GALACTIC BARS AS ALIGNED EXCENTRIC ORBITS

V.L. POLYACHENKO

Astronomical Council of the USSR Academy of Sciences
Moscow USSR

The simplest analytical theory of bars made of stars in excentric orbits is suggested below (such a possibility was mentioned in [1]). Approximately, the subsystem of particles with elongated orbits may be described as consisting of hard "needles" elongated along radii, slowly (compared with radial oscillations of particles) rotating due to orbit precession. As a first approximation, we shall consider the needles mentioned to be infinitesimally thin ones. Besides, let us assume for simplicity that the radial energy of stars is fixed: $E=E_0$. Then we may introduce the distribution function of such needles, $f=f(\varphi, \Omega)$, so that $dn = f(\varphi, \Omega)d\varphi d\Omega$ is the number of needles within the angles $(\varphi, \varphi+d\varphi)$, rotating with the angular velocities $\Omega = \dot{\varphi}$, in the range $(\Omega, \Omega+d\Omega)$. Let us write now the collisionless kinetic equation in the form $\partial f/\partial t + \Omega\, \partial f/\partial\varphi + P\partial f/\partial\Omega = 0$, where $P=d\Omega/dt=d^2\varphi/dt^2=M_{tot}/I$; M_{tot} is the total torque acting on the needle, I is the needle's inertia momentum relative to the disk center.

The reference frame where the final bar is in steady state rotates relative to the inertial framework with the angular velocity $\Omega_p= \overline{\Omega}$, where $\overline{\Omega}$ is the mean angular velocity of star's orbit precession. In this framework, we may take $f=F(\Omega^2/2 +\phi)$ with $P=-\partial\phi/\partial\varphi$. If we assume that the function F is a Maxwellian: $F(\Omega)= \exp[-\Omega^2/\Omega_T^2]/\pi\Omega_T$, we obtain the nonlinear integral equation for the function $\Pi= \exp[-2(\phi-\phi_0)]-1$:

$$\mathrm{Ln}[1+\Pi(\varphi)]=\frac{G}{I\pi\Omega_T^2}\int_{-\pi/2}^{\pi/2}K(\varphi-\varphi')\Pi(\varphi')d\varphi', \qquad K(\alpha) = \int_{-a}^{a}\frac{dxdy\,\mu(x)\mu(y)}{\sqrt{x^2+y^2-2xy\cos\alpha}} \qquad (1)$$

$\mu(r)$ is the linear mass density of the needle, 2a is its length (we assume for simplicity the needle to be symmetric).

In the linear approximation, we obtain from (1) the simple integral equation with the solution $\Pi \approx \exp(im\varphi)$ which occurs under certain conditions on the value of $\lambda = G/I\pi\Omega_T^2$ (see [2] for more details; in particular it is shown in [2] that just bar-mode (m=2) demands the maximum dispersion of precession velocities Ω_T for stabilisation).

It is also possible to find analytically the first nonlinear correction to the linear value of λ: $\lambda = \lambda_0 +A_m \Pi^2$. For this purpose, we should rewrite the left side of equation (1) as $\Pi -\Pi^2/2+\Pi^3/3$, and then perform usual calculations. As a result, one can find

$$A_m=\frac{1}{8K_1}.(2 - \frac{2}{1-\lambda_0 K_0} - \frac{1}{1-\lambda_0 K_2}), \qquad K_n=\int_{-\pi/2}^{\pi/2}K(\alpha)\exp(in\alpha)d\alpha$$

As for the numerical calculations for strongly nonlinear case (as well as for different F-functions), they will be published elsewhere.

References

[1] Lynden-Bell D., Kalnajs A.J. (1972) M.N.R.A.S. **157**, 1
[2] Polyachenko V.L. (1989) Pis'ma Astron. Zh. (USSR) **15**, 890

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