ADIABATIC OSCILLATIONS OF SOLAR MODELS WITH A HIGH-Z CONVECTIVE CORE*

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Abstract. Normal mode spectra and neutrino counting rates are calculated for a set of chemically-inhomogeneous solar models. Each model has a core with a high concentration of heavy elements; high opacity makes the core convective. The structure of the envelope is that of the standard model. It is shown that (1) the spectrum of *g* modes becomes less densely separated than that of the standard model, which simplifies the problem of interpreting 160-min oscillations; (2) low neutrino counting rates may be achieved for a low initial helium concentration in the core; (3) the models do not contradict the frequency spacing of global 5-min oscillations.

1. Introduction

In connection with the solar neutrino problem, various non-standard models of the structure and evolution of the Sun have been proposed by different authors over the past decade. One of them is the solar model containing a core with an unusually high concentration of iron-group metals and a low initial concentration of helium, proposed by Hoyle (1975). The high opacity caused by these metals makes the core convective, and low neutrino emission arises, because of the high hydrogen concentration maintained at the center and because the convective mixing of ^{7}Be on a timescale less than ^{7}Be destruction time minimizes the importance of reaction ^{7}Be $(p, \gamma)^8$B. The luminosity of such a model remains almost constant over the whole history of the Earth – the expectation is in better agreement with recent paleoclimatological data than is the standard model, which requires the solar constant to have been significantly lower during the Earth's early history.

In the present paper, a set of models of this type with different core masses is analysed. Theoretical normal mode spectra are calculated and compared with observations.

2. Models

The structure of the models beyond a convective core is taken to be that of the standard model. The standard model calculated by Abraham and Iben (1971) with $Z = 0.0149$, $Y = 0.253$, and the model of the outer convective zone constructed by Spruit (1974) with a depth of 198 000 km, are used. The part of the standard model below some given radius (R_{core}) is replaced by a adiabatic convective core of a different chemical composition.

* Proceedings of the 66th IAU Colloquium: *Problems in Solar and Stellar Oscillations*, held at the Crimean Astrophysical Observatory, U.S.S.R., 1–5 September, 1981.

Solar Physics **82** (1983) 215–222. 0038–0938/83/0821–0215$01.20.

The degree of ionization of heavy elements is assumed to be constant throughout the core. This assumption simplifies the calculations significantly because the density and pressure distributions in the core may be calculated using the Lane–Emden function $\theta(\xi)$ for polytrope index 1.5:

$$\rho = \rho_c \theta^{3/2}, \qquad p = p_c \theta^{5/2}, \tag{1}$$

$$\frac{10\pi r^4 p(r)}{GM^2(r)} = \frac{\theta^{5/2}}{(\mathrm{d}\theta/\mathrm{d}\xi)^2}, \qquad \frac{4\pi r^3 \rho(r)}{M(r)} = -\frac{\xi\theta^{3/2}}{\mathrm{d}\theta/\mathrm{d}\xi}. \tag{2}$$

The pressure and density distributions in the core are then determined by the conditions of pressure and mass continuity at the core boundary by using dimensionless expressions (2). The discontinuity in density at the core boundary was found to be dynamically stable for all the models; the value of $\rho(R_{\mathrm{core}} - 0)/\rho(R_{\mathrm{core}} + 0)$ is in the range from 1.06 for $M_{\mathrm{core}} = 0.05\, M_\odot$ to 1.9 for $M_{\mathrm{core}} = 0.9\, M_\odot$. At this stage the models are ready for the calculation of adiabatic normal mode spectra.

The condition of temperature continuity determines the mean molecular weight of the core material:

$$\bar{\mu}_{\mathrm{core}} = \bar{\mu}_{\mathrm{envelope}} \frac{\rho(R_{\mathrm{core}} - 0)}{\rho(R_{\mathrm{core}} + 0)}. \tag{3}$$

Three parameters of chemical composition X, Y, and Z of core material are then calculated from three conditions:

$$\begin{aligned} &X + Y + Z = 1, \\ &2X + \tfrac{3}{4}Y + aZ = 1/\bar{\mu}_{\mathrm{core}}, \\ &L = L_\odot. \end{aligned} \tag{4}$$

The degree of ionization of heavy elements is represented by parameter a which is in the range from $a = 0$ for zero ionization to $a = \frac{1}{2}$ for total ionization. The results, as will be shown, are only weakly dependent on the degree of ionization.

It was found that the condition of solar luminosity may be satisfied for a core mass up to $0.5\, M_\odot$ in the calculated series of models. After the values of X, Y, Z have been determined (there may be two solutions), the neutrino counting rates are computed.

The high mass-number elements in core material are assumed to be heavier than Mg, the usual C, N, O, and Ne being removed by α additions (Hoyle, 1975). Hence, there will be no ^{13}N or ^{15}O neutrinos from the core. Convective circulation time is short enough (Hoyle, 1975) to assume that all nuclear species (except deuterium) are distributed uniformly throughout the core. Nuclear reaction rates given by Fowler *et al.* (1974) were used in the computation of luminosity and neutrino fluxes.

3. Normal Mode Spectra

Adiabatic non-radial oscillations were calculated by numerical integrations of the usual fourth-order differential system, taking into account the perturbation of gravitational potential. The form of the equations and the method of calculation are the same as given in Vorontsov and Zharkov (1978).

Special attention was paid to the accurate computation of the radial eigenfunctions, because the modes associated with the discontinuity in density were expected to appear in the theoretical spectra. It was found, however, that these 'discontinuity' modes cannot be distinguished in the spectra for low l values because of their strong interaction with a number of g modes of close periods. The situation is illustrated by Figure 1, where radial displacement functions of quadrupole ($l = 2$) oscillations are shown for a model with $M_{\text{core}} = 0.4\ M_{\odot}$. The relative amplitudes at the core boundary versus mode number are shown in Figure 1b. This dependence has a maximum, but there is no unique 'discontinuity' mode. The situation for the models of a different core mass is the same. The 'discontinuity' modes become strongly pronounced in theoretical spectrum only for $l \approx 10$ and higher.

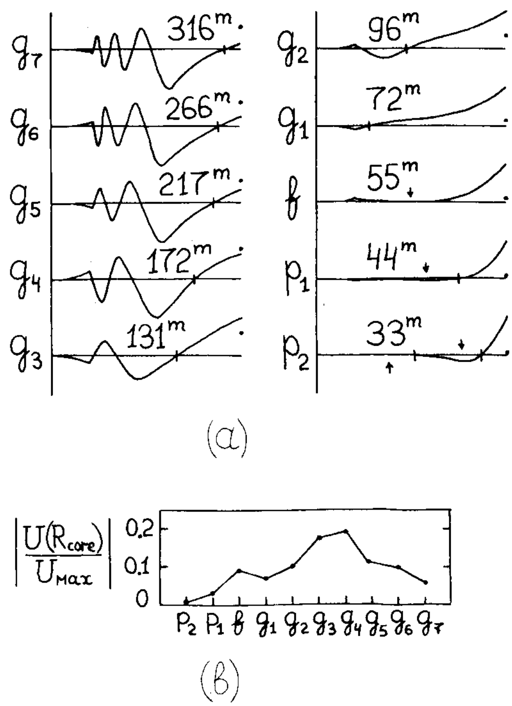

Fig. 1. The quadrupole ($l = 2$) oscillations for a model with $M_{\text{core}} = 0.4\ M_{\odot}$ ($R_{\text{core}} = 0.22\ R_{\odot}$). (a) The radial displacement functions $U(r)$ and periods in minutes. For each mode, the horizontal axis corresponds to the radius, from the center (left) to the surface. Relative amplitudes of tangential displacements at the surface are shown by dots. The extremes in eigenfunctions are shown by arrows when needed. (b) The relative amplitudes at the core boundary for different modes.

The periods of quadrupole oscillations for all the models are given in Figure 2. The limiting case $R_{\text{core}} = 0$ corresponds to the standard model. When the core radius increases, the spectrum of periods of g modes becomes more and more widely separated with respect to that of the standard model which, in general, simplifies the problem of the interpretation of 160-min solar oscillations. Possible identifications are shown in Figure 2 by arrows, but since solar luminosity may be achieved only for $M_{\text{core}} < 0.5\ M_{\odot}$, identification only with a g_3 or higher order g mode is allowed. The preferential excitation of a single g mode may be due to relatively higher amplitudes in the core (Figure 1b).

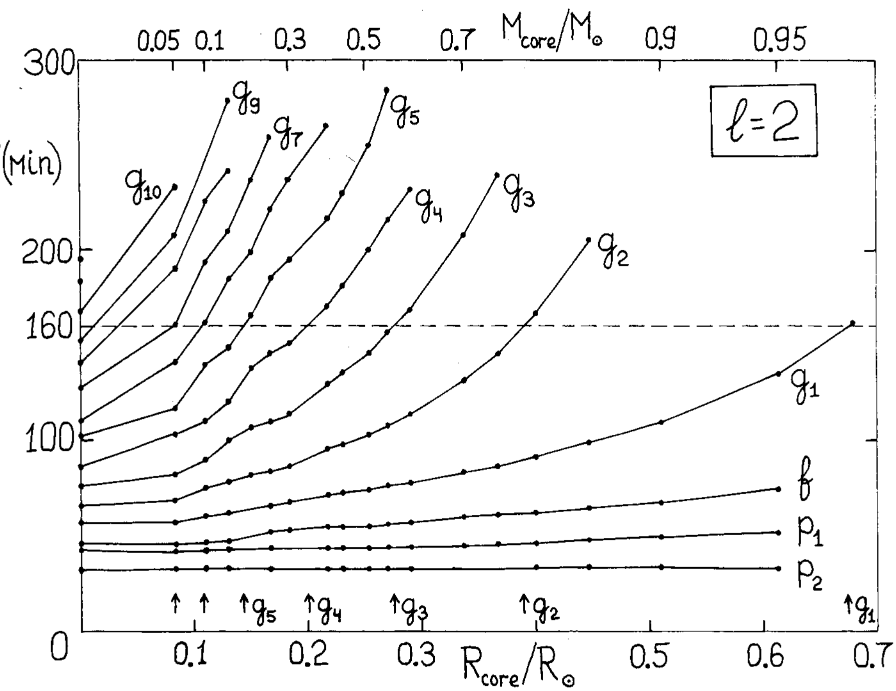

Fig. 2. Periods of quadrupole ($l = 2$) modes versus core radius. Possible identifications of the 160-min period are indicated.

The periods of dipole ($l = 1$) oscillations are given in Figure 3. The radial oscillations were also calculated; their periods (Figure 4) are only weakly-dependent on the core radius, like the periods of acoustical oscillations on the whole.

The high-order p mode frequency spacings were estimated using the asymptotic expression (Vandakurov, 1967)

$$\Delta \nu = \left[2 \int_0^{R_\odot} \frac{\mathrm{d}r}{c} \right]^{-1}, \tag{5}$$

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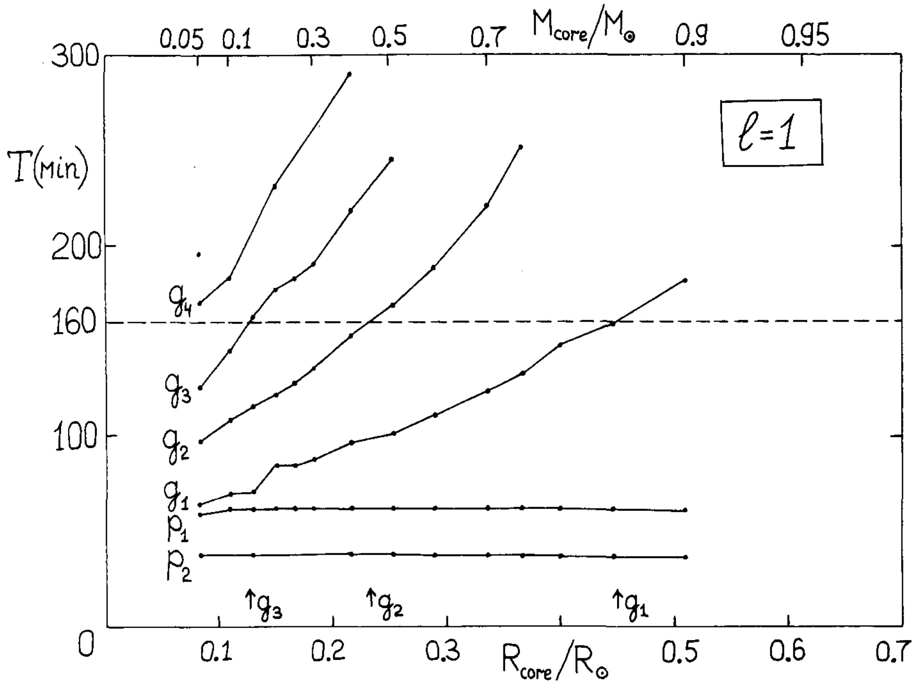

Fig. 3. Periods of dipole (l = 1) modes.

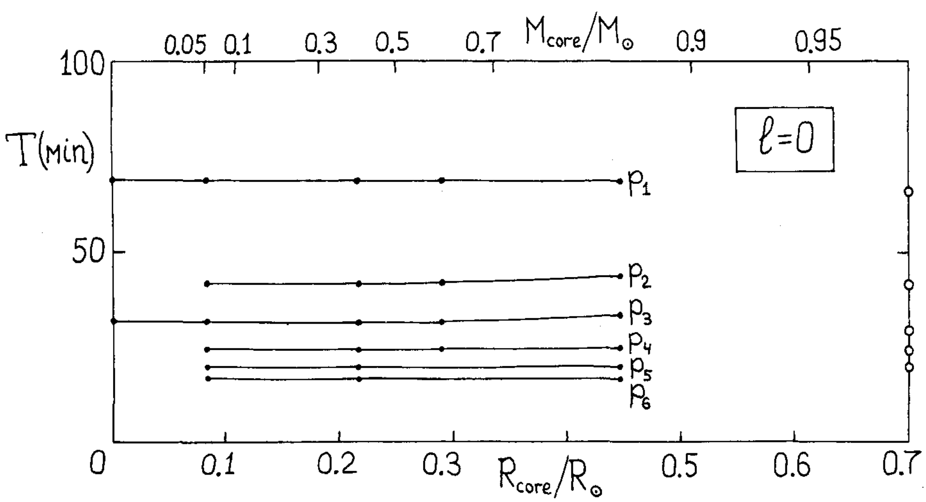

Fig. 4. Periods of radial (l = 0) modes.

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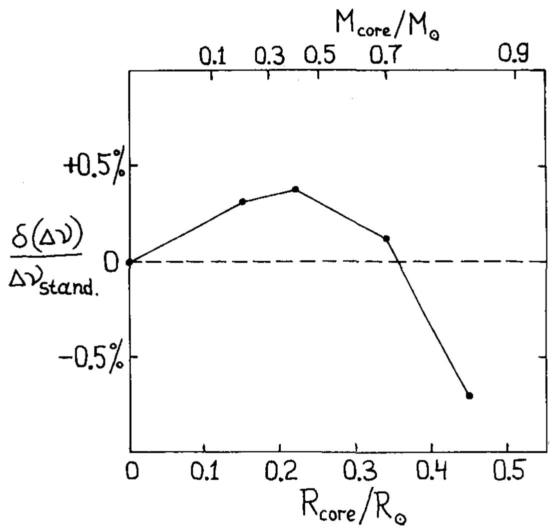

Fig. 5. The deviation of the high-order p-mode frequency spacing from that of the standard model.

where c is the adiabatic sound speed. The results are shown in Figure 5. Only the relative deviations of $\Delta\nu$ from the prediction of the standard model are given, because the influence of the solar atmosphere on the absolute value of $\Delta\nu$ was not included in the calculations. The small deviations obtained indicate that the models of the type under consideration show no evident contradiction with the frequency spacing of global 5-min solar oscillations, taking into account that the experimental spacing seems to be in agreement with the standard model predictions (Christensen-Dalsgaard and Gough, 1980).

4. Neutrino Counting Rates

The core compositions, luminosities and neutrino counting rates in ^{37}Cl detector are listed in Tables I and II for the models with $M_{core} = 0.4\,M_\odot$ and $M_{core} = 0.2\,M_\odot$. The results are given for the different degrees of ionization of heavy elements in the core. The presented values of $L/L_\odot$ deviate from unity in some cases because the same values of helium content were used for the different degrees of ionization in order to examine the influence of the assumed degree of ionization on the results. The limiting cases of total and zero ionization were tested in order to make sure that this influence is rather small.

The neutrino absorption cross-sections given by Bahcall (1979) were used in computations. The results indicate that low neutrino counting rates, comparable with observational limits, may be obtained for a low helium concentration in the core. The less Y values in Tables I and II are their lower limits corresponding approximately to a zero helium content for the core material at its origin (Hoyle, 1975). The increase of Y leads to the increase of neutrino flux while the accompanying effect on luminosity is rather small.

TABLE I

The model with $M_{core} = 0.4\,M_\odot$ ($R_{core} = 0.22\,R_\odot$, central temperature $T_c = 16.1 \times 10^6$ K, central density $\rho_c = 95$ g cm^{-3})

X	Y	Z	$L/L_\odot$	$\sum \phi_i \sigma_i$ (SNU)
		Zero ionization		
0.69	0.10	0.21	1.1	2.7
0.67	0.15	0.18	1.1	3.8
		60% ionization		
0.65	0.10	0.25	1.0	2.5
0.64	0.15	0.21	1.0	3.5
		Total ionization		
0.62	0.10	0.28	0.9	2.3
0.61	0.15	0.24	0.9	3.3

TABLE II

The model with $M_{core} = 0.2\,M_\odot$ ($R_{core} = 0.15\,R_\odot$, central temperature $T_c = 15.2 \times 10^6$ K, central density $\rho_c = 114$ g cm^{-3})

X	Y	Z	$L/L_\odot$	$\sum \phi_i \sigma_i$ (SNU)
		Zero ionization		
0.68	0.20	0.12	1.0	2.3
0.66	0.25	0.09	1.0	2.7
		60% ionization		
0.66	0.20	0.14	1.0	2.2
0.64	0.25	0.11	0.9	2.6
		Total ionization		
0.64	0.20	0.16	0.9	2.1
0.63	0.25	0.12	0.9	2.5

5. Conclusions

The set of models with a high-Z convective core which was examined is not unique: similar sets with a different helium concentration in the envelope and/or with a different structure of the outer convective zone may be constructed. Some simplifying assump-

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tions, like that of homogeneous ionization of core material, were also used. Certain general conclusions for all the models of this type can, nevertheless, be formulated:

(a) the neutrino counting rates within the observational limits can be achieved for a low initial helium concentration;

(b) the gravity mode spectrum is more widely separated than that of the standard model, which can simplify the problem of interpretation of 160-min oscillations;

(c) the luminosity of such a model is approximately constant over the Earth's history, which is due to convective mixing of the products of nuclear reactions;

(d) the models do not contradict the usual interpretation of the global 5-min oscillations of the Sun.

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