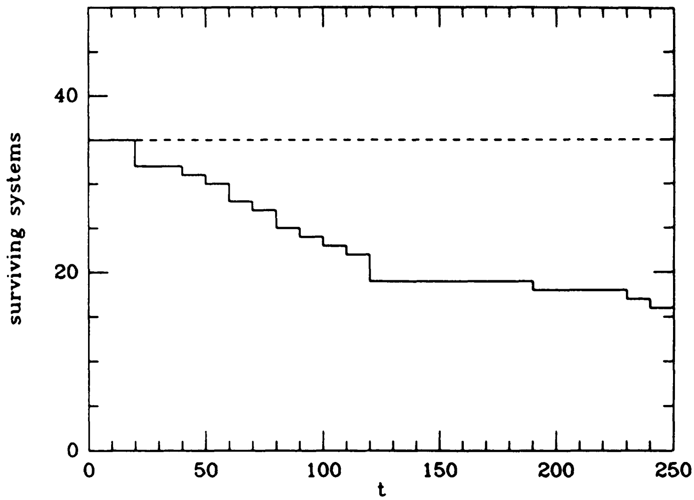

Figure 2: Number of surviving systems as a function of time.

are three types of energy: the internal binding energy of the binaries (thick solid line); energy carried away by the escapers (dotted line); and energy available to the system. The latter is represented by the difference between the solid and the dotted lines, since the binding energy has been offset vertically by a quantity equal to the total energy of the system (constant; dashed line). The energy available to the system increases with time, as energy is poured off by the hardening of binaries. When it becomes positive (i.e., the solid and dotted lines in Fig. 3a cross each other), the system is unbound.

In Fig. 3b we show the variation with time of the core radius, averaged over all 35 runs. The core radius is defined here as in von Hoerner (1963). There seems to be no indication of core collapse in the initial phase of the evolution, and the core radius increases steadily with time. This effect is probably related to the small number of particles, as it is known (Aarseth et al 1974) that for $N > 100$ the core radius decreases initially. We are currently extending our study to larger values of N, hoping to be able to set limits on the minimum number of particles for which the core collapse can take place.

Although general patterns of behaviour can be found, there is a large scatter when we consider individual runs. For this reason we emphasize the need for good statistics, especially for small values of N. In the forthcoming work - with N in the range 10-100 - we are also trying to draw the initial conditions from the same ensemble as for N=8, in order to compare more readily the results for different values of N.

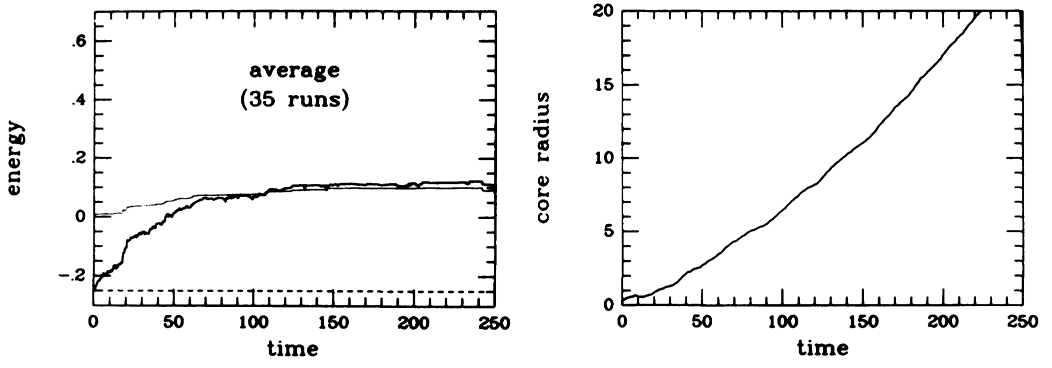

Figure 3: (a) Energy balance as a function of time, averaged over 35 runs with 8 equal mass particles. Units are such that $G = 1$, $M_{tot} = 1$ and $E_{tot} = -0.25$. The difference between the solid and the dotted line is the (negative) total energy of the system, except that frozen in internal degrees of freedom of the binaries or carried away by the escapers.
(b) Evolution of the core radius, averaged over 35 runs. There is no indication of a collapse phase before the expansion.

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