*Proceedings of the Edinburgh Mathematical Society* (1995) **38**, 543–546 ©

# BOOK REVIEWS

KHUKHRO, E. I. *Nilpotent groups and their automorphisms* (de Gruyter Expositions in Mathematics Vol. 8, de Gruyter, Berlin, New York 1993) 252pp., 3 11 013672 4, about £90.

This is an excellent coverage, by a leading expert in the field, of the use of Lie ring methods in the theory of (mostly!) finite $p$-groups. The book falls into two sections. The first is on Linear Methods, and is a very good textbook which could stand alone as such. The exposition starts *ab initio*, defining groups, rings, modules, isolators, Lie rings associated with lower central series and the like. Here are proved such classical results as the fact that the associated Lie rings of groups of prime exponent $p$ satisfy the $(p-1)$-st Engel condition, and the nilpotency of soluble Engel Lie rings.

The second section is devoted to the subject indicated by the title. Chapter 4 contains proofs of the theorem of Higman, Kostrikin and Kreknin on regular automorphisms, with extensions to almost regular automorphisms. These results are applied in Chapter 5 to give a proof of Higman's theorem establishing a bound for the nilpotency class of groups admitting a regular automorphism of prime order. Then comes the beautiful result that every finite $p$-group admitting an automorphism of order $p$ with exactly $p^m$ fixed points contains a subgroup of $(p,m)$-bounded index and $p$-bounded class. Some highlights from the highly technical Chapter 6 (*Nilpotency in varieties of groups with operators*) are as follows. Let $M_p$ be the variety of operator groups consisting of all groups with a splitting automorphism $\phi$ of prime order $p$, which means that the operator identities $x^{\phi^p}=x$, $xx^{\phi}\dots x^{\phi^{p-1}}=1$ hold. Corollary 6.4.2 states that soluble groups in $M_p$ are nilpotent, while 6.4.5 establishes the positive solution for the Restricted Burnside Problem in $M_p$: the locally nilpotent groups in $M_p$ form a subvariety, that is, for each $d$ the nilpotency classes of $d$-generator nilpotent groups in $M_p$ are $(d,p)$-bounded. Chapter 7 contains another proof of this latter result. For finite $p$-groups, the study of splitting automorphisms of prime order is equivalent to the study of groups different from their Hughes subgroups. Highlights here are as follows. Another version of the RBP in $M_p$ is that every $d$-generator finite $p$-group admitting a partition contains a subgroup of index $p$ which is nilpotent of $(d,p)$-bounded class not more than $f(d-1,p)$, where $f$ is the function implicit in the above result confirming RBP for $M_p$. There are many other interesting Hughes-type results in this chapter, indeed too many to mention here. Finally, in Chapter 8, the focus is on nilpotent $p$-groups admitting automorphisms of general $p$-power order having fixed-point set of given order, and includes the latest results of Shalev and the author.

The exposition throughout is clear, despite the technical nature of the subject; the author's pleasant style has been enhanced by J. C. Lennox's assistance with the pitfalls of English. Each chapter contains a final section of *Comments*, giving insights, overviews and reflections on proof. Altogether, this is a book to be recommended for its wealth of detail, covered extremely sketchily in this review, of a difficult area of nilpotent group theory.

J. WIEGOLD

HÖRMANDER, L. *Notions of convexity* (Progress in Mathematics Vol. 127, Birkhäuser, Basel, Berlin, Boston 1994) 424 pp., 3 7643 3799 0, £38.

Lars Hörmander is the author of two well-known books: *The analysis of linear partial* *differential operators* (Springer-Verlag, Berlin 1983–85) [ALPDO] and *An introduction to the complex analysis in several variables* (North-Holland, Amsterdam 1990) [CASV]. The book under review overlaps slightly with both of these books and depends in places on proofs in [ALPDO], though the full definitions and many examples are given here.

Convexity in the title of this book is to be interpreted in a wide sense, particularly pertaining to differential operators. This viewpoint starts by observing that affine functions $f$ from $\mathbb{R}^n$ to $\mathbb{R}$ are the solutions of $\frac{d^2 f}{dx^2}=0$ and that harmonic functions $F$ from $\mathbb{R}^n$ to $\mathbb{R}$ are solutions of $\sum \frac{\partial^2 F}{\partial x_i^2}=0$. A convex function on $\mathbb{R}$ is a function whose graph lies below the chord between any two points. This compares a convex function with the affine functions on any interval that dominate the function on the boundary of the interval. A function $F$ from $\mathbb{R}^n$ to $\mathbb{R}$ is subharmonic if for each compact subset $K$ and each harmonic function $h$ on $K$ with $h \geq F$ on the boundary $\partial K$ the inequality $h \geq F$ in $K$ holds. Of course there are continuity and domain conditions omitted from the simplified description above.

The basic results and examples of convex functions and convex sets are thoroughly developed for finite-dimensional spaces with examples and some applications. For example, the inequality of Fenchel and Alexandrov on mixed volumes of convex subsets of $\mathbb{R}^n$ is proved and used to deduce the Brunn–Minkowksi inequality. Similar care and detail are used in the discussion of subharmonic functions and plurisubharmonic functions from an open set $X$ contained in $\mathbb{R}^n$ into $[-\infty, \infty)$. Representations, inequalities and exceptional sets are covered. In the fifth chapter he discusses $G$-subharmonic functions, where $G$ is a subgroup of $GL_n(\mathbb{R})$, and shows that convex, subharmonic and plurisubharmonic functions correspond to the full, the orthogonal and complex linear groups respectively.

The book is well written with good examples and frequent discussion of the case $n=2$ to help the reader. For the full discussion on subharmonic and plurisubharmonic functions it is essential to have [ALPDO] and [CASV] at hand. This book should be in every mathematics library.

A. M. SINCLAIR

MORTON, K. W. and MAYERS, D. F. *Numerical solution of partial differential equations* (Cambridge University Press, Cambridge 1994), 227 pp., hardcover: 0 521 41855 0, £35.00, paperback: 0 5214 2922 6, £13.95.

Partial differential equations (PDES) are the most widely used models for many scientific, engineering and economic problems. Unfortunately most PDES of any practical interest cannot easily be solved analytically due to nonlinearities and complex geometric domains. The only recourse to solve these problems is by numerical methods.

This book, which can be seen as an introduction and a complement to the classical text of Richtmyer and Morton (1967), is a self-contained account of the theory and applications of mainly finite difference methods to the numerical solution of PDES. The book is aimed at final year undergraduate and first year postgraduate mathematics students but could easily be used with engineering and computer science students with a rudimentary knowledge of PDES.

Parabolic equations in one dimension are introduced first and through the use of a discrete maximum principle a simple error analysis is given. The extensions of these methods to two and three-dimensional and nonlinear problems are discussed. Hyperbolic problems are treated extensively and here discrete Fourier analysis is used not only for deducing necessary stability conditions but also to analyse phase and amplitude errors. At this point the authors formalise the notions of consistency, convergence and stability for linear initial boundary value problems. A readable account of what is often a confusing area for students is given, including Lax's equivalence theorem uniting the concepts of consistency, stability and convergence. The main difficulty with establishing convergence is a proof of stability. Necessary conditions can be