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ROBUST GUARANTEED COST CONTROL FOR DESCRIPTOR SYSTEMS WITH MARKOV JUMPING PARAMETERS AND STATE DELAYS

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Abstract

This paper deals with robust guaranteed cost control for a class of linear uncertain descriptor systems with state delays and jumping parameters. The transition of the jumping parameters in the systems is governed by a finite-state Markov process. Based on stability theory for stochastic differential equations, a sufficient condition on the existence of robust guaranteed cost controllers is derived. In terms of the LMI (linear matrix inequality) approach, a linear state feedback controller is designed to stochastically stabilise the given system with a cost function constraint. A convex optimisation problem with LMI constraints is formulated to design the suboptimal guaranteed cost controller. A numerical example demonstrates the effect of the proposed design approach.

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1. Introduction

Descriptor systems capture the dynamic behaviour of many natural phenomena, and have applications in many fields, such as network theory, robotics, and so on (see for example [6, 7, 14, 19, 20] and the references therein). Descriptor systems are also referred to as singular systems, implicit systems, generalised state-space systems or semi-state systems. Many results on descriptor systems have been proposed and various methods obtained (see for example [6, 7, 14, 19, 20] and the references therein).

Stochastic modelling has come to play an important role in many branches of science and industry. Much research has been focused on the Markov jumping linear system (see for example [3, 5, 8, 9] and the references therein). Nevertheless, these

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results are only applicable to Markov jumping linear systems without delays. More recently, increasing attention has been focused on stochastic systems with time delays (see, [1, 4, 11–13]). References [1, 12, 13] presented stability criteria of stochastic systems with time delays. Robust control of uncertain stochastic systems with time delays was considered in [1, 4, 11].

Robust guaranteed cost control for time-delay systems has been the focus of much attention [2, 15–18, 21, 22]. Some useful results have been established. These results can be classified into two cases: the continuous-time case ([2, 15–17, 21]) and the discrete-time case [18] and [22]. Guaranteed cost control for uncertain descriptor systems was considered in [23, 24] based on the LMI method. However, little attention has been paid to robust guaranteed cost control for linear descriptor systems with state delays and Markov jumping parameters.

In this paper, motivated by the results in [21], we address the robust guaranteed cost control of a class of descriptor systems with state delays and Markov jumping parameters based on the LMI method. The transition of the jumping parameters in the systems is governed by a finite-state Markov process. The class of systems is a hybrid class of systems with two components in the vector state. The first component refers to the mode and the second one to the state. The mode is described by a continuous Markov process with finite state space. The state in each mode is denoted by a stochastic differential equation. The synthesis problem proposed here is to design a memoryless state feedback control law such that the closed-loop system is regular, impulse free, stochastically stable independent of delay and satisfies the proposed guaranteed cost performance.

2. Problem formulation

Consider the following descriptor time-delay systems with Markov jumping parameters:

$$\begin{cases} E(r(t))\dot{x}(t) = A(r(t),t)x(t) + A_1(r(t),t)x(t-d) + B(r(t),t)u(t), \\ x(t) = \varphi(t), \quad \text{for all } t \in [-d, 0], \end{cases} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$, and $u(t) \in \mathbb{R}^r$ are the state vector and the control vector, respectively. Here d represents the discrete state time-delay and $\varphi(t) \in L_2[-d, 0]$ is a continuous vector-valued initial function. The random parameter $r(t)$ represents a continuous-time discrete-state Markov process taking values in a finite set $N = \{1, 2, \dots, s\}$ and having the transition probability matrix $\Pi = [\pi_{ij}]_{i,j \in N}$. The transition probability from mode i to mode j is defined by

$$P\{r(t+\Delta = j) \mid r(t) = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & i \neq j, \\ 1 + \pi_{ii}\Delta + o(\Delta), & i = j, \end{cases}$$

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where $\Delta > 0$ satisfies $\lim_{\Delta\to 0}(o(\Delta)/\Delta) = 0$. Here $\pi_{ij} > 0$ is the transition probability from mode i to mode j $(i \neq j)$ and satisfies $\sum_{i\neq j}\pi_{ij} = -\pi_{ii}$.

The matrix $E(r(t)) \in \mathbb{R}^{n\times n}$ may be singular with rank $E(r(t)) = n_E(r(t)) \leq n$, and $A(r(t),t)$, $A_1(r(t),t)$ and $B(r(t),t)$ are matrix functions of the random jumping process $\{r(t)\}$. For simplicity of notation, we let $A_i(t)$ represent $A(r(t),t)$ when $r(t) = i$. For example, $A_1(r(t),t)$ is denoted by $A_{1i}(t)$, and so on. Further, for each $r(t) = i \in N$, it is assumed that the matrices $A_i(t)$, $A_{1i}(t)$ and $B_i(t)$ can be described by the following form:

$$A_i(t) = A_i + \Delta A_i(t), \quad A_{1i}(t) = A_{1i} + \Delta A_{1i}(t), \quad B_i(t) = B_i + \Delta B_i(t), \tag{2.2}$$

where A_i, A_{1i} and B_i are known real coefficient matrices with appropriate dimensions. Time-varying uncertain matrices $\Delta A_i(t)$, $\Delta B_i(t)$ and $\Delta A_{1i}(t)$ are assumed to be of the form

$$\begin{bmatrix}\Delta A_i(t) & \Delta B_i(t) & \Delta A_{1i}(t)\end{bmatrix} = D_i F_i(t)\begin{bmatrix}E_{1i} & E_{2i} & E_{3i}\end{bmatrix},$$

where D_i, E_{1i}, E_{2i} and E_{3i} are known constant real matrices of appropriate dimensions, which represent the structure of uncertainties, and $F_i(t)$ is an unknown matrix function with Lebesgue measurable elements and satisfies $F_i^T(t)F_i(t) \leq I$.

For convenience, it is assumed that the system has the same dimension at each mode and the Markov process is irreducible.

Consider the following nominal unforced descriptor system of (2.1) with a state delay:

$$\begin{cases} E_i\dot{x}(t) = A_i x(t) + A_{1i}x(t-d), \\ x(t) = \varphi(t), \quad \text{for all } t \in [-d, 0]. \end{cases} \tag{2.3}$$

Let x_0, r_0, and $x(t,\varphi,r_0)$ be the initial state, initial mode and the corresponding solution of system (2.3) at time t respectively. We have the following definition.

DEFINITION 1. System (2.3) is said to be stochastically stable if for all $\varphi(t) \in L_2[-d,0]$ and initial mode $r_0 \in N$, there exists a matrix $M > 0$ such that

$$E\left\{\int_0^\infty \|x(t,\varphi,r_0)\|^2\,dt \;\middle|\; r_0, x(t) = \varphi(t),\ t \in [-d,0]\right\} \leq x_0^T M x_0.$$

The following definition can be regarded as an extension of the definition in [20].

DEFINITION 2. (1) System (2.3) is said to be regular if $\det(sE_i - A_i)$, $i \in N$ is not identically zero.

(2) System (2.3) is said to be impulse free if $\deg(\det(sE_i - A_i)) = \operatorname{rank} E_i$, $i \in N$.

(3) System (2.3) is said to be admissible if it is regular, impulse free and stochastically stable.

Similar to [4], it is also assumed in this paper that for all $\varsigma \in [-d, 0]$, there exists a scalar $h > 0$ such that

$$\|x(t+\varsigma)\| \leq h\|x(t)\|. \tag{2.4}$$

Associated with the systems (2.1) is the cost function

$$J = E\left\{\int_0^\infty \left[x^T(t)Q(r(t))x(t) + u^T(t)R(r(t))u(t)\right]dt\right\}, \tag{2.5}$$

where the cost weighting matrices $Q(r(t))$ and $R(r(t))$ are symmetric positive definite matrices for each $r(t)$.

When the following state feedback controller law

$$u(t) = K_i x(t), \quad \text{when } r(t) = i,\ i \in N, \tag{2.6}$$

is applied to the system (2.1), the closed-loop system is obtained in the following form:

$$\begin{cases} E_i\dot{x}(t) = (A_i + B_iK_i + D_iF_i(t)(E_{1i} + E_{2i}K_i)]x(t) \\ \qquad\qquad + (A_{1i} + D_iF_i(t)E_{3i})x(t-d), \\ x(t) = \varphi(t), \quad \text{for all } t \in [-d, 0]. \end{cases} \tag{2.7}$$

Based on Definition 2, we have the following definition.

DEFINITION 3. Consider system (2.1). If there exist a control law $u^*(t)$ and a positive scalar J^*, for all uncertainties, such that the closed-loop system (2.7) is admissible and the closed-loop value of the cost function (2.5) satisfies $J \leq J^*$, then J^* is said to be a robust guaranteed cost and $u^*(t)$ is said to be a robust guaranteed cost control law for (2.1).

With the above description, the problem to be solved in this paper can be stated as follows.

PROBLEM 1 (Robust guaranteed cost control). *Given system* (2.1), *determine a memoryless state feedback controller* (2.6) *such that the control law is a robust guaranteed cost control law.*

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3. Main results

Before presenting the main results, we introduce the following lemma which is to be used in the proof of the main results.

LEMMA 3.1 ([20]). *The system*

$$\begin{cases} E\dot{x}(t) = Ax(t) + A_1x(t-d), \\ x(t) = \varphi(t), \quad \text{for all } t \in [-d, 0] \end{cases}$$

is said to be admissible if there exist a matrix P and a symmetric positive definite matrix Q such that $E^T P = P^T E \geq 0$ and $A^T P + P^T A + P^T A_1 Q^{-1} A_1^T P + Q < 0$.

Based on stochastic Lyapunov stability theory, we have the following theorem.

THEOREM 3.2. *System* (2.3) *is admissible if there exist matrices P_i, $i \in N$, and symmetric positive definite matrices S_i, $i \in N$, such that*

$$E_i^T P_i = P_i^T E_i \geq 0 \quad \text{and} \tag{3.1}$$

$$\Sigma_i = \begin{bmatrix} \Pi_i & P_i^T A_{1i} \\ A_{1i}^T P_i & -S_i \end{bmatrix} < 0, \quad i = 1, \dots, s, \tag{3.2}$$

where $\Pi_i = A_i^T P_i + P_i^T A_i + \sum_{j=1}^{s} \pi_{ij} E_j^T P_j + S_i$.

PROOF. Based on Definition 2 and Lemma 3.1, it follows from (3.1)–(3.2) that system (2.3) is regular and impulse free. Next, the proof of stability is given. Let the mode at time t be i, that is, $r(t) = i \in N$, and consider the following positive definite function as a stochastic Lyapunov-Krasovskii function of system (2.3):

$$V(x(t), r(t) = i) = x^T(t) E_i^T P_i x(t) + \int_{t-d}^{t} x^T(t) S_i x(t)\, dt,$$

where S_i is a symmetric positive definite matrix and P_i is a matrix satisfying (3.1). The weak infinitesimal operator L of the stochastic process $\{r(t), x(t)\}$, $t \geq 0$, is given by

$$\begin{aligned} LV&\big(x(t), r(t) = i\big) \\ &= \lim_{\Delta \to 0} \frac{1}{\Delta}\big[E\{V(x(t+\Delta), r(t+\Delta)) \mid x(t), r(t) = i\} - V(x(t), r(t) = i)\big] \\ &= x^T(t)\left[A_i^T P_i + P_i^T A_i + \sum_{j=1}^{s} \pi_{ij} E_j^T P_j + S_i\right] x(t) + 2x^T(t) P_i^T A_{1i} x(t-d) \\ &\quad - x^T(t-d) S_i x(t-d). \end{aligned} \tag{3.3}$$

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The following inequality is obviously true:

$$2x^T(t)P_i^T A_{1i}x(t-d) \le x^T(t)P_i^T A_{1i}S^{-1}A_{1i}P_i x(t) + x^T(t-d)S_i x(t-d). \quad (3.4)$$

Therefore, it follows from (3.3) and (3.4) that $LV(x,t) \le x^T(t)\Sigma_i x(t)$, where

$$\Sigma_i = \Pi_i + P_i^T A_{1i}S_i^{-1}A_{1i}P_i.$$

On the other hand, for $x \neq 0$ and each mode i, we have

$$V\big(x(t), r(t) = i\big) \ge x^T(t)E_i^T P_i x(t) \ge 0.$$

Note that $\Sigma_i < 0$, thus

$$\frac{LV(x(t), r(t) = i)}{V(x(t), r(t) = i)} \le \frac{x^T(t)\Sigma_i x(t)}{x^T(t)E_i^T P_i x(t)} \quad \text{for } x \neq 0.$$

Letting $\beta = -\min\left[\lambda_{\min}(\Upsilon_i)/\lambda_{\max}(E_i^T P_i)\right] > 0$, we have

$$\frac{LV(x(t), r(t) = i)}{V(x(t), r(t) = i)} \le -\beta.$$

Similar to [4], using Dynkin's formula and the Gronwall-Bellman lemma, we obtain for each $i \in N$, $E\left\{V(x(t), r(t) = i) \mid \varphi, r_0 = i\right\} \le e^{-\beta t}V(x_0, i)$. Then

$$\begin{aligned} &E\left\{V(x(t), r(t) = i) \mid \varphi, r_0 = i\right\} \\ &= E\left\{x^T(t)P_i x(t) + \int_{t-d}^{t} x^T(t)S_i x(t)\,dt \;\middle|\; \varphi, r_0 = i\right\} \\ &= E\left\{x^T(t)P_i x(t) \mid \varphi, r_0 = i\right\} + E\left\{\int_{t-d}^{t} x^T(t)S_i x(t)\,dt \;\middle|\; \varphi, r_0 = i\right\} \\ &\le e^{-\beta t}V(x_0, i). \end{aligned}$$

Thus we have $E\left\{x^T(t)E_i^T P_i x(t) \mid \varphi, r_0 = i\right\} \le e^{-\beta t}V(x_0, i)$.

Let $\theta_m = \min_{i\in N}\left\{\lambda_{\min}(E_i^T P_i)\right\}$, and applying Fubini's theorem, this yields

$$\theta_m E\left\{\int_0^T x^T(t)x(t)\,dt \;\middle|\; \varphi, r_0 = i\right\} \le -\frac{1}{\beta}\left[e^{-\beta T} - 1\right]V(x_0, i).$$

Taking the limit as $T \to \infty$, and using (2.4), we obtain

$$\lim_{T\to\infty} E\left\{\int_0^T x^T(t)x(t)\,dt \;\middle|\; \varphi, r_0 = i\right\} \le x_0^T M x_0,$$

where

$$M = \frac{1}{\theta_m \beta}\left[\theta_M + dh^2\lambda_{\max}(S_i)\right]I, \quad \theta_M = \max_{i\in N}\left\{\lambda_{\max}(E_i^T P_i)\right\}.$$

From Definition 2, it results that system (2.3) is admissible. □

REMARK 1. Theorem 3.2 gives a sufficient condition for system (2.3) to be admissible. When it has no Markov jumping parameters, the system (2.3) reduces to a descriptor system with state delays which is studied in [20]. It is easy to show that Theorem 3.2 coincides with [20, Theorem 1]. Therefore, the results presented in Theorem 3.2 can be viewed as an extension of results from descriptor systems with state delays to descriptor systems with state delays and Markov jumping parameters.

Based on Theorem 3.2, we give the following sufficient condition on the existence of robust guaranteed cost controllers for system (2.1).

THEOREM 3.3. *Consider system* (2.1) *with cost function* (2.5). *There exists a memoryless state feedback controller* (2.6) *that solves the addressed robust guaranteed cost control problem if there exist matrices* P_i, $i = 1, \dots, s$, *and symmetric positive definite matrices* S_i, $i = 1, \dots, s$, *such that*

$$E_i^T P_i = P_i^T E_i \geq 0 \quad \text{and} \tag{3.5}$$

$$\begin{bmatrix} \Gamma_i + Q_i + K_i^T R_i K_i & P_i^T (A_{1i} + D_i F_i(t) E_{3i}) \\ (A_{1i} + D_i F_i(t) E_{3i})^T P_i & -S_i \end{bmatrix} < 0, \quad i = 1, \dots, s, \tag{3.6}$$

where

$$\begin{aligned} \Gamma_i = {} & [A_i + B_i K_i + D_i F_i(t)(E_{1i} + E_{2i} K)]^T P_i \\ & + P_i^T [A_i + B_i K_i + D_i F_i(t)(E_{1i} + E_{2i} K)] + \sum_{j=1}^{s} \pi_{ij} E_i^T P_j + S_i. \end{aligned} \tag{3.7}$$

PROOF. It follows from Theorem 3.2 and Definition 2 that the closed-loop system (2.7) is admissible for all uncertainties if there exist matrices P_i, $i = 1, \dots, s$, and symmetric positive definite matrices S_i, $i = 1, \dots, s$, such that (3.5) and the following inequalities hold:

$$\begin{bmatrix} \Gamma_i & P_i^T (A_{1i} + D_i F_i(t) E_{3i}) \\ (A_{1i} + D_i F_i(t) E_{3i})^T P_i & -S_i \end{bmatrix} < 0, \quad i = 1, \dots, s.$$

From (3.6), the closed-loop system (2.7) is admissible. On the other hand, from Theorem 3.2 and (3.6), we have

$$LV(x(t), r(t) = i) < x^T(t)(-Q_i - K_i^T R_i K_i) x(t) < 0. \tag{3.8}$$

Based on cost function (2.5) and (3.8), we obtain

$$\begin{aligned} J &= E\left\{ \int_0^\infty \left[x^T(t) Q(r(t)) x(t) + u^T(t) R(r(t)) u(t) \right] dt \right\} \\ &= E\left\{ \int_0^\infty x^T(t) [Q(r(t)) + K^T(r(t)) R(r(t)) K(r(t)] x(t)\, dt \right\} \end{aligned}$$

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$$
\begin{aligned}
&< -\int_0^{\infty} LV(x(t), r(t))\,dt \\
&= -E\left\{\lim_{t\to\infty} V(x(t), r(t))\right\} + V(x_0, r_0).
\end{aligned} \tag{3.9}
$$

As the closed-loop system (2.7) is stochastically stable, it follows from (3.9) that $J < V(x_0, r_0)$. From Definition 3, we may conclude that a robust guaranteed cost for system (2.1) can be given by

$$
J^* = x_0^T E_{r_0}^T P_{r_0} x_0 + \int_{-d}^{0} x^T(t) S_{r_0} x(t)\,dt. \qquad \square
$$

In the following, based on the above condition for the existence of robust guaranteed cost controllers, a design method for such controllers is given.

THEOREM 3.4. *Consider system* (2.1) *with cost function* (2.5). *Then there exists a memoryless state feedback controller* (2.6) *that solves the addressed robust guaranteed cost control problem if there exist matrices* P_i, $i = 1, \dots, s$, *and symmetric positive definite matrices* S_i, $i = 1, \dots, s$, *such that*

$$
E_i^T P_i = P_i^T E_i \geq 0 \quad \text{and} \tag{3.10}
$$

$$
\begin{bmatrix}
\Psi_i & P_i^T B_i & P_i^T D_i & P_i^T A_{1i} & P_i^T B_i & (E_{1i} + \frac{1}{2\alpha_i} E_{2i} B_i^T P_i)^T \\
B_i^T P_i & -\alpha_i I & 0 & 0 & 0 & E_{3i}^T \\
D_i^T P_i & 0 & -\beta_i I & 0 & 0 & 0 \\
A_{1i}^T P_i & 0 & 0 & -S_i & 0 & 0 \\
B_i^T P_i & 0 & 0 & 0 & -4\alpha_i^2 R_i^{-1} & 0 \\
E_{1i} + \frac{1}{2\alpha_i} E_{2i} B_i^T P_i & E_{3i} & 0 & 0 & 0 & -\beta_i^{-1} I
\end{bmatrix} < 0 \tag{3.11}
$$

for some positive constants α_i *and* β_i, $i = 1, \dots, s$, *where*

$$
\Psi_i = P_i^T A_i + A_i^T P_i + Q_i + S_i + \sum_{j=1}^{s} \pi_{ij} E_j^T P_j.
$$

In this case, the memoryless state feedback

$$
u^*(t) = \frac{1}{2\alpha_i} B_i^T P_i x(t) \tag{3.12}
$$

is a robust guaranteed cost control law and

$$
J^* = x_0^T E_{r_0}^T P_{r_0} x_0 + \int_{-d}^{0} x^T(t) S_{r_0} x(t)\,dt \tag{3.13}
$$

is a robust guaranteed cost for system (2.1).

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PROOF. Define

$$\Omega = \begin{bmatrix} (A_i + B_iK_i)^T P_i + P_i^T(A_i + B_iK_i) \\ \quad + \sum_{j=1}^{s} \pi_{ij} E_j^T P_j + S_i + Q_i + K_i^T R_i K_i & P_i^T A_{1i} \\ A_{1i}^T P_i & -S_i \end{bmatrix}, \quad i = 1, \dots, s.$$

Then (3.2) is equivalent to

$$\Omega + \begin{bmatrix} P_iD_i \\ 0 \end{bmatrix} F_i(t) \begin{bmatrix} E_{1i} + E_{2i}K & E_{3i} \end{bmatrix} + \begin{bmatrix} E_{1i} + E_{2i}K & E_{3i} \end{bmatrix}^T F_i^T(t) \begin{bmatrix} P_iD_i \\ 0 \end{bmatrix}^T < 0.$$

By applying [10, Proposition 2.2], for any scalars $\beta_i > 0$, $i = 1, \dots, s$, and all $F_i(t)$ satisfying $F_i^T(t)F_i(t) \le I$, the following inequality holds:

$$\begin{aligned} &\begin{bmatrix} P_i^TD_i \\ 0 \end{bmatrix} F_i(t) \begin{bmatrix} E_{1i} + E_{2i}K & E_{3i} \end{bmatrix} + \begin{bmatrix} E_{1i} + E_{2i}K & E_{3i} \end{bmatrix}^T F_i^T(t) \begin{bmatrix} P_i^TD_i \\ 0 \end{bmatrix}^T \\ &\quad \le \beta_i^{-1} \begin{bmatrix} P_i^TD_i \\ 0 \end{bmatrix} \begin{bmatrix} P_i^TD_i \\ 0 \end{bmatrix}^T + \beta_i \begin{bmatrix} E_{1i} + E_{2i}K & E_{3i} \end{bmatrix}^T \begin{bmatrix} E_{1i} + E_{2i}K & E_{3i} \end{bmatrix}. \end{aligned} \tag{3.14}$$

Based on Schur complement results and Theorem 3.3, it follows from (3.11), (3.12) and (3.14) that a robust guaranteed cost for system (2.1) is given by (3.13) under the memoryless state feedback (3.12). □

REMARK 2. Theorem 3.4 provides a design approach for robust guaranteed cost controllers in terms of LMIs, which can be solved by the LMI toolbox in MATLAB. The design of a robust guaranteed cost controller for linear time-delay systems is considered in [16] and [21]. The results presented in Theorem 3.4 can be viewed as an extension of results from linear time-delay systems to descriptor systems with state delays and Markov jumping parameters.

REMARK 3. The solution of LMIs (3.10) and (3.11) parametrises the set of stochastic guaranteed cost controllers. This parametrised representation can be used to design the robust guaranteed cost controller with some additional performance constraints. Similar to [21], the suboptimal guaranteed cost control law can be determined by solving a certain optimisation problem, as presented in the following theorem.

THEOREM 3.5. *Consider system* (2.1) *with cost function* (2.5), *and suppose that the initial conditions r_0 and x_0 are known. If the following optimisation problem*:

$$\min_{P_i, S_i, \alpha_i \text{ and } \beta_i} J^* \quad \text{s.t. (3.10) and (3.11)} \tag{3.15}$$

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has a solution P_i, S_i, α_i, *and* β_i, $i = 1, \dots, s$, *then the control law of form* (3.12) *is a suboptimal guaranteed cost control law for system* (2.1), *where*

$$J^* = x_0^T E_{r_0}^T P_{r_0} x_0 + \operatorname{tr}\left(\int_{-d}^{0} x(t)x^T(t)\,dt\,S_{r_0}\right). \tag{3.16}$$

PROOF. It follows from Theorem 3.4 that the control law of form (3.12) constructed in terms of any feasible solution P_i, S_i, α_i and β_i, $i = 1, \dots, s$, is a robust guaranteed cost control law. It follows from

$$\int_{-d}^{0} x^T(t)S_{r_0}x(t)\,dt = \int_{-d}^{0} \operatorname{tr}\left(x^T(t)S_{r_0}x(t)\right)dt = \operatorname{tr}\left(\int_{-d}^{0} x(t)x^T(t)\,dt\,S_{r_0}\right)$$

that (3.16) holds. Therefore, the problem of suboptimal guaranteed cost control is turned into the minimisation problem (3.15). □

4. A numerical example

Consider the following two-mode descriptor systems with uncertainties and state delays. For mode 1, the system matrices are given by

$$E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -5 & 1 \\ 1 & 4 \end{bmatrix}, \quad A_{11} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad E_{11} = E_{12} = E_{13} = I, \quad Q_1 = I, \quad R_1 = 0.5I.$$

For mode 2, the system matrices are given by

$$E_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2 & 0 \\ 0 & -6 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix},$$
$$D_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad E_{21} = E_{22} = E_{23} = I, \quad Q_2 = I, \quad R_2 = 0.5I.$$

The time delay is $d = 0.1$, the initial state is $\varphi(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$ and the initial mode is $r_0 = 1$. The transition probability matrix Π is given by:

$$\Pi = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}.$$

After all these parameters are given, we can then obtain a suboptimal guaranteed cost control law for this system.

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Selecting $\alpha_1 = \alpha_2 = 10$, $\beta_1 = \beta_2 = 1$, applying Theorem 3.5, and solving the optimisation problem (3.15) by using the MATLAB LMI toolbox, one gets

$$P_1 = \begin{bmatrix} 0.1986 & 0 \\ -1.0703 & -5.1863 \end{bmatrix}, \qquad S_1 = \begin{bmatrix} 0.0277 & 0.0014 \\ 0.0014 & 8.1991 \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 5.2088 & -4.0571 \\ 0 & 8.0011 \end{bmatrix}, \qquad S_2 = \begin{bmatrix} 6.3565 & -3.6646 \\ -3.6646 & 38.6054 \end{bmatrix}.$$

Then using (3.12), we can obtain the following state feedback gain matrices for the two modes:

$$K_1 = \begin{bmatrix} -0.0436 & -0.2593 \\ 0.0971 & -0.5186 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 0.5209 & -0.0057 \\ 0.2604 & 0.5973 \end{bmatrix}$$

and the corresponding suboptimal guaranteed cost is $J^* = 0.2013$.

5. Conclusions

In this paper, a robust guaranteed cost control via state feedback for a class of uncertain descriptor time-delay systems with Markov jumping parameters is studied using the LMI method. The uncertainty is time-varying and assumed to be norm-bounded. Memoryless guaranteed cost controllers are designed in terms of a set of linear coupled matrix inequalities. The proposed state feedback control law guarantees that the closed-loop system is regular, impulse free, stochastically stable and satisfies the proposed guaranteed cost performance. The suboptimal guaranteed cost controller is designed by solving a certain optimisation problem. A demonstrative example shows the effect of the proposed approach.

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