# Reviews

**Algebra and geometry, an introduction to university mathematics** (second edition) by Mark V. Lawson, pp. 424, £49.99 (paper), ISBN 978-0-36756-303-5, CRC/Taylor and Francis (2021)

This book aims to provide a bridge between school mathematics and university mathematics centred on themes from algebra and geometry. In doing so, it seeks to build on intuitions already developed, making them rigorous through an introduction to formal proofs, as well as pointing the way to new ideas that will be met in the years ahead.

The text is split into two main parts. 'Ideas' is a reference section on techniques of proof (especially induction), set-theoretic foundations (from basics to countability) and general notions of abstract algebra. The second part, 'Theories', uses these ideas in the context of number theory, complex numbers, polynomials, matrices and vectors, focusing on solving linear equations, polynomial equations and quadratic forms in 2 or 3 variables.

Number theory encompasses the fundamental theorem of arithmetic, Bézout's theorem and the Euclidean algorithm (including Blankinship's version of this using elementary row operations (EROs)), modular arithmetic, linear Diophantine equations and continued fractions. There is an analogous development for polynomials as far as properties of irreducible polynomials, a very full account of partial fractions (and their use in integration) via GCDs and a description of the solution of cubics and quartics by radicals. Matrix theory is developed from scratch, with a full discussion of solving systems of linear equations using EROs, invertibility and diagonalisation. Vectors and their various products are reprised with a notably full treatment of determinants (from algebraic, geometric and axiomatic viewpoints). This section of work is crowned by the principal axes theorem in 2- and 3-dimensions with full details of the associated classification of conics and quadrics.

Although full advantage is taken of the geometric intuitions available in 2- and 3-dimensions, proofs are given by methods that will readily generalise. Each section contains numerous worked examples and mainly routine exercises, but there are some harder ones flagged with asterisks, including the invitation in Question 11 on page 24 to prove the Collatz $3x + 1$ conjecture! There is a 156-item bibliography that I initially thought was intimidating until I read the text closely and could appreciate how skilfully the author had used and blended his sources.

On one level then, the content is pretty much as expected, but this introductory text has a number of unusual, and unusually attractive, features. First, boxes are used to highlight harder proofs and provide glimpses ahead to more advanced topics (for example, $P = NP$?, the Banach-Tarski paradox, Gödel's theorems, and Moore-Penrose inverses); there are also 15 "essays" describing more immediate developments of the topics treated. All of these serve to introduce the novice to wider mathematical vistas. Second, the author emphasises the historical roots of the subject throughout as a reminder that mathematical ideas are not preserved in aspic but are constantly evolving. This respect for the past includes respect for older algebra texts and, as one who cut his mathematical teeth as a teenager on the miscellaneous examples at the end of Hall and Knight's *Higher algebra*, I inwardly cheered the comment that, "Chrystal's books [*Algebra* I & II] … are full of good sense and good mathematics. They can be read with profit today." Also, applications of algebra and geometry to cryptography and calculus are flagged to highlight the fact that mathematics is not rigidly compartmentalised. Finally, and most attractively, there is a lot of the author's personality in the text. There are numerous

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witty asides and footnotes and much wise advice. For example (page 54), on the definition $gf(x) = g(f(x))$, "This is the Big-endian definition. I regret to inform you that there is also a Little-endian definition where $fg$ is computed as $((x)f)g$. I cannot stop this sort of thing, but at least I can warn you of the hazards ahead"; (page 183, on Euler's formula), "Some authors are so worried by not being able to prove the above results rigorously at this stage that they simply *define* $e^{i\theta} = \cos\theta + i\sin\theta$. One wonders how such people can sleep at night"; and (page 204), "The sequence—complex numbers, real numbers, rational numbers, integers and natural numbers—represents a progressive simplification in our notion of number and a progressive complication in the algebra that results.".

*Algebra and geometry* successfully meets its aims. It has a reassuringly large overlap with familiar ideas from school mathematics but reappraises them in a readable yet rigorous manner. It introduces readers to the style of abstract reasoning that will be the staple of pure mathematics courses at university. It also includes plenty of nuggets that can be savoured after a first reading (such as the construction of the real numbers via equivalence classes of Cauchy sequences of rationals, and the proofs of the generalised associativity and Cantor-Schröder-Bernstein theorems). I shall happily recommend this book to prospective undergraduate mathematicians and warmly welcome it to the growing shelf of recent bridging texts such as [1, 2, 3, 4].

*References*

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2. M. Gould & E. Hurst, *Bridging the gap to university mathematics*, Springer (2009).
3. T. Körner, *Calculus for the ambitious*, Cambridge University Press (2014).
4. M. Liebeck, *A concise introduction to pure mathematics* (3rd edition), (Chapman & Hall/CRC Press) (2010).

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**Anachronisms in the history of mathematics** edited by Niccolò Guicciardini, pp. 366, £110 (hard), ISBN 978-1-10883-496-4, Cambridge University Press (2021)

Mainstream historians learn early the dangers of applying present-day thinking and concepts to the study of the past. To what extent do the same dangers apply to the history of mathematics? After all, a common, if not uncontroversial, claim for the special status of mathematics is that its content is independent of time or culture. This book derives from a symposium held in Pasadena in 2018; in what is a relatively new discipline, there is a welcome sense of freshness to the papers, although there are also quite a lot of 'isms'.

The editor Niccolò Guicciardini argues that, although anachronistic thinking has obvious dangers, it is not always wrong or unhelpful in mathematics:

> I do not share the skepticism, sometimes even derision, so commonly felt by professional historians when they accuse mathematicians who turn to history of producing work that is hopelessly naïve because it is often based on anachronistic translations and evaluations.

And in his introduction: