5

Arrows That Misfire

Précis. *The Representation View provides a strategy for determining whether time itself has an arrow, but most purported arrows fail to establish this.*

The 'arrow of time' is a phrase coined by physicist Arthur S. Eddington, to express that time is asymmetric:[1]

> I shall use the phrase 'time's arrow' to express this one-way property of time which has no analogue in space. It is a singularly interesting property from a philosophical standpoint. (Eddington 1928, p.69)

Let us try to get to the bottom of what that 'one-way property' is and determine what evidence there is that time has it. Figure 5.1 summarises some famous responses. And, of course, the list goes on.

A study of these arrows should proceed with caution, as the possibility of illusion looms. Our naïve human senses often detect phenomena that appear asymmetric in time when they are not. For example, a book will slide to a stop on a tabletop, but never the reverse: it does not spontaneously begin sliding. But, when that experience is carefully described in terms of dynamical systems, we find that the description invariably omits degrees of freedom in a way that hides an underlying temporal symmetry.

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[1] The Fellowship of Trinity College, Cambridge seem to have been concerned about the arrow of time: Trinity physicist Eddington was preceded 20 years earlier by Trinity philosopher McTaggart (1908, p.474), who wrote at the end of his famous article: "what is that quality, and is it a greater amount of it which determines things to appear as later, and a lesser amount which determines them to appear as earlier, or is the reverse true?" One wonders how much serious concern a denier of the reality of time can muster about its direction.

Arrow	**Example**
Thermodynamic	Dissipating gas
Radiation	Expanding electromagnetic waves
Quantum measurement	Dynamical state reduction
Cosmological	Cosmic expansion
T violation	Kaon quark flavour mixing
Spacetime	Intrinsic temporal orientation
Causal/dependency	Causes leading to effects

Figure 5.1 Some arrows of time.

This chapter will begin by identifying what is required to have an arrow of time, in the sense that time itself has an asymmetry, through an application of what I call the Representation View.[2] In particular, I will propose that any adequate account of an arrow of time should show both how time itself can be asymmetric and some plausible evidence that supports this. I will argue that, once that background is clarified, most of what is commonly referred to as an 'arrow of time' fails to be a time asymmetry in this sense. The failure can happen in at least three ways: by resorting to *heuristics*; by relying on *boundary conditions*; or by describing a physical system with *missing information*. Those who make use of these techniques may still manage to produce a good explanation of our asymmetric experiences. However, they do not establish an arrow of time.

Section 5.1 will set out what I take to be needed to establish an asymmetry of time itself. The remaining sections will review many of the main contenders for an arrow of time in physics and identify the ways in which each falls short. These include: the radiation arrow (Section 5.2), the arrow of statistical mechanical entropy increase (Section 5.3), cosmological arrows (Section 5.4), quantum collapse (Section 5.5), and causal structure (Section 5.6). I reserve a longer discussion of equilibrium thermodynamics for Chapter 6, where I argue that it falls short as well. Chapter 7 will then present what I take to be the most promising arrow of time: the time reversal symmetry violation that arises from the weak interactions in particle physics.

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[2] Recall that the Representation View was introduced in Section 2.3.

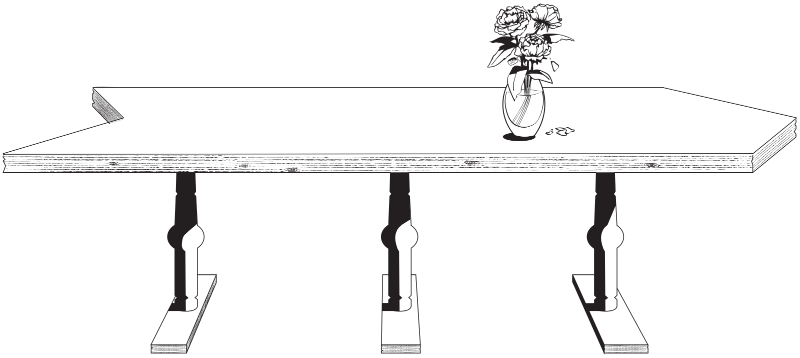

Figure 5.2 Price's table: the asymmetry of the table, like the asymmetry of time, is distinct from the asymmetric placement of items.

5.1 Seeking the Arrow of Time Itself

According to Price (2011, p.292), understanding the arrow of time requires answering a number of questions: "Is time anisotropic *at all*, and how could we tell if it is? What could constitute good grounds for taking it to be so, and do we have such grounds?" I will propose answers to these questions in this chapter and in the rest of the book. But first, let me highlight a general principle that they inspire:

Spacetime–Evidential link: An account of the asymmetries of time or space must identify both a sense in which time or space *itself* is asymmetric, as well as some plausible empirical evidence that supports that asymmetry.

The difficulty we face in the special case of temporal symmetry is to link it to experience, so that we stand a chance of having reasonable empirical evidence for or against it. In his seminal book, *Time's Arrow and Archimedes' Point*, Price (1996, p.16) characterises the difficulty in a picturesque way: think of time as analogous to a table and the evolution of material systems as analogous to the placement of items on it. Then, the question of whether or not 'time itself' is asymmetric is conceptually different from whether the evolution of a material system is – just as the question of whether the table itself is asymmetric is conceptually different from whether the items on it are asymmetrically placed (Figure 5.2). Our most direct experience is of the evolution of material systems. But, how is this linked to the structure of time itself?

The link that I propose involves a shift of focus in how time is often described by philosophers. Instead of speaking about a temporal 'axis' or coordinate variable, I propose to focus on the structural, functionalist aspects of time and speak instead of time *translations*, following the view motivated in Section 2.4.1. Price himself often makes use of the former, writing for example that, “the contents of the block universe appear to be arranged asymmetrically with respect to the temporal axis” (Price 1996, p.17). But, as I argued in Section 2.4, time is much more than that: it has rich relational, topological, and other structural properties including time translations, with elements of the form $t =$ time shift by two hours, rather than $t =$ two o’clock.

We will correspondingly interpret the statement that ‘time has an arrow’ to mean that the structure $\mathbb{T}$ used to describe time translations has an asymmetry, in that $\tau : t \mapsto -t$ is not an automorphism of that structure. And, to replace Price’s notion of the ‘contents’ of time, we will adopt the Representation View, developed in Section 2.3: we postulate a representation of time translations $\varphi : \mathbb{T} \rightarrow \mathrm{Aut}(M)$ amongst the automorphisms of a state space M, in order to give meaning to the notion of time evolution among physical states. According to the Representation View, this is required of any theory that is deserving of the name ‘dynamical’. A representation is what encodes the structure of time translations and their symmetries in the context of a dynamical theory.

On this view, Price’s table is not quite the right analogy. That table might give the impression that the symmetries of the contents of time are totally independent of the symmetries of time itself. In contrast, on the Representation View, these two are deeply linked: a representation is a homomorphism, which ensures that the symmetries of time translate to a ‘homomorphic copy’ amongst the symmetries of state space. So, given a representation of time translations, the structure of time is projected down onto state space, like the shadow of a tabletop on the floor (Figure 5.3). This suggests that, by studying dynamical asymmetries in a representation

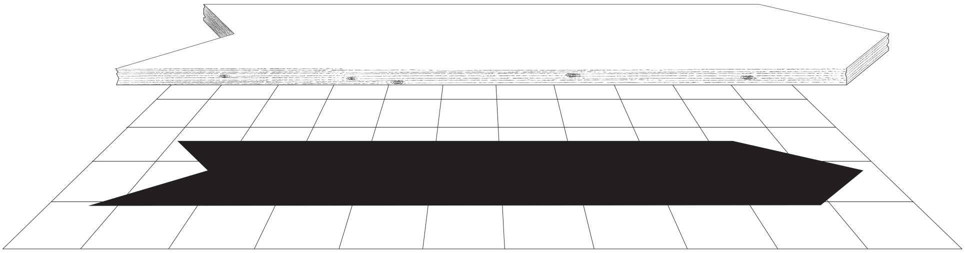

Figure 5.3 The structure of time is projected onto a state space by a representation, the way the structure of a table is projected onto the floor by its shadow.

of time translations, it may be possible to infer an asymmetry or arrow of time itself. That is the kind of asymmetry that I will argue for in detail in Chapter 7. But, in this chapter and the next, let me first review of some competing 'arrows' of time.

In the next sections, I will introduce five phenomena that are commonly referred to as 'arrows of time'. I will argue that, at least in their current formulations, these approaches do not yet pass muster as time asymmetries, because they fail to make the Spacetime–Evidential link in a satisfactory way. In each case, this failure happens for one or more of the following reasons:

1. *Heuristic misfire:* making essential use of an informal extra-theoretical judgement, which is not justified by any well-supported physics.
2. *Boundary Condition misfire:* postulating contingent initial or boundary conditions that pick out one particular class of trajectory as special.
3. *Missing Information misfire:* omitting essential information about the time development of a physical system, which if restored would also restore time reversal symmetry.

The first fails to make the Spacetime–Evidential link because of a lack of adequate evidence for a time asymmetry; the second and third fail to make it because they do not establish an asymmetry in the structure of time translations and so cannot be used to infer that time itself is asymmetric.

Let me emphasise going forward that each of these misfires may be associated with an important area of research in the foundations of physics. However, their significance for our purposes is that they do not to establish an arrow of time itself.

5.2 The Radiation Arrow

An oscillating charge is associated with an outgoing shell of electromagnetic radiation, which expands with phase velocity equal to the speed of light.[3] The phenomenon is analogous to the circular ripples of a water wave when a stone is dropped in a pond. However, like the ripples in a pond, we never seem to observe this radiation in the reverse form of an inward-collapsing shell, as illustrated in Figure 5.4. Is this an arrow of time?

This question was discussed in a correspondence between Planck (1897a) and Boltzmann (1897) and then more famously in a debate between Ritz (1908) and Einstein (1909) in the journal *Physikalische Zeitschrift*. It has

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[3] Thomson (1907, p.217) gave an early derivation of this result, which he identified as analogous to the mechanism producing "Röntgen radiation" or X-rays.

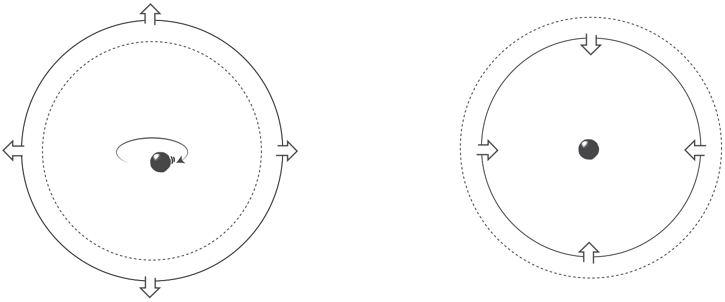

Figure 5.4 An expanding radiation shell (left) is easily produced from an oscillating charge, while a collapsing one (right) is not.

recently generated renewed philosophical interest.[4] The phenomenon is particularly puzzling because the laws of electromagnetism are time reversal invariant: both the outgoing radiation wave and its time reverse are possible! Ritz argued that the observed asymmetry is due to an additional time asymmetric law of nature, while Einstein maintained that it is just a matter of special boundary conditions: roughly speaking, one would have to begin with a ring of charges oscillating in perfect unison in order to produce a collapsing circular wave, just like a water wave. Their conclusions were summarised in a joint statement by Ritz and Einstein (1909).

The boundary condition that Einstein identifies – that the experiment begins with a charge oscillation, rather than ending with one – is not evidence for an asymmetry of time itself. Whenever time reversal is a dynamical symmetry, two oppositely-directed representations of time translations are always possible. In the case of the expanding wave, the reverse time development is equally well picked out by the possibility that the experiment *ends* with a charge oscillation and begins with a boundary condition that gives rise to a collapsing wave. Neither is evidence for an asymmetry of the time translations of electromagnetism, any more than specifying a piece at the boundary of a jigsaw puzzle makes its development asymmetric: the piece could be specified either at the beginning or at the end (Figure 5.5). As a result, no link is made to an asymmetry of time itself. In our three categories of failed arrows of time, this is called a Boundary Condition misfire.

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[4] Popper (1958) seems to have independently arrived at a similar idea. Davies (1977, Chapter 5), Price (1996, Chapter 3), and Zeh (2007, Chapter 2) give classic discussions. Compare also the Ritz-like position of Frisch (2000, 2005, 2006) to the responses of Earman (2011), North (2003), Norton (2009) and Price (2006).

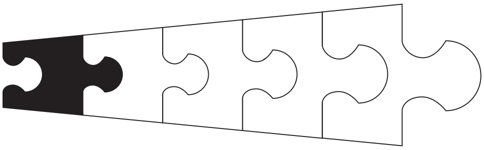

Figure 5.5 Specifying a boundary condition, like specifying a jigsaw puzzle boundary piece, may determine a development from either the beginning or end.

Of course, something might still seem to be missing, at least because most human beings seem to prefer *explanations* that place the oscillating charge at the beginning and the wave ring at the end. But, this by itself is what I have called a Heuristic misfire: it is an extra-theoretic judgement, which is not justified by any well-supported physics.

So, let's try again: one might instead respond by an appeal to statistical facts: in our universe, isolated oscillating charges that produce expanding advanced waves are statistically more likely, whereas coordinated rings of oscillating charges producing collapsing waves are unlikely. However, as Price has convincingly argued, this just moves the heuristic judgement to a different place, through the application of a temporal "double standard":

> [T]here would be advanced waves, despite the improbability, if conditions at the center were as they are when we look at the 'normal' case in reverse: in other words, if wave crests were converging, stones being expelled, and so on. The normal case shows that the statistical argument does not exclude these things as we look toward (what we call) the past. To take it to exclude them as we look toward (what we call) the future is thus to apply a double standard, and to fall into a logical circle – to *assume* what we are trying to *prove*. (Price 1996, p.57)

I agree. In our universe, an oscillating charge and a coordinated ring of oscillating charges are equally likely: if the first occurs, the second occurs as well, for example when the waves produced by the single oscillating charge are absorbed into the environment.[5] More generally, the time reversal invariance of electromagnetism guarantees that if one solution is possible, then the time reversed solution is possible as well. One might like to add that an oscillating charge is more likely to occur 'earlier than' a coordinated ring of charges in time. But, in which temporal direction are we looking when we say this? If we respond, "towards (what we normally call) the future",

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[5] Penrose (1979, p.590) points out an exception, that radiation from a star might escape forever into space without absorption; but, likewise, there might also be the reverse phenomenon of source-free radiation from the big bang or a white hole that gets absorbed by a star.

then we have just assumed what we were trying to prove, that there is a preferred direction of time for formulating such statements.[6]

What may still demand explanation is why, when we represent time evolution in either direction, there appear to be unequal numbers of expanding and collapsing waves at every instant. But, this is an example of an inhomogeneity in space, rather than an asymmetry in time. The study of such inhomogeneities in the early universe is an important and active area of research in modern physics, which we will discuss shortly. But, it is not evidence for an arrow of time itself.

So, let us instead turn to the Ritz strategy in this debate, which is to reject that time reversal is a symmetry of the dynamics and instead stipulate a new law of nature that is asymmetric in time. This new law is supposed to be similar to the original laws, except that it restricts solutions to those that satisfy an early-time boundary condition. If such a statement can be made precise, then it might well provide a new dynamical theory in which time reversal symmetry fails. We will discuss theories like that in Chapter 7. But, in the present context, we do not have such a theory. Moreover, most agree that the origin of electromagnetic (EM) asymmetry lies elsewhere. The latter statement might be called, *Earman's Conjecture:*

> I will mention a more general conjecture: any EM asymmetry that is clean and pervasive enough to merit promotion to an arrow of time is enslaved to either the cosmological arrow or the same source that grounds the thermodynamic arrow (or a combination of both). But much more work would be needed before I would be willing to make this conjecture with any confidence. (Earman 2011, p.524)

Thus, even on the Ritz approach, the radiation arrow at best reduces to one of the other arrows, either dynamical, cosmological, or thermal. So, let us turn to those other arrows instead.

5.3 The Arrow of Statistical Mechanics

A well-known folklore going back to Boltzmann (1896, 1898) says that the increasing entropy of the universe, as observed in processes like melting ice, is what determines time's arrow (Figure 5.6). Reichenbach formulates

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[6] Price (1991, 1996) argues further that temporal symmetry is supported here by the adoption of the Wheeler and Feynman (1945) absorber theory, where each contribution to an electromagnetic field between a source and an absorber is half-advanced and half-retarded, but adjusting the proposal by dropping the requirement that every source has a perfect absorber. My own sympathies still lie with the photon: as Penrose (1979, p.590) points out, a theory that so tightly constrains the electromagnetic field seems "unfairly biased against the poor photon, not allowing it the degrees of freedom admitted to all massive particles!".

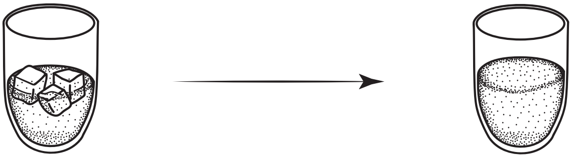

Figure 5.6 The ‘arrow’ of increasing entropy.

this as the statement: “positive time is the direction toward higher entropy” (Reichenbach 1956, p.54). This statement can be interpreted in two ways: either in terms of classical thermodynamics, or in terms of statistical mechanics. Classical equilibrium thermodynamics does not aim to describe the fundamental constituents of reality; and, I will give an extended argument in Chapter 6 that it does not establish an arrow of time. So, in this section, I will focus on the case of statistical mechanics. It faces all three of the misfires identified at the outset of this chapter.

5.3.1 Boundary Condition Misfires

Statistical mechanics is formulated with either classical or quantum mechanics as its basis. For this reason, it is possible to give a rigorous representation of time translations in this theory, which is usually time reversal invariant (or at least CPT invariant). A variety of well-known strategies can then be used to argue that, on a certain ‘coarse grained’ level of description, the system can be expected to evolve towards a higher entropy state until it reaches equilibrium. Boltzmann’s *Stoßzahlansatz* (‘assumption of molecular chaos’) in classical statistical mechanics is perhaps the most famous, but a variety of master equations in classical and quantum statistical mechanics also describe the irreversible evolution of probability distributions.[7] Such arguments invariably postulate a special boundary condition in the form of a probability distribution on microstates, or on macrostates, or both, for example to render the state of the universe to be far out of equilibrium in the first microseconds after the big bang.[8]

Thus, like the radiation arrow, this approach to statistical mechanics suffers from a Boundary Condition misfire: a time asymmetry only seems to appear because of special initial or boundary conditions and not because

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[7] See Zeh (2007): §3.1.1 for a discussion of the former, and §§3.1–3.2 for the latter.

[8] This approach is the subject of a great deal of philosophical debate: see Albert (2000, Chapter 3), Callender (2010), Earman (2006), Penrose (1979, §12.2.3), Penrose (1994), Price (1996, Chapter 2), Price (2004), Wallace (2013, §4.2), and Zeh (2007, §3.1.1).

of an arrow of time itself. Indeed, a Price-style argument can be applied to the statistical mechanical arrow as well, which of course Price gives:

> [W]e don't take the *final* microstate to explain the *initial* ordered state. But then by what right do we propose that an *initial* microstate can explain a *final* macrostate? In practice, of course, we are inclined simply to help ourselves to the principle that the past explains the future, but what could possibly justify that inclination here, where the temporal asymmetry of the universe is what we are seeking to explain? (Price 1996, p.42)

One might try to resist Price's conclusion here, by insisting on the logic of Boltzmann's *Stoßzahlansatz*. That is: one should expect all states to produce a maximum entropy state, both to the past and to the future, and so a low entropy past is enough to produce the increasing entropy of our universe. In contrast, a high entropy future is not, since it only produces other high entropy macrostates. Maudlin has responded to Price in this way, suggesting that the process by which macrostates "produce" higher entropy requires a temporal direction.[9]

I won't dwell on the formal issues with the Boltzmann approach, which, as many have noted, does not involve rigorous argumentation.[10] My concern is rather with two overarching concerns about the Boltzmann picture: the origin of the coarse-graining, and the role of time translations. These lead to the next two misfires.

5.3.2 Heuristic Misfires

A coarse-graining in statistical mechanics is an equivalence relation on microstates, which defines what it means to be in the same macrostate. Without this, one cannot define either equilibrium or entropy in statistical mechanics. A typical approach is to fix a set of 'macroscopic' observables $\{A_i\}$ on the underlying classical or quantum state space and define an equivalence relation on pairs of states by the relation of 'being assigned the same value by all the observables A_i'; this partitions the space into

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[9] Maudlin (2007, p.134–5) writes: "Even though the laws themselves might run perfectly well in reverse . . . we cannot specify an independent, generic constraint on the final state that will yield (granting the final macrostate is typical) ever decreasing entropy in one direction. . . . This sort of explanation requires that there be a fact about which states produce which. That is provided by a direction of time: earlier states produce later ones".

[10] See Price (1996, p.40), or Sklar (1993, §7.II.1) for a classic argument that the *Stoßzahlansatz* is usually false. Earman (2006) argues that the whole low-entropy-past approach is "not even false" on the scale of the universe as a whole; and D. Wallace (2011, "The logic of the past hypothesis", Unpublished manuscript, http://philsci-archive.pitt.edu/8894/) and (2013) argues that it is redundant.

macrostates.[11] Intuitively, two states are equivalent if we have no way to observe a distinction between them using our collection of macroscopic observables. But, how do we choose the set of observables on which to base this judgement?

Unfortunately, the formalism of statistical mechanics alone does not provide any advice about this question. The problem is, moreover, a serious one, because different choices of observables give rise to different definitions of equilibrium and entropy: thus, the definition of the 'statistical mechanical arrow' depends on it. Indeed, Rovelli (2017) has argued on this basis that the statistical mechanical arrow of time is fundamentally perspectival: formally speaking, there are choices of observables that lead to oppositely-directed arrows of time.

Penrose (1979, p.588) notes that the problem of choosing a set of observables or coarse-graining is "fraught with difficulties", in spite of the fact that by a standard convention, "entropy is a concept that may be bandied about in a totally cavalier fashion!" In the absence of any well-motivated choice for fundamental physics, we face having to embrace this fact: that the approach to equilibrium in statistical mechanics depends on a choice of observables, which is at worst arbitrary and at best a contingent fact about the observations currently available to us. Thus, this arrow suffers from a Heuristic misfire: it does not describe an asymmetry of time itself but rather depends on extra-theoretical judgements that lack an evidential basis in physics.

5.3.3 *Missing Information Misfires*

The final misfire arises from a peculiarity of Boltzmann's argument, which is that his *Stoßzahlansatz* makes no mention of the time parameter characterising the approach to equilibrium. As a result, it leaves out some important physical facts about equilibrium, such as the relaxation time required to achieve it. This leaves us with a rather poor explanation of the approach to equilibrium in our universe. But, more importantly for our purposes, if no time translations are associated with the approach to equilibrium, then there is no way to associate it with the structure of time through a representation. Boltzmann's approach by itself lacks the information needed to establish an arrow of time itself.

So, a more appropriate interpretation of Maudlin's "production" process for high entropy states is one that includes a time parameter, like the

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[11] Conversely, every partition of a state space into macrostates can be associated with a set of observables with this property. See Wills (Forthcoming) for a philosophical discussion of this approach in the context of particle identity debates and Rovelli (2017) for an argument that this makes the statistical mechanical arrow 'perspectival'.

Zwanzig (1960) projection formalism, among others.[12] These approaches to the arrow of time are of a fundamentally different form: they invariably produce a representation of time translations in terms of a dynamical evolution operator on probability distributions, known in differential form as a *master equation*. These approaches provide a much more empirically adequate account of the approach to equilibrium. However, they also generally formalise the fact that we are leaving out information. As Wallace (2013, §3) has pointed out, the fact that such equations 'project out' information over time is what ultimately leads to their time asymmetry. So, this is an example of a Missing Information misfire: an arrow of time only appears to arise because some facts are being ignored.

Statistical mechanics is widely identified as the source of the arrow of time. However, there are devils in the details: this supposed arrow misfires in all three of the ways I have identified above, including heuristic additions, boundary conditions, and missing information. Each of these might be fruitfully used in a physical model. But, none of them provide evidence for an arrow of time.

5.4 Cosmological Arrows

The cosmological arrow, in its simplest form, is the observation that the universe on the largest scale is expanding rather than contracting. In a slightly more interesting sense, it is the additional fact that structure formation occurs in an asymmetric way, with great gases clumping together to form galaxies, stars, and planets, but not generally the reverse. Insofar as general relativity is time reversal invariant, these asymmetries are characterised by special boundary conditions: at an early moment, the universe was extremely dense, as well as nearly homogeneous and isotropic, with just enough inhomogeneity to yield clumping and structure formation. At the other end of time, it appears that the universe will continue to expand forever towards an ever-more lifeless and diluted state. One of the great achievements of twentieth-century cosmology is the confirmation of this description of our universe.

5.4.1 Boundary Condition Misfire

However, when the cosmological arrow is put this way, the Price-style objection applies, just as it did in electromagnetism and in statistical mechanics.

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[12] See Zeh (2007, Chapter 3), who argues that Boltzmann's *Stoßzahlansatz* can be viewed in the same spirit as modern statistical mechanical master equations.

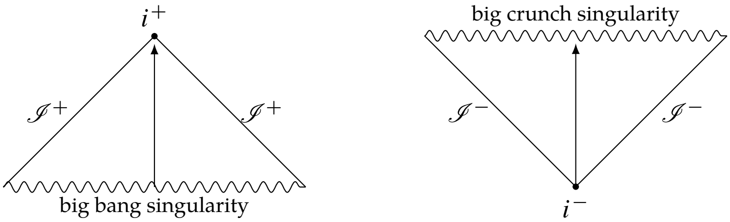

Figure 5.7 Conformal diagrams for a big bang (left) and big crunch (right) universe.

The original model of an expanding universe – called *FLRW spacetime*, for Friedman, Lemaître, Robinson and Walker – is completely time reversible: each of the two available temporal orientations define 'future-directed' developments in opposite directions. This is illustrated in Figure 5.7 using a Penrose conformal diagram, which accurately represents lightcone structure but warps distances so as to allow the representation of infinitely long curves. The first development in the illustration begins with a big bang and then dissipates on its way towards future timelike infinity i^+. The second begins dissipated at past timelike infinity i^- and collapses to a big crunch.[13] We cannot say which represents the 'correct' future-directed development without begging the question as to the direction of time. Again, Price says as much:

> [U]ntil we find some reason to think otherwise, we should take the view that it is not an objective matter which end of the universe is the 'bang' and which end is the 'crunch'. (Price 1996, p.84)

Let me emphasise that I do not take this to mean that modern cosmologists are confused: far from it. As far as I can see, cosmologists simply use the phrase 'time's arrow' to refer to a completely different issue: not whether cosmic time translations can be represented in opposite directions but whether they couple in an appropriate way to statistical mechanical entropy as we ordinarily understand it. That is the subject of the next subsection.

5.4.2 *Statistical Mechanical Coupling*

The question of how the cosmic expansion couples to statistical physics is tied up with serious foundational issues, as Penrose (1979) has pointed out.

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[13] Of course, these two descriptions are not generally related by an isometry, i.e. the spacetime is not 'temporally anisotropic' in the sense of Earman (1974, p.29). I will discuss this more in Chapter 7.

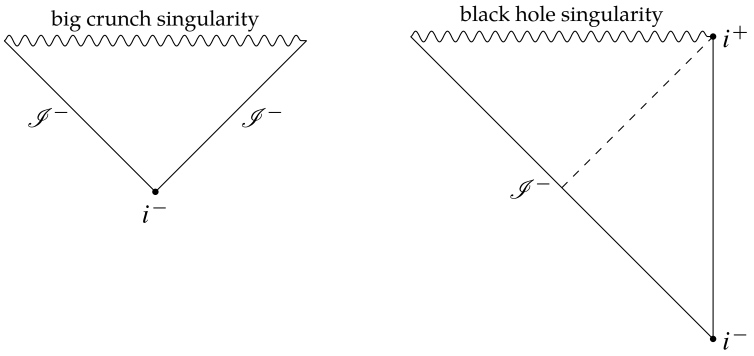

Figure 5.8 The geometry of a big crunch is similar to that of a black hole.

For example, suppose that statistical mechanical entropy were coupled so as to increase with cosmic expansion and therefore to decrease with cosmic contraction. The environment inside a black hole event horizon is formally quite similar to a collapsing universe, as illustrated in the Penrose diagrams of Figure 5.8. So, on this coupling, statistical mechanical entropy should decrease inside the black hole as well. This can even happen near an event horizon where the geometry is not so strange, with water unmelting and glasses unbreaking, among all manner of bizarre things:

> Suppose that experiments are performed by the astronaut for a period while he is inside the hole. The behaviour of his apparatus (indeed, of the metabolic processes within his own body) is entirely determined by the conditions at the black hole's singularity (assuming that behaviour is governed by the usual hyperbolic-type differential equations) – as, equally, it is entirely determined by the conditions at the big bang. The situation inside the black hole differs in no essential respect from that at the late stages of a recollapsing universe. If one's viewpoint is to link the local direction of time's arrow directly to the expansion of the universe, then one must surely be driven to expect that our astronaut's experiments will behave in an entropy-decreasing way (with respect to 'normal' time). (Penrose 1979, pp.598–9)

Here Penrose is using the phrase 'time's arrow' as I have suggested, to refer to the arrow of entropy increase. His aim is to state a potential difficulty with assuming it is aligned with the geometry of expansion: this implies the geometry of collapse is aligned with entropy decrease. A great deal of effort has gone into the description of this coupling as part of the search for new physics, which prominently includes the no-boundary proposal of Hartle and Hawking (1983) as well as the Weyl curvature hypothesis of Penrose (1979).

Price has been a prominent critic of the former, charging that Hawking's conjecture does not produce a convincing arrow of time, although it appears that Hawking was never convinced.[14] In my view, there is room for agreement between the two parties: Price is right that Hawking's account is compatible with the possibility of 'no arrow' in the sense of oppositely-directed representations of cosmic time translations; but Hawking's proposal has a different purpose, to establish an 'arrow' in the sense of a plausible thermodynamic coupling.

So, the cosmological arrow as described either suffers from a Boundary Condition misfire or else from all the misfires of thermodynamics, if we view it as fundamentally defined by this coupling. In this form, it does not provide the kind of asymmetry needed to establish an arrow of time itself.

5.4.3 Baryogenesis

There is another aspect of the cosmological arrow that may yet provide a plausible source of time asymmetry. A boundary condition by itself does not determine an arrow of time. But, it may be an indication of something that does, like an instance of dynamical symmetry breaking.

For example, if the boundary condition for the big bang is a highly symmetric state, which is symmetric even in the distribution of baryonic (ordinary) matter as compared to antimatter, then some mechanism is needed to explain how the current universe came to be dominated by the former. This is called *baryogenesis*. One obvious proposal is that some symmetry-violating interaction led to more ordinary matter than antimatter. This is called a *charge conjugation symmetry violation*. Even a small amount would help explain baryogenesis, if it could be shown to be magnified by the subsequent cosmic evolution.

Sakharov (1967) pointed out that baryogenesis would have to involve more than just charge conjugation symmetry violation. Its combination with the parity transformation, denoted CP, would have to be violated as well, since otherwise baryon symmetry violation could still happen in equal amounts for both matter and antimatter.[15] So, assuming that CPT is a symmetry of whatever this underlying theory is, time reversal symmetry would be violated as well.

How this process works is a matter of active research, since the Sakharov conditions are clearly at best necessary conditions for baryogenesis.[16]

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[14] Price (1989) published his critique in the journal *Nature* and later expanded on it in his book (Price 1996, Chapter 4). As Brown (2000, p.335) noted in his review of the book, the former article "essentially accused Hawking of sleight of hand".

[15] These transformations are discussed in more detail in our discussion of CPT in Chapter 8.

[16] See White (2016) for a recent introduction.

However, this means that, according to one prominent road to understanding the cosmological arrow of time, it reduces it to a dynamical arrow: namely, a time asymmetry in the dynamical theory governing early universe interactions. I will argue that this kind of time asymmetry really can provide a plausible arrow of time. We will return to the arrow of time in dynamical theories in Chapter 7.

5.5 Quantum Collapse

Quantum theory famously has two predictive laws: unitary (Schrödinger) evolution and the statistical Born rule. It is often suggested that the former is usually time reversal invariant, while the latter is not. However, on the most direct reading, this is incorrect: unitary evolution is time symmetric if and only if the Born rule is. Let me begin by clarifying what I mean by that.

5.5.1 Harmony between Schrödinger and Born

Suppose that the dynamics of quantum theory is given by a strongly continuous unitary representation $\mathcal{U}_t$ of the time translation group $(\mathbb{R},+)$ on a Hilbert space. According to the Born rule, if an initial state ψ evolves unitarily for some time t, then the probability of finding it in the state ϕ is given by

$$\Pr(\psi \xrightarrow{t} \phi) = |\langle \phi, \mathcal{U}_t \psi \rangle|^2. \tag{5.1}$$

Following our discussion of Section 3.4, consider the time reversed representation of this set-up: if the time reversed state $T\phi$ evolves under the same dynamics for a time t, then the probability of finding the system in the time reversed state $T\psi$ is given by

$$\Pr(T\phi \xrightarrow{t} T\psi) = |\langle T\psi, \mathcal{U}_t T\phi \rangle|^2 = |\langle \psi, T^* \mathcal{U}_{-t} T\phi \rangle|^2. \tag{5.2}$$

By an elementary theorem of Hilbert space theory (cf. Messiah 1999, §XV.2 Theorem II), the probabilities in Eqs (5.1) and (5.2) are equal for all states ψ, ϕ if and only if $T\mathcal{U}_t T^* = e^{i\theta}\mathcal{U}_{-t}$ for some arbitrary phase factor $e^{i\theta}$. But, this phase disappears in the description of true quantum states as 'rays' or one-dimensional subspaces.[17] So, the equality of these equations really

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[17] Another way to look at this is to note that the phase factor θ can be eliminated by redefining the Hamiltonian generator of $\mathcal{U}_t = e^{-itH}$ by $H \mapsto H - \theta$. Since the Hamiltonian only has physical significance up to an additive constant, this adjustment makes no difference to the predictions of quantum theory.

holds if and only if $T\mathcal{U}_t T^* = \mathcal{U}_{-t}$. That is just what it means to say that time reversal is a dynamical symmetry in this representation (see Section 4.1.3). In other words, Schrödinger evolution is symmetric in time if and only if the statistical predictions of the Born rule are time symmetric: the two rules are in happy harmony.

In Chapter 7, I will argue that quantum theory does provide evidence for a true arrow of time, through the time reversal symmetry violation associated with electroweak theory. In line with what I have just said about the harmony between its two rules, this means that both Schrödinger evolution and the Born rule are time asymmetric in the weak interactions.

However, there are at least three other ways that quantum theory is often said to have an arrow, even when its unitary dynamics and statistical rule are time symmetric. These are: the collapse of the quantum state, decoherence, and branching universes. Although each of these ideas is associated with an important research programme, each of them also misfires as an arrow of time. Let me briefly review why that is.

5.5.2 Collapse

Von Neumann (1955, Chapter 6) postulated that quantum measurement is associated with a 'state reduction' or 'collapse' process. This led to a research programme that supplements or replaces quantum theory with a new dynamical law, which describes how each quantum state will irreversibly evolve to an eigenstate of some observable during the measurement process. The two most prominent approaches to this are the 'GRW' or 'flash' theory, due to Ghirardi, Rimini, and Weber (1986), and its continuous analogue 'CSL' or 'continuous spontaneous localisation', due to Pearle (1989). I will refer to these collectively as *collapse theories*.[18]

Collapse theories are often regarded as being asymmetric in time. To illustrate the idea in simple terms, let me adopt a picturesque example that follows Rovelli (2016). Imagine a quantum state undergoing Schrödinger evolution $\psi(t) = \mathcal{U}_t\psi$, at least approximately and which describes a wavefunction expanding around some point p in space. After evolving this way for a duration of time, the wavefunction collapses to a small shell around a different point q, according to the dynamics of some collapse theory. What is the time reverse of this process? It is a system that begins as a small, outward moving shell centred around p, which spontaneously

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[18] For an introduction, see Bassi and Ghirardi (2003); Bassi, Lochan, et al. (2013); Ghirardi and Bassi (2020).

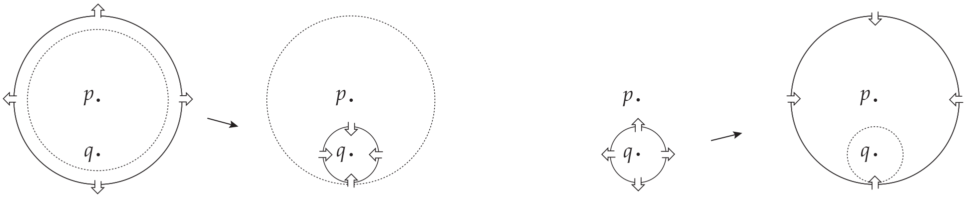

Figure 5.9 A collapse process (left) and the time reversed description (right).

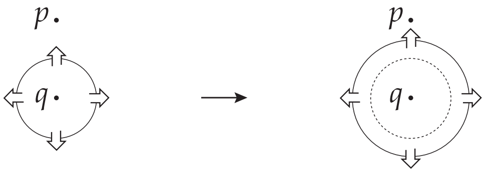

Figure 5.10 Correct evolution from the time reversed initial condition.

'jumps' to a large shell centred at q, which it then collapses down towards, as shown in Figure 5.9.

But, no approximation of quantum theory will describe the system this way: an outward moving shell around q will continue to expand around q, with no jumping, at least until another collapse event occurs (Figure 5.10). So, this concept of 'collapse' appears to inevitably introduce a time asymmetry. Moreover, the unitary dynamics in this example (and hence the Born rule) are time symmetric! So, the origin of the time asymmetry arises from the addition of a dynamical collapse law.

What does this say about the arrow of time? The answer depends on the status of collapse theories. I will argue in Chapter 7 that, if a dynamical theory is time reversal violating, and if that dynamical theory is accurate, then it provides evidence of a true asymmetry in time. So, if a collapse theory turns out to be both accurate and time asymmetric, then this is good evidence for an arrow of time.

However, neither of these two requirements have been settled in the case of collapse theories. Rovelli argues that this disharmony between the symmetries of the unitary dynamics and collapse is so egregious that we should reject the latter, and any metaphysics that goes with it.[19] Of course,

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[19] In particular, Rovelli (2016, p.1232) takes this as evidence against a certain kind of realism about the quantum state, writing, "But if the state is taken to be real, the fact that it behaves in a non T-invariant way, when everything we measure about the (classical and quantum) world is T-invariant, sounds illogical".

some of this decision might be left up to experiment: collapse theory is associated with some novel empirical predictions, like a slow increase of energy.[20] However, no decisive evidence in its favour has yet been produced, and so its status as a replacement for quantum theory remains unclear. Regarding the time asymmetry of collapse: this claim can be challenged as well. For example, Bedingham and Maroney (2017) propose a framework for collapse theory in which it is time reversal invariant, arguing that the apparent time asymmetry is the result "not of an inherent asymmetry in the dynamics, but of the time asymmetric use of boundary conditions" (Bedingham and Maroney 2017, p.692). Thus, the implications of collapse theories for the arrow of time are at best unsettled and at worst a Boundary Condition misfire.

5.5.3 Decoherence

Many of the unusual statistical observations associated with quantum phenomena can be explained using ordinary quantum theory without a collapse postulate. For example, consider the double-slit experiment, in its classic presentation by Feynman (1963b, §1): a beam of electrons is directed at a thin metal plate with two small holes in it, producing an interference pattern on a surface located on the other side. This arises from the fact that the quantum system emerging from the plate is a superposition of two positions. But next, suppose that we 'watch' which hole each electron has passed through, by placing a light source on the side where they emerge, which scatters in different ways depending on which hole the electron passes through. Then the interference pattern disappears, and one observes a concentration of electrons in front of each hole.

The phenomenon of *decoherence* explains the difference between these two observations using ordinary unitary (Schrödinger) evolution. The central idea is to consider how the electron subsystem interacts with its environment. In the first experiment, electrons emerging from the metal plate are more or less isolated from their environment. But, once a macroscopic light source is introduced, the electrons interact with that object in complicated ways, which include a spectacularly large number of degrees of freedom. Decoherence is a body of work showing that, when such interactions are accounted for in the unitary dynamics, the probability of a superposition

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[20] This was pointed out in the original paper of Ghirardi, Rimini, and Weber (1986). See Bassi, Lochan, et al. (2013) for a more recent discussion of the experimental basis for collapse theories.

is suppressed to a vanishingly small value, with respect to some basis.[21] As a result, the probability of observing an interference pattern becomes vanishingly small as well, thus explaining the difference between the two observations.

This suppression of interference is sometimes referred to as the "time arrow of decoherence" (cf. Zeh 2007, p.16). However, this usage of the term should not be understood as referring to an asymmetry in time itself. If the interaction of a subsystem with its environment is time reversal invariant, as most such interactions are, then decoherence is completely symmetric in time: if there is a representation in which interference becomes suppressed towards the future, then there is one in which it becomes suppressed towards the past as well.[22] Of course, the founders of decoherence theory were well-aware of this and of the reason for the apparent time asymmetry:

> Environments are notorious for having large numbers of interacting degrees of freedom, making extraction of lost information as difficult as reversing trajectories in a Boltzmann gas. (Zurek 1991, p.40)

In other words, the apparent time asymmetry of decoherence is very similar to that of statistical mechanics. In particular, it is a Missing Information misfire: an arrow only appears when we neglect information about the environment as 'lost'. It is a remarkable fact that so much quantum behaviour can be understood in this way. However, having limited access to information is just a fact about the human condition rather than evidence for a true time asymmetry. Indeed, if those environmental degrees of freedom are restored, then whenever the interactions are time reversal invariant, there is a perfectly adequate representation of time translations in the opposite direction and in which spontaneous coherence occurs.

5.5.4 *Branching*

On the Everett interpretation, also called the 'many worlds' or 'multiverse' approach to quantum theory, the universe is imagined to be a great branching tree, with each path through the tree corresponding to a familiar world of definite measurement outcomes.[23] The strange phenomena of quantum superposition in some basis is then imagined to arise from facts

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[21] The modern study of decoherence was launched by the works of Griffiths (1984), Gell-Mann and Hartle (1990), Zeh (1970), and Zurek (1981). For an introduction, see Bacciagaluppi (2020) or Joos et al. (2013).

[22] A similar point is emphasised by Hagar (2012, pp.4603–4).

[23] The many worlds approach is due to Everett III (1957); for a modern defence, see Wallace (2012).

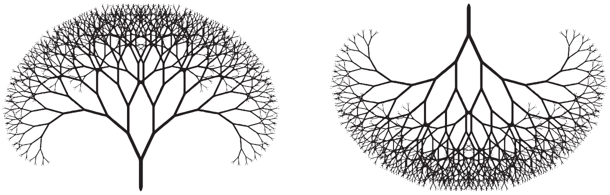

Figure 5.11 Everettian 'branching' happens to the future (left) but not to the past (right).

about a multiplicity of branches: for example, in the double-slit experiment described above, each electron travels through both holes in the metal plate, but on different branches of the tree.

Since the branches of the Everettian tree appear to 'grow' towards the future but not the past, as in Figure 5.11, this interpretation is sometimes said to be associated with a time asymmetry. Indeed, Rovelli (2016, §) presents this as evidence against the Everett interpretation, given that most quantum interactions are time reversal invariant (or at least CPT invariant).[24] If an Everettian did add a time asymmetric structure to quantum theory of this kind, then in my language it would be a Heuristic misfire: it is an addition to quantum theory that is not justified by any well-supported physics.

However, in modern approaches to the Everett interpretation, such as the one presented by Wallace (2012), Everettian branching is understood in a different way. Branching is not a part of the fundamental quantum dynamics, and so it is not associated with a representation of time translations at the level of fundamental microscopic behaviour. Instead, it is used to help explain the emergence of definite outcomes relative to the experience of a macroscopic observer. Indeed, Wallace (2012, Chapter 3) places decoherence at the foundation of branching: its suppression of superpositions in some basis is taken to imply that there is a preferred basis for measurement outcomes, each one corresponding to a 'branch' in the Everettian tree.[25]

But, as I have argued above, this is a Missing Information misfire when it comes to the arrow of time: it only appears to be an asymmetry from the perspective of an observer who ignores environmental degrees of freedom.

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[24] CPT invariance is discussed in detail in Chapter 8.

[25] Wallace (2012, p.87) summarises how successive suppression due to decoherence can create a branching structure: "[I]f the state evolves from a basis vector to a superposition of such basis vectors, and if each of those evolves into a superposition of different basis vectors so that no two such superpositions interfere with one another – then we would have branching (relative to that basis, at any rate)".

This is not a problem for Wallace, whose aim here is to help explain the emergence of the macroscopic world. But, neither is it a basis for the arrow of time.

5.6 Causal Structure

Our final discussion is about the supposed 'arrow of causation'. Philosophers often locate the arrow of time in the statement that all effects occur later than their causes, or in some more general asymmetric dependence relation such as that of Lewis (1979). However, most philosophers of physics are sceptical of this, in the absence of any well-defined physical property corresponding to 'causation'. This led Russell (1912) to decry causation as "a relic of a bygone age", Norton (2003) to demote it to a "folk science", and Stephen Hawking to report being "very disappointed" by Reichenbach's book, *The Direction of Time*. Hawking's basic concern was that, if causes and effects are states associated with a time reversal invariant dynamics, then they cannot be asymmetric in time:

> I was very disappointed. It was rather obscure, and the logic seemed to be circular. It laid great stress on causation, in distinguishing the forward direction of time from the backward direction. But in physics, we believe there are laws that determine the evolution of the universe uniquely. So if state A evolved into state B, one could say that A caused B. But one could equally well look at it in the other direction of time, and say that B caused A. So causality does not define a direction of time. (Hawking 1994, p.346)

This characterisation of Reichenbach's argument is not entirely fair: Reichenbach's idea is, just as Hawking suggests, to begin with a dynamical theory that is time reversal invariant. Reichenbach then uses "causal connections" in a temporally symmetric way as a synonym for this dynamics, applying phrases like "A is causally connected to B" to refer to states A and B that lie on the same trajectory of a dynamical theory, without saying which is a 'cause' and which is an 'effect' (Reichenbach 1956, §5). However, to describe the "direction" of such a trajectory, he then goes on to propose we use the increase of statistical mechanical entropy. For our purposes, this approach falters because, as we have seen in Section 5.3, it falls prey to each of the three misfires identified at the outset of this section.

Since the appearance of Reichenbach's book, one of his characterisations of the arrow has become the subject of much discussion, known as the Principle of the Common Cause. In its simplest form, the principle says that every pair of correlated events admit an event in their common past (a 'cause') that increases their probability and screens off the correlation between them.[26] This principle can be used to state the existence of a time asymmetry, so long as it includes the further postulate that there is no common cause in the future. Philosophers sometimes refer to this kind of asymmetry as a 'fork asymmetry'. However, whether it is evidence for an arrow of time depends on how we interpret it. Price (1996, Chapter 6) argues that the Principle of the Common Cause and other purported 'forks' just amount to further boundary conditions.[27] In that case, this approach amounts to a Boundary Condition misfire.

On the other hand, Penrose and Percival (1962, p.616) elevate a version of Reichenbach's principle to the status of a "basic statistical law, which is asymmetric in time". If we view that law as associated with a dynamical theory, which admits a representation of time translations without a reverse representation, then this is a different situation, which I will call a 'dynamical arrow'. This sort of case, I claim, is more promising, and it will be dealt with in Chapter 7.

5.7 Summary

This section has presented a brief review of some commonly identified 'arrows of time'. Nearly all of them involve a great deal of interesting physics and philosophy. However, most of them also misfire as arrows of time. They may be Boundary Condition misfires, as when one identifies oscillating charges as to the 'past' of their absorbed radiation; they may be Heuristic misfires, as when one introduces a set of observables to pick out a notion of entropy associated with the statistical mechanical arrow; or they may be Missing Information misfires, as when one ignores environmental degrees of freedom during the decoherence process. All such arrows fail to make the Spacetime–Evidential Link and thus fail to establish an asymmetry of time itself.

There are two more famous arrows that were omitted from our discussion: the arrow of electroweak interactions and the arrow of classical equilibrium thermodynamics. I have left out electroweak theory because, as I will argue in Chapter 7, it provides an arrow that does not misfire! Electroweak

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[26] Formally, a common cause C for events A and B has four independent properties: the 'screening-off' conditions $p(A \cap B|C) = p(A|C)p(B|C)$ and $p(A \cap B|C^{\perp}) = p(A|C^{\perp})p(B|C^{\perp})$, and the 'relevance' conditions $p(A|C) > p(A|C^{\perp})$ and $p(B|C) > p(B|C^{\perp})$. This formulation is due to Salmon (1978, 1984). For a recent introduction to this large literature see Hofer-Szabó, Rédei, and Szabó (2013). A classic book on the causal modelling approach is Spirtes, Glymour, and Scheines (2001, Chapter 3).

[27] This includes in particular the "asymmetry of counterfactual dependence" that Lewis (1979) takes to generalise the causal asymmetry.

interactions provide the most compelling evidence available that time itself has an arrow. In contrast, I have left out classical thermodynamics because my thesis, that there is no thermodynamic arrow in any non-trivial sense, may sound so preposterous to some that it deserves its own discussion. So, if I may be forgiven for this transgression, let me turn to that discussion next.