DISCUSSION PANEL

The last afternoon included a Panel Discussion with R. Cowsik, B. Mandelbrot, J. Primack, E. Salpeter (discussion editor), H. Sato and N. Vittorio. To avoid overlap with Jim Peebles' summary talk on the same afternoon, this discussion mainly concentrated on a few points which were not emphasized greatly during the meeting. A summary of each contribution is given below, in alphabetical order.

(A) X-RAY BACKGROUND - AN IMPORTANT COSMOLOGICAL SIGNAL

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High degree of isotropy [1] (< 25 fluctuations over 5° × 5°) and a precise thermal bremsstrahlung spectrum [2] with $T \sim 40$ keV characterises the X-ray background over 5-300 keV; the early indications of these features and the high emission measure were responsible for its recognition as an important cosmological signal and the suggestion that it could be the emission from a hot intergalactic plasma [3]. Discussions of possible contributions of discrete sources like the quasars and BL-Lac objects to the background indicate that their contributions could be significant [4-8] only below a few keV. The key questions are: (1) When and how was the intergalactic medium heated [9-12], and (2) What is the level of clumpiness of the hot gas [10], and much effort has been devoted to answer these questions.

An important source of energy for heating the gas to $T = (1+Z)40$ keV could be the gravitational energy of infall of the baryonic gas (into the potential wells of dark matter) at the time of galaxy formation [9] and it has been noted that the typical potential depths of voids and superclusters are adequate [13]. Indeed if $\Omega_B = 0.1$ and all these baryons do fall through potentials of ~ 10 keV then there would be too many X-rays [14]. A way out of this difficulty is suggested by the fact that the relic microwave background acts as a strong coolant [11], through "Inverse–Compton" scattering and if the baryons with $\Omega_B = 0.1$ are smoothly distributed they could not have been heated earlier than $Z = 4$. With clumping one can construct models [15] where a few percent of the baryons are in regions of high enough densities to cool through bremsstrahlung emission and the rest are compton cooled in tenuous regions at $Z > 4$. The Sunyaev-Zeldovich effect in the region of such hot spots would in principle be observable, even though the average distortion of the microwave background is negligible ($y < 0.001$).

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The high degree of isotropy requires that at least ~ 10^6–10^7 sources are present in the sky [1,16,17]. Such a requirement is not inconsistent with the hypothesis that the X-ray background has been generated by the baryonic gas falling into the potential wells created by clouds of dark-matter of cluster and supercluster dimensions, at $Z \simeq 5$.

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(B) NEUTRINO MASS ESTIMATES FROM SN 1987-A

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The explosion of the ~ $30M_\odot$ star, Sanduleak - 69 292, in the LMC as a supernova on 23 February 1987 and the concomitant observation of neutrino induced events in kilo-tonne detectors deep under the earth brought in a new era in neutrino astronomy and confirmed

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dramatically the qualitative predictions of supernova theory. The interest in this study, especially in interpreting the spread in arrival times of the neutrinos as due to neutrino mass effects has been so great that there has been a spate of papers cited extensively in the recent Fermilab preprint! The importance of finite neutrino mass to cosmology [2-4] and to the study of large scale distribution of dark matter [5] is not new. Many studies have shown that they may play a dramatic role in the formation of large scale structures and in the dynamics of the galaxies [6-10]. At this conference we have had poster [11] showing the correlation between the arrival times and energies of the neutrino events. The events appear to be forming two bands about $t = 1/2\ t_s (m_i/E_\nu)^2$ with $m_1 \sim 4$ eV and $m_2 \approx 22$ eV. If indeed the masses of the neutrinos are as indicated, they alone would contribute an $\Omega_{weak} = 0.75$. One might also recall that such neutrino masses fit in very well with those anticipated in the earlier dynamical studies of dark matter [5-10]. One must however add a cautionary note here - the statistics of events are meagre, the detector thresholds are comparable to the neutrino temperatures and relative time calibration between different detectors is lacking. In view of this it is not possible to assert the neutrino masses in a model in independent way with a confidence exceeding 80%.

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FRACTAL LARGE SCALE STRUCTURES AND CROSSOVER TO HOMOGENEITY: THE MASS-RADIUS FUNCTION VERSUS THE CORRELATIONS, AND THE MEASUREMENT OF THE CORRELATION RANGE

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ABSTRACT. The existence of a fractal zone in the distribution of galaxies in unquestioned (even though some writers state this fact in different words). The issue is whether this fractal zone is at least as deep as the horizon of observation R_{max} or (to the contrary) crosses over to a homogenous zone at a distance $R_{cross} < R_{max}$. The statistical tests must avoid prejudging between these possibilities. This requires unprecedented care. If a finite crossover does occur, R_{cross} must be defined realistically. Roughly speaking, R_{cross} should be the size of the largest significant structures (such as the voids). Other definitions are possible, but the definition used by Peebles (1980) seems to fail to satisfy the intuitive requirements (in addition to raising other problems, see below). If the test yields $R_{cross} < R_{max}$, the value of R_{cross} must be estimated without built-in bias against $R_{cross} = \infty$.

One neutral summary of the evidence is the mass radius function $M(R)$. Its derivative, divided by $4\pi R^2$, is a conditional occupation probability; it is better in some ways, but less acceptable in other. Arguments will be given against replacing it further by the covariance function or, worse, by either variant of the renormalized correlation functions. While it is proper in more conventional statistics, renormalization appears to be ill-adapted to the case when the fractal range is significant.

1. INTRODUCTION: TWO TALES OF HOW THE WORLD BEGAN.

The introduction restates in fanciful style the two basic models of fractal galaxy clustering, as advanced in Mandelbrot (1982 [Chapters 32, resp., 33 to 39], 1988).

• THE SEEDING OF THE HEAVEN. In the beginning, the heaven was a void. And the Master of Matter, Light and Life proclaimed, Let there be matter: and matter was. It was one point. And the Master proclaimed, Let matter be seeded over the heaven, and let every small part of the heaven be just like every other small part and like every large part. And two archangels set forth hopping; wherever the alighted, they left a pinch of matter and then resumed their journey as at its beginning. And the parts of the heaven were all made just alike. And the Master was everywhere: dwelling in every pinch of matter; and the heaven looked the same from every point where the Master dwelt.

• THE PARTING OF THE HEAVEN. In the beginning, the heaven was filled with matter. And the Master of Matter, Light and Life proclaimed, Let matter part away. Let it remove itself to form voids without number, and Let every small part of heaven be just like every other small part and like every large part. And matter removed itself, and the Master was everywhere, dwelling in every place that was not in a void; and the heaven looked the same from every point where the Master dwelt.

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2. ON DIVERSE STATISTICAL SUMMARIES OF THE DATA.

The present discussion will continue in terms of a fractally uniform measure: for example, of a uniform measure on either of the fractal dusts defined by the above tales. The resulting models can now be called *unifractal*. But it is nearly as easy to place a fractal but non uniform measure on the same set. I have introduced and developed the notion of *multifractal measure* long ago, mostly in 1972-1976, see Mandelbrot (1988), but it has not acquired a large following until recently.

3. THE THEORETICAL MASS-RADIUS FUNCTION.

In a wide class of random fractals with crossover at R_{cross}, including the preceding models, the degree of non homogeneity is best described by the mass radius function $M(R)$ = mass within a radius R around a randomly selected galaxy.

$$M(R) = F(R)R^D \qquad \text{for } R < R_{cross}$$
$$M(R) = (4\pi/3)\delta R^3 + \text{ (a fluctuation } \sim \sqrt{(4\pi/3)\delta R^3}\,) \qquad \text{for } R > R_{cross}.$$

The additive fluctuation factor for $R > R_{cross}$ is familiar to everyone, but the multiplicative fluctuation factor for $R < R_{cross}$ is familiar to few. The very important prefactor $F(R)$ is a stationary random function of $\rho = \log R$; it is almost sure that $F(R)$ does not take the values 0 and ∞, and it is usually safely away from these bounds. The variability of $F(R)$ or $\log R(R)$ around their expectations is one aspect of a fractal's "lacunarity". For example, as $\langle F^2 \rangle$ increases, the fractal's lacunarity also increases (Mandelbrot (1982) Chapters 34 and 35).

To appreciate what happens on the way from $M(R)$ to the correlation, it is best to take several distinct steps in succession.

4. THE THEORETICAL LOCAL CONDITIONAL DENSITY WITHIN THE VOLUME BETWEEN TWO SPHERES OF RADII R and $R + \Delta R$.

$$\text{This density is } (\Delta M/4\pi R^2)\Delta R = \begin{cases} (1/4\pi)R^{D-3}[DF(R) + \dfrac{\Delta F(R)}{R\Delta R}\,] & \text{for } R < R_{cross}. \\ \delta + \text{ (a fluctuation term)} & \text{for } R > R_{cross}. \end{cases}$$

5. THE EXPECTED CONDITIONAL DENSITY.

The expectation is taken over different origins with the same R. The expectation $\langle F(R) \rangle$ is positive and finite, and is independent of R. The expectation $\langle \Delta F(R) \rangle$ vanishes, because the prefactor is a stationary random function. On a plot of log (density) as function of $\log R$, the two regimes give straight lines of respective slopes $-D$ and 0 . The crossover occurs near the point where these straight lines intersect each other. In truncated fractal models I know well, the crossover between these two regimes is quite sudden. The corresponding expected mass radius plot is the plot of an integral, hence the crossover is far more gradual.

6. THE EMPIRICAL OR SAMPLE CONDITIONAL DENSITY.

It is an unavoidable feature that a given portion of space, e.g., the volume covered by an catalogue, intersects a small relative number of spheres of small radius, but a large relative number of spheres of large radius. When the sample average is carried over all sphere origins, this creates the serious complication that the individual sample values fail to be independent. Hence, the fluctuations in F and ΔF average out less well than if the samples had been independent. One deals with a reduced "effective sample size", whose value depends upon R . More precisely, when R is

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well below R_{max}, the effective sample size is near the actual one, and the conditional density is reliable. But for R close to R_{max}, the effective sample size is small, and the conditional density is affected by the fluctuation F. In addition, the sample values for two large values R' and R'' involve positively correlated errors, the correlation depending on the range of dependence of the prefactor $F(R)$ viewed as function of $\log R$. For example, while the expected conditional density decreases up to $R = R_{cross}$, the sample density may well actually increase in the range below R_{cross}.

7. COVARIANCES.

The theoretical covariance cov(R) is obtained by multiplying the conditional density by $M(\Delta R)$ and averaging. When $R >> \Delta R$, the fluctuations of $F(\Delta R)$ and $F(R)$ are independent, and these quantities average out independently.

8. THE CORRELATIONS.

In statistics, the correlation is the expression

$$\frac{\text{covariance}(R) - \text{covariance}(\infty)}{\text{covariance}(0)}.$$

By the Cauchy-Schwarz inequality, this quantity is ≤ 1 in absolute value. However. Peebles (1980) renormalizes differently, and works with the ill-explored expression

$$\frac{\text{covariance } (R)}{\text{covariance } (\infty)} - 1 = \frac{\text{covariance } (R)}{\langle \text{ global density } \rangle^2} - 1.$$

This quantity can exceed 1, and the radius R^* where it equals 1 is taken by Peebles (1980) as a substitute for R_{cross}. Alternative definitions from the same data would yield R_{cross} equal to several times R^*. In addition, the global density is a very elusive notion in the truncated fractal case, because it is very dependent on the depth of investigation R_{max}. A statistician would estimate cov(∞) on the same body of data as cov(R), and take $\text{cov}(\infty) = \text{cov}(R_{max})$. When $R_{cross} < R_{max}$, this yields $C(\infty) = C(R_{cross})$. However, renormalization projects the fluctuation of cov(R_c) onto all the values of cov(R_{cross}) for $\Delta R < R < R_{cross}$, making it dubious.

CONCLUSION.

Many statistically dubious steps enter in the procedures customarily used to reach the conclusion that $R_{cross} < R_{max}$, therefore this conclusion is not persuasive. The definition of R^* greatly understates the range of significant correlation. Analogous criticisms, with a fresh analysis of the data, are made by Pietronero (1987).

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COMMENTS ON COSMIC STRINGS *VS.* INFLATION AND ON COLD DARK MATTER IN AN OPEN UNIVERSE

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ABSTRACT. I briefly discuss challenges to two current dogmas: inflation and $\Omega = 1$. It is very difficult to generate a network of cosmic strings in particle physics models that also have an epoch of cosmic inflation, but models that generate such a network during an inflationary era may be the only cosmic string models with large voids. Cold dark matter in an open universe with $\Omega \approx 0.2$ implies larger cluster–cluster correlations, larger streaming velocities, and an earlier epoch of galaxy formation than the standard $\Omega = 1$ biased CDM model; but consistency with the latest $\Delta T/T$ constraints probably requires a cosmological constant.

1. COSMIC STRINGS *VS.* INFLATION

In particle physics models with cosmic strings, the string network forms as the temperature drops below a characteristic energy M_s. Below that temperature, the associated complex scalar field ϕ_s, which I will call the "stringon," acquires a non-vanishing vacuum expectation value (of order M_s) and therefore a phase. The phases are different in causally disconnected regions, and strings (along which $\Re\phi_s(\mathbf{r}) = \Im\phi_s(\mathbf{r}) = 0$: two equations in three dimensions determining a linear structure — a string) go through points such that the phase happens to make a complete circle $(0 \rightarrow 2\pi)$ along a path encircling the point.

If Newton's constant times the mass per unit length of the string μ satisfies $G\mu \approx 10^{-6}$, loops of cosmic string may accrete sufficient matter to form galaxies and clusters by the present epoch without violating the observational constraints on the microwave background anisotropy.[1)] Since $G\mu \approx (M_s/M_{Planck})^2$, this implies that $M_s \approx 10^{16}$ GeV. The problem is that this is much larger than the reheating temperature reached after the inflationary era ends.

In all inflationary models constructed to date which are consistent with the constraint that the fluctuations are not too large (*i.e.*, $(\delta\rho/\rho)_{Horizon} \lesssim 10^{-4}$), inflation is driven by a special gauge-scalar field, the "inflaton" ϕ_i, which is very weakly coupled both to itself and to all other fields. As a result, the reheating temperature is very low, $T_{RH} \approx 10^6 - 10^{10}$ GeV. This is much lower than M_s, so the cosmic string network will not form — a conclusion which has been pointed out by several authors. [2–4)]

As usual, sufficient cleverness may be able to overcome the problem. By coupling together the scalar fields responsible for inflation and for the cosmic strings, the inflaton ϕ_i and the stringon ϕ_s, it is possible to generate a cosmic string network even in inflationary models.[3] There is a small range of parameters for which the string network looks like that of the usual string scenario, but the more generic possibility is that the string network forms during the inflationary era and is very inhomogeneous. In this case, most of space is empty of strings and therefore of galaxies. Since we live in a galaxy we are in one of the non-empty patches, but we expect to be surrounded by a very inhomogeneous distribution of galaxies with very large voids. This might even be an attractive feature of this scenario, since in the usual scenario string loops, and hence galaxies, form everywhere to a first approximation, and it is hard to see how voids as large as those observed in the galaxy distribution could form. But the inhomogeneity of the galaxy distribution is constrained by observations of distant objects (radio sources, QSOs, optical galaxies, X-ray background)[5] and it remains to be seen what these constraints imply for such inflationary cosmic string models.

2. COLD DARK MATTER IN AN OPEN UNIVERSE

Although the biased $\Omega = 1$ CDM model has now become "standard," as evidenced for example by a number of the contributions at this meeting, we should also consider the possibility that the universe is open, with Ω as low as 0.2.[6] Recent measurements[7] suggesting $\Omega \approx 1$ are very difficult and uncertain; my own reading of the available data with as little theoretical prejudice as I can muster leads me to conclude that $\Omega \approx 0.2$ is as likely as $\Omega = 1$.

CDM in a open universe has a number of potentially attractive features:

- More large-scale power, especially if the ratio of baryons to dark matter is fairly large. This in turn implies [8,9]
    - larger streaming velocities v_{bulk}.
    - larger cluster–cluster correlations ξ_{cc}.
    - possibly (bigger) voids with (less) biasing.
- Earlier galaxy formation.
- Lower small-scale velocity.
- Smaller halos.[10]

There are also some potentially less attractive features:

- Fewer halos with large rotation velocities.[11]
- Fewer rich clusters with high velocity dispersion.[6]
- Large microwave background radiation anisotropy,[12] which is probably inconsistent with the new Owens Valley measurements discussed in these Proceedings unless there is a fairly large cosmological constant such as is necessary to make a low Ω universe consistent with inflation.[13,8,9]

Why does CDM with low Ω lead to more large scale power in the fluctuation spectrum? Because the bend in the CDM fluctuation spectrum — where the spectrum, which for low M is nearly flat ($\delta M/M \approx$ constant), tips over toward the primordial spectrum (which is presumably of the Zeldovich form $\delta M/M \propto$

$M^{-2/3}$) — corresponds to the transition at scale factor R_{eq} from the radiation-dominated era when DM fluctuations grow only logarithmically[14] to the matter-dominated era where DM fluctuations grow linearly with the scale factor. Since lower Ω corresponds to a later transition to matter dominance ($R_{eq} \propto (\Omega h^2)^{-1}$), the fluctuation spectrum is flatter to higher masses M. But since the spectrum is normalized at fairly low mass (*e.g.*, $\delta M/M = 1$ at $8h^{-1}$ Mpc, where h is the Hubble parameter in units of 100 km s^{-1} Mpc^{-1}), this means that there is more large-scale power. Increasing the fraction of baryons gives even more large-scale power, since the baryons contribute to the growth of fluctuations above the baryon-photon Jeans mass ($\approx 10^{17} M_\odot$) but are a dead weight below.[9] Why consider CDM if Ω is low enough to be (almost) consistent with the standard nucleosynthesis constraints on an entirely baryonic universe? Because adiabatic fluctuations of galaxy size Silk damp away in such a model, while galaxy formation with CDM + baryons may have many of the attractive features of the standard CDM model.

Particle physics has yet to explain why the cosmological constant is not enormous. Maybe there is a reason why Ω_{baryon}, Ω_{DM}, and $\Omega_\Lambda = \Lambda/3H^2$ are all comparable. In our present state of ignorance, I think it is best to keep an open mind about the values of both fundamental cosmological parameters Ω and Λ.

I am grateful to my colleagues George Blumenthal, Avishai Dekel, Sandra Faber, and David Seckel, as well as to many of the participants at this Symposium, for helpful discussions. This work was partly supported by research grants from the National Science Foundation and from the University of California, Santa Cruz.

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A FEW MORE COMMENTS

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I will mention a few different minor points, but I first want to make a plug for more future observations of the type pioneered by Art Wolfe and his colleagues: As noted by Wally Sargent, the ''damped wing'' Ly_α absorption lines for distant quasars give information on some type of intervening medium with large neutral hydrogen column density (say, 10^{19} to more than 10^{21} H cm^{-2}). Results so far refer mainly to the redshift range $z \sim 2$ to 3, a controversial era regarding galaxy formation. I regard these lines of special interest for two reasons: (1) One gets a lot of information for each system--not only the absorption redshift and column density, but also the heavy element abundance and the velocity dispersion along the line of sight. (2) The results indicate something ''ordinary''--intervening disk galaxies--rather than some ''exotic'' medium, but learning about galaxies at $z > 2$ is exciting enough. One aspect of ''ordinary-ness'' is already pretty firm and already has significant consequences: The absorption lines are quite narrow, just as for present-day disks, which (a) already rules out the possibility of proto-elliptical galaxies having had a uniform HI distribution (which would have had a much larger velocity dispersion) and (b) will impose restrictions on the violence with which additional material was accreted onto proto-galaxies. These observations make it likely that galaxy disks had larger gas masses when young, which is not surprising. They may also indicate that the specific angular momentum was also larger, which would be more surprising, but this will require more observational statistics for clarification.

On one topic this meeting has expressed ''optimism by default'' regarding the final outcome for some uncertain observational numbers: We have assumed that the age of our Galaxy, t_{Gal}, will turn out to be less than $t_{0,1}$, the age of the cosmological model witlh zero cosmological parameter Λ and unit density parameter Ω. However, in spite of our collective hopes, it might yet turn out that (a) the Hubble parameter H_o is near 100 (rather than 45 or 75, say) and (b) uncertainties in chemical abundances, etc. might push present-day estimates for stellar evolution ages (and hence t_{Gal}) up rather than down. If the horror of $t_{Gal} > t_{0.1}$ should befall us, are there other ways out or do we really need models with a non-zero value for Λ and with Ω substantially smaller than unity? Such models would have to have a substantial ''coasting period'' near some $z > 1$. This would have one benign side-effect of pushing down the predicted value for microwave background fluctuations, but a long coasting period should have more direct observational consequences. Could detailed galaxy observations at $z \sim 0.5$ to 1, plus less detailed observations at $z \sim 2$ to 3, rule out (or even suggest) models with long coasting periods, even before the numerical value of H_o settles down?

One minor point regarding the possibility of dark matter near the sun being "stellar population III" of the low mass variety: It is usually assumed that, if this is the case at all, the objects are of really low mass--Jupiters rather than brown dwarfs--and that we have to deal with bimodal star formation. There is now a possibility that the minimum mass for hydrogen burning may be larger for some kind of non-homologous star formation than for homologous contraction. This raises the possibility of population III stars of "ordinary mass", $\sim 0.1\, M_{\odot}$ but having escaped hydrogen burning.

We have heard convincing evidence that there is an increased density of galaxies in the region of space where velocity anomalies call for a "great attractor". It is not clear, however, whether the contrast in "light density" is sufficiently large, if the "great attractor" has the same mass to light ratio as "typical" superclusters. There may not be a numerical discrepancy but--if there is--it may only mean that M/L fluctuates from case to case even for regions of a given density enhancement $\delta\rho/\rho$: If we accept "biased galaxy formation", i.e. if M/L varies with $\delta\rho/\rho$, this is likely to be only an average relation anyway, so we should expect fluctuations. Internal velocity dispersions for any galaxy clusters (or even virialized groups) near the "great attractor" would be very useful to get another handle on M/L.

A CONTRIBUTION TO THE DISCUSSION

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Fractal dimension as to the galaxy distribution has been pointed out. However, it was introduced to analyse a pattern of spatial distribution. If the energy distribution in the expanding universe had a fractal dimension by nature, the cosmic expansion would be changed from the conventional one, on which I will comment here [1].

Let us assume that the total energy with a radius r is given by a fractal dimension D as $E(<r) \propto r^D$. Then the density averaged over the volume with a radius r is $<\rho>_r \propto E(<r)/r^3 \propto r^{D-3}$. Here we assume that the density in the right-hand side of the expansion equation should be the density averaged over the horizon volume. Since a local density is size-dependent in case of fractal dimension, it is not trivial what kind of density should be put in there. The above treatment is an assumption. To solve the expansion equation, the thermodynamical relation about the change of energy by expansion is necessary, which is written as $E(<r) \propto a(t)^{-\beta}$. $\beta = 0$ for dust, $\beta = 1$ for radiation, $\beta = -1$ for string and $\beta = -3$ for bulk vacuum.

The expansion equation is written as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G <\rho> \propto \frac{t^D}{t^3} \frac{1}{a^\beta} .$$

If $\beta \neq -3$ or $D \neq 1$,

$$a \propto t^{\frac{D-1}{\beta+3}} .$$

For $\beta = -3$, $a \propto \exp t^{\frac{D-1}{2}}$ and we have the inflation for $D = 3$. For $D = 1$ and $\beta \neq -3$, $a \propto (ln\ t)^{\frac{2}{\beta+3}}$ and $a \propto t$ if $\beta = -3$.

For $\beta = 0$, $H = (D-1)/3t$ and the models of $D < 3$ do not help the "age problem". By the same reason of a slowness of the expansion, the density fluctuation grows faster with $(a(t)$, e.g. $\rho \propto a^{\frac{1+\sqrt{\eta}}{2}}$ for $D = 2$ and $\beta = 0$.

Recent favourite story of structure formation in the universe is to start from scalar field before the inflation. In most scenarios, however, the field energy is assumed to be dissipated away completely into the incoherent hot particles, which cluster by gravity after their cooling. Here, we can imagine a different scenario as follows: the scalar field could be very clumpy, the energy density may have a fractal dimension and the field energy dissipates away only partially, not completely. Large scale clumpiness could lead to the multiple universes or the infinitely-recycling of universe production [2]. The smaller scale clumpiness may have remained to create the large scale structure [3]. Such scenario of "structure from structure" would be contrasted with the conventional scenario of "structure from nothing (except quantum fluctuation)". In fact, the field energy cannot disappear completely, for example, by a topologaical constraint. Cosmic strings are an example of such remnants.

Generation of density perturbation by a cosmic string is described by an accretion onto a spherical shell with mass m_s [4]. In the linear regime, the perturbation with a scale containing mass M is written as

$$\ddot{\delta}_M + 2H\dot{\delta}_M - 4\pi G\rho\delta_M = \frac{m_s}{M} G\rho\theta(t - t_{cross}(M)) ,$$

where $4\pi\rho(t_{cross}(M))r_s^3 = M$. Here we note that perturbations in new proper scales are continuously generated differently from the case where the equation is homogeneous or source-free.

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Contribution to the Panel Discussion

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A flat cold dark matter dominated universe seems to provide the most efficient scenario for forming galaxies: gravitational instability can develop in the cold weakly interacting component well before matter–radiation decoupling, and density fluctuations can grow until today. Because of both these effects it is possible to form galaxies with the minimum initial amplitude for density fluctuations. This is crucial for being consistent with the observed smoothness of the cosmic microwave background (CMB). Assuming that galaxies trace the overall density field, the quadrupole anisotropy expected in this scenario is $Q = 2.9 \cdot 10^{-(4+1.285n)}$, where n is the primordial spectral index (see, e.g., Vittorio, Mattarrese, Lucchin, 1987). Any observational upper limit on the quadrupole anisotropy implies a lower limit on the value of the primordial spectral index n. The results from the RELIC experiment discussed here by Dr.Strukov are extremely exciting. The upper limit on the CMB quadrupole anisotropy , $Q < 3 \cdot 10^{-5}$, sets quite a strong constraint on the value of the primordial spectral index: $n > 0.8$. The observational upper limit is even tighter if in analyzing the data, one assumes that the primordial density fluctuations have a Zel'dovich spectrum (n=1). In this case $Q < 1.6 \cdot 10^{-5}$, just above the theoretical prediction for the unbiased cold dark matter scenario. If galaxies do not trace the mass, as has been proposed on theoretical grounds (see, e.g., Bardeen, Bond, Kaiser and Szalay, 1986), then the theoretical predictions for the quadrupole anisotropy will be reduced by a factor $2 \rightarrow 3$. In a massive neutrino dominated universe, with a Zel'dovich density fluctuation spectrum, $Q = 7.5 \cdot 10^{-6}(1 + z_{NL})$. Then consistency with the observational upper limit requires that the redshift of nonlinearity $z_{NL} \lesssim 1$.

All the current experiments for searching CMB anisotropies are differential, based on a beam–switching technique. This implies that the observations provide a measurement of the first derivative (single subtraction experiment) or of the second derivative (double subtraction experiment) of the CMB temperature field. One expects the CMB temperature distribution on the sky to be a 2-D random gaussian field. From the theoretical point of view it is possible to statistically predict the entire pattern of the microwave sky (Sazhin, 1985, Vittorio and Juskiewicz,1987, Bond and Efstathiou, 1987). The temperature pattern expected on large angular scales differs qualitatively from the pattern expected on small angular scales. For scale–invariant density fluctuations, the effective coherence angle for the large scale anisotropy is the antenna beamwidth θ. A change in θ changes the whole pattern of the large scale temperature distribution (i.e., the number of hot spots, their angular diameters, etc.). On the other hand, on small scales there is a preferred angular scale. The last scattering surface has a finite thickness, which subtends an angle $\alpha \simeq 8'\Omega_0$. Because of this, temperature fluctuations on angular scales $\lesssim \alpha$ are smeared out and the temperature distribution exhibits a characteristic coherence angle $\sim \alpha$. It is possible to show that the typical hot spot angular size is of the order of the coherence angle (see, e.g., Vittorio and Juszkiewicz, 1987): either the beam size θ on large angular scales or the smearing angle α on small scales. In order not to look at the same hot spot with both beams in a double beam experiment, the beam separation should be

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much larger than the maximum of either the beam size or the smearing angle. The sky coverage in the fine–scale experiment is usually rather limited, with the number of pairs of pixels hardly exceeding ~ 10 pairs. If the coherence angle is $\sim 10'$, experiments with beam separation $\lesssim 10'$ are likely to strongly suppress any signal. The fine–scale temperature fluctuations may have escaped detection so far because of use of too small beam separation and limited sky coverage. The former problem makes the latter (undersampling) more severe. On large angular scales, both sky coverage and beam separation are sufficient. However, the granularity (i.e. number of hot spots) of the large scale maps is expected to increase if the beam size is decreased. This would increase the necessary integration time, thereby favoring satellite experiments as RELIC, or the planned COBE and RELIC II.

Dr.Lasenby reported the high sensitivity observations of the CMB made at an angular scale of 8^o, with a large number of independent fields of view. The observed anisotropy in the sky emission is $\delta T/T = 3.7 \cdot 10^{-5}$ and it may possibly be the direct imprint of the primordial density perturbations. Assuming that the density fluctuations were scale-invariant, one expects a quadrupole anisotropy (Davies et al. 1987, Scaramella and Vittorio, 1987) $Q = 8.9 \cdot 10^{-5}$, which exceeds by a factor $\gtrsim 2$ the RELIC upper limit. If both of these results are confirmed, then the primordial density fluctuation spectrum had to be steeper than the Zel'dovich one (i.e., $n > 1$). This would be a crucial test for the inflationary theories of the early universe.

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