Fourth Meeting, 11th February 1898.

J. B. Clark, Esq., M.A., F.R.S.E., President, in the Chair.

On a Geometrical Problem.

By R. Guimarães.

Stewart, a Scotch geometrician, gave in 1763 the demonstration of the following theorem: "If we divide the base of a triangle into two segments by a straight line going through the vertex, the sum of the squares of the two sides, multiplied each by the non-adjacent segment, is equal to the product of the base multiplied by the square of the straight line plus the rectangle contained by the two segments."

Recently, the Belgian geometer, Clément Thiry,* presented under various forms the formula which represents Stewart's theorem, drawing from it numerous classical propositions. As for us, we have used this theorem to resolve many geometrical problems.†

At present we wish to show that Stewart's theorem enables us to resolve easily the following problem: "To draw a circle touching another given circle and passing through two given points."

The enunciation of Stewart's theorem ‡ most useful for our purpose is as follows: Three points A, B, C in a straight line being given, the distances between them and any given point O are satisfied by

$$OA^2 . BC + OB^2 . CA + OC^2 . AB + AB . BC . CA = 0 \quad (1)$$

where we must consider Euler's identity

$$AB + BC + CA = 0.$$

* Sur le théorème de Stewart (*Revue de l'Instruction Publique*), Bruxelles, 1887; applications remarquables du théorème de Stewart et théorie du barycentre, Bruxelles, 1891.

† *El Progreso Matemático*, Zaragoza, 1892, pp. 62, 94, 124.

‡ Clément Thiry, *Op. Cit.*, p. 6.

https://doi.org/10.1017/S0013091500032302 Published online by Cambridge University Press

Now the potencies a, b, c of the points A, B, C relatively to the circle O are, R being the radius of the circle,

$$a = OA^2 - R^2, \qquad b = OB^2 - R^2 \qquad c = OC^2 - R^2$$

which transform equation (1) into

$$a \, . \, BC + b \, . \, CA + c \, . \, AB + AB \, . \, BC \, . \, CA = 0 \quad - \quad - \quad (2)$$

But $CB \, . \, CA = c$, if C be the point where the tangent common to the two circles meets AB.

Thus equation (2) becomes

$$a \, . \, BC + b \, . \, CA = 0$$

or

$$\frac{CA}{CB} = \frac{a}{b} \quad - \quad - \quad - \quad - \quad - \quad (3)$$

Thus the point C is determined and the problem solved. If the problem is possible, A and B must both be inside or both outside the circle O.

Mr Muirhead suggests the following Solution.

A and B are the given points, DEF the given ⊙, and ABD the required ⊙

In virtue of the equality of angles indicated in Fig. 15, we have

$$\frac{\text{Power of A}}{\text{Power of B}} = \frac{AD \, . \, AF}{BD \, . \, BE}$$

$$= \frac{AD^2}{BD^2} = \frac{CA}{CB}$$

Discussion on Euclid's Definition of Proportion.

Papers by Prof. Gibson and Mr W. J. Macdonald.

https://doi.org/10.1017/S0013091500032302 Published online by Cambridge University Press