Glasgow Math. J. **55** (2013) 123–130.
doi:10.1017/S0017089512000389.

RICCI SOLITONS AND CONTACT METRIC MANIFOLDS

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(Received 14 August 2011; accepted 8 March 2012; first published online 2 August 2012)

Abstract. We study on a contact metric manifold $M^{2n+1}(\varphi, \xi, \eta, g)$ such that g is a Ricci soliton with potential vector field V collinear with ξ at each point under different curvature conditions: (i) M is of pointwise constant ξ-sectional curvature, (ii) M is conformally flat.

2000 *Mathematics Subject Classification.* 53C15, 53C25, 53D10

1. Introduction. By a Ricci soliton we mean a Riemannian metric together with a vector field (M, g, V) and a constant λ that satisfies

$$\pounds_V g + 2S + 2\lambda g = 0, \tag{1}$$

where $\pounds_V$ denotes the Lie derivative along V, S is the Ricci tensor. Obviously, a trivial Ricci soliton is an Einstein metric with V zero or Killing. Thus, a Ricci soliton may be considered as an apt generalisation of an Einstein metric. A Ricci soliton is said to be shrinking, steady and expanding as λ is negative, zero and positive, respectively. If $V = -\nabla f$ (where f is a smooth function on M), then equation (1) can be written as

$$\nabla\nabla f = S + \lambda g,$$

and is known as a gradient Ricci soliton. For background on Ricci solitons and their interaction to Ricci flow, we refer to Cao-Zhu [**6**] and Chow–Knoff [**9**]. We also remark that a Ricci soliton on a compact manifold is a gradient Ricci soliton (see [**14**]).

Recently, there has been a rising interest in the study of a contact metric manifold whose metric is a Ricci soliton. In this direction, Sharma [**15**] proved that *if the metric g of K-contact manifold is a gradient soliton, then it is shrinking and the metric g is Einstein–Sasakian.* This result has been generalised by Ghosh et al. [**12**] for a (κ, μ)-space (see [**3**]). Moreover, Sharma–Ghosh [**16**] studied Sasakian 3-metric as a Ricci soliton and proved that *it is expanding and homothetic to the standard Sasakian metric on the Heisenberg group nil^3.* On the other hand, on a contact metric manifold, one may think of another type of a Ricci soliton in which the vector field V is collinear with the Reeb vector field ξ or $V = \xi$. In this direction, Sharma [**15**] proved that *if a K-contact metric g is a Ricci soliton with V pointwise collinear with ξ, then V, a constant multiple of ξ and g, is Einstein.* We now recall the following results of Cho [**7**] and Cho–Sharma [**8**].

THEOREM (CHO–SHARMA). *If a contact metric g of a compact contact metric manifold M is a Ricci soliton with potential vector field V collinear with ξ, then g is Einstein.*

THEOREM (CHO). *A contact Ricci soliton is shrinking and is Einstein K-contact.*

Here we generalise the last two results and prove.

THEOREM 1. *Let $M^{2n+1}(\varphi, \xi, \eta, g)$ be a contact metric manifold such that g is a Ricci soliton with a non-zero potential vector field V collinear with ξ at each point. If M is of pointwise constant ξ-sectional curvature, then it is Einstein K-contact and the soliton is shrinking. Moreover, if M is complete, then M is the compact Einstein–Sasakian.*

In [**16**], Sharma–Ghosh introduced a new class of contact metric manifold whose curvature tensor R satisfies

$$R(X, \xi)\xi = \kappa(X - \eta(X)\xi) + \mu(hX),$$

which can also be written in terms of the Jacobi operator $l = R(., \xi)\xi$ as

$$l = -\kappa\varphi^2 + \mu h, \tag{2}$$

for real constants κ, μ and $h = \frac{1}{2}\pounds_\xi\varphi$. We call this manifold as the Jacobi (κ, μ) contact manifold. This type of manifold may be considered as a generalisation of (κ, μ)-contact manifold, introduced and studied by Blair et al. [**3**], and defined by

$$R(X, Y)\xi = \kappa(\eta(Y)X - \eta(X)Y) + \mu(\eta(Y)hX - \eta(X)hY).$$

It is easy to observe that a Jacobi (κ, μ) includes K-contact (for which $k = 1$ and $h = 0$) and the (κ, μ)-contact manifolds. Unlike a (κ, μ)-contact manifold, the associated CR-structure on the Jacobi (κ, μ)-contact manifold need not be integrable. On the other hand, a straightforward computation shows that like (κ, μ)-contact metric structures, the Jacobi (κ, μ)-contact metric strucures are also invariant under a D-homothetic deformation:

$$\bar{\eta} = a\eta, \bar{\xi} = \frac{1}{a}\xi, \bar{\varphi} = \varphi, \bar{g} = ag + a(a-1)\eta \otimes \eta.$$

Examples of a Jacobi (0,0)-contact structure (i.e. $l = 0$) are the normal bundles of integral submanifolds of a Sasakian manifold (see [**1**], p. 153). Applying D-homothetic deformation to the Jacobi (0,0)-contact structure, one can easily (see [**17**]) obtain the Jacobi $(1 - a^{-2}, 2 - 2a^{-1})$-contact structures.
Using Theorem 1, we prove the following.

COROLLARY 1. *If the metric of a Jacobi (κ, μ)-contact manifold $M^{2n+1}(\varphi, \xi, \eta, g)$ is a Ricci soliton with a non-zero potential vector field V collinear with ξ at each point, then it is Einstein and K-contact. In addition, if M is complete, then M is compact Einstein–Sasakian.*

Now we turn our attention to conformally flat contact metrics. Conformal flatness has been studied by several authors in the framework of contact metric manifolds. Generalising the result of Tanno [**18**], Blair–Koufogiorgos [**2**] proved that a conformally flat contact metric manifold with $Q\varphi = \varphi Q$ (where Q is the Ricci operator associated with the Ricci tensor, i.e. $S(X, Y) = g(QX, Y)$) is a space form. Extending this further, Ghosh et al. [**11**] proved that a conformally flat contact metric manifold satisfying $Q\xi = (Trl)\xi$ and $K(\xi, X) + K(\xi, \varphi X)$ is a function independent of X orthogonal to ξ

and is of constant curvature. But it is shown in [**13**] that the same conclusion can be drawn without restriction on sectional curvatures.

Recently, Ghosh (see [**10**]) considered a real hypersurface of a complex space form satisfying

$$(\pounds_\xi g)(X, Y) + 2S(X, Y) + 2\lambda g(X, Y) = 0$$

for all vector fields X, Y orthogonal to ξ. This is known as a generalised *η-Ricci soliton*. Thus, as a generalisation of *a contact Ricci soliton* [**7**] as well as a generalised *η-Ricci soliton*, in the framework of contact metric manifold, one may consider equation (1) for all vector fields X, Y orthogonal to ξ. We call this as a *generalised Ricci soliton*. For a *contact Ricci soliton*, it is easy to observe that $Q\xi = -\lambda\xi$ (see equation (9) in which $f = 1$) and hence by the result of Gouli-Andreou and Tsolakidoua [**13**] we see that *a conformally flat contact Ricci soliton is a space form* (see [**7**]). But for a generalised Ricci soliton this is not true. Thus, we are motivated to study conformally flat contact metric manifold whose metric is a *generalised Ricci soliton*. Precisely, we prove the following.

THEOREM 2. *Let $M^{2n+1}(\varphi, \xi, \eta, g)$, $n > 1$ be a contact metric manifold such that g is a generalised Ricci soliton with a non-zero potential vector field V collinear with ξ at each point. If M is conformally flat, then it is of constant curvature* 1.

2. Preliminaries. By a contact manifold we mean a $(2n + 1)$-dimensional smooth manifold M that carries a global 1-form η such that $\eta \wedge (d\eta)^n$ is non-vanishing everywhere on M. For a given contact 1-form η there exists a unique vector field ξ, called the Reeb vector field such that $d\eta(\xi, X)= 0$ and $\eta(\xi) = 1$. Polarising $d\eta$ on the contact sub-bundle $\eta = 0$, one obtains a Riemannian metric g and a (1,1)-tensor field φ such that

$$d\eta(X, Y) = g(X, \varphi Y), \eta(X) = g(X, \xi), \varphi^2 = -I + \eta \otimes \xi, \tag{3}$$

where g is called an associated metric of η and (φ, η, ξ, g) is a contact metric structure. Following [**1**] we recall two self-adjoint operators $h = \frac{1}{2}\pounds_\xi\varphi$ and $l = R(., \xi)\xi$ that satisfy $h\xi = 0 = l\xi$. The tensors h, $h\varphi$ are trace-free and $h\varphi = -\varphi h$. For a contact metric manifold we also have the following formulas (for details we refer Blair [**1**]):

$$\nabla_X\xi = -\varphi X - \varphi hX. \tag{4}$$

$$l - \varphi l\varphi = -2(h^2 + \varphi^2). \tag{5}$$

$$\nabla_\xi h = \varphi - \varphi l - \varphi h^2. \tag{6}$$

$$Trl = S(\xi, \xi) = 2n - Trh^2. \tag{7}$$

$$(div(h\varphi))X = g(QX, \xi) - 2n\eta(X). \tag{8}$$

Formula (8) appears in Blair–Sharma [**4**]. A contact metric structure is said to be K-contact if ξ is Killing with respect to g, equivalently, $h = 0$, or $Tr.l = 2n$. The contact structure on M is said to be normal if the almost complex structure on $M \times R$ defined by $J(X, f d/dt) = (\varphi X - f\xi, \eta(X)d/dt)$, where f is a real function on $M \times R$, is integrable. A normal contact metric manifold is called a Sasakian manifold. Sasakian manifolds are K-contact and 3-dimensional K-contact manifolds are Sasakian. The sectional curvature $K(\xi, X)$ of a plane section spanned by ξ and a vector field X

https://doi.org/10.1017/S0017089512000389 Published online by Cambridge University Press

orthogonal to ξ is called ξ-sectional curvature, where as the sectional curvature $K(\xi, \varphi X)$ of a plane section is spanned by ξ and φX, where X is orthogonal to ξ.

3. Proof of the results. Before entering into the proof of Theorem 1 we first prove the following lemma.

LEMMA 1. *On a contact metric manifold $M^{2n+1}(\varphi, \xi, \eta, g)$, if a function f depends only on the direction of ξ, then it is constant on M.*

Proof. By the hypothesis we see that $((\varphi X)f) = 0$ for all vector field X on M. Therefore, taking φX instead of X and recalling (3), we can write $df = (\xi f)\eta$. Applying d to this equation, using the Poincare lemma provides

$$(X(\xi f))\eta(Y) - (Y(\xi f))\eta(X) + 2(\xi f)g(X, \varphi Y) = 0.$$

Choosing X, Y orthogonal to ξ, the above equation immediately gives $\xi f = 0$. Hence, f is constant.

Proof of Theorem 1. Since M is of pointwise constant ξ-sectional curvature, we have

$$g(R(X, \xi)\xi, X) = \kappa(p)g(X, X)$$

for some function $\kappa(p)$ and for any tangent vector field X orthogonal to ξ at $p \in M$. Polarising the last equation and using the symmetries of curvature tensor, it is easy to observe that the foregoing equation is equivalent to

$$lX = -\kappa\varphi^2 X.$$

Making use of this in (5) and (6), we get $h^2 = (\kappa - 1)\varphi^2$ (where $Trl = 2n\kappa$) and $\nabla_\xi h = 0$. Moreover, the last equation implies that

$$\nabla_\xi h^2 = h(\nabla_\xi h) + (\nabla_\xi h)h = 0$$

and hence by (7), $\xi Trl = -\xi Trh^2 = 0 = \xi\kappa$. Next, by hypothesis we have $V = f\xi$ and V is non-zero. Therefore, f is non-zero on M. Taking covariant derivative of this along an arbitrary vector field X and using (4) we obtain $\nabla_X V = (Xf)\xi - f(\varphi X + \varphi hX)$. By virtue of these equations, the soliton equation (1) becomes

$$2S(X, Y) + \{(Xf)\eta(Y) + (Yf)\eta(X)\} + 2fg(h\varphi X, Y) + 2\lambda g(X, Y) = 0. \tag{9}$$

Substituting $X = Y = \xi$ in equation (9) and recalling (7), we get

$$Trl + \lambda + \xi f = 0. \tag{10}$$

Contracting equation (9) we also have

$$r + (2n + 1)\lambda + \xi f = 0. \tag{11}$$

Combining this with (10) yields

$$r - Trl + 2n\lambda = 0. \tag{12}$$

Next, substituting $Y = \xi$ in equation (9) and using (10) it follows that

$$2Q\xi + (\lambda - Trl)\xi + Df = 0, \tag{13}$$

for all vector fields X in M and D is the gradient operator of g. Operating (13) by φ gives

$$2g(Q\varphi X, \xi) + \varphi Xf = 0. \tag{14}$$

Replacing X by φX and Y by φY in equation (9), we get

$$\varphi Q\varphi X + fh\varphi X + \lambda\varphi^2 X = 0, \tag{15}$$

for all vector field Y on M. Operating (15) by φ and then replacing X by φX shows that

$$QX - g(QX, \xi)\xi - \eta(X)Q\xi + (Trl)\eta(X)\xi - \lambda\varphi^2 X + fh\varphi X = 0. \tag{16}$$

Differentiating equation (16) along an arbitrary vector field Y, using (4) and then contracting the resulting equation over Y, taking into account equation (8) and $(div\varphi^2) = 0$ (follows from (3) and (4)), we get

$$\begin{aligned}&\frac{1}{2}(Xr) - g((\nabla_\xi Q)X, \xi) - g(Q\varphi X + Qh\varphi X, \xi) + f(Trh^2)\eta(X)\\&- \frac{1}{2}(\xi r)\eta(X) + (\xi Trl)\eta(X) + ((h\varphi X)f) + f\{g(QX, \xi) - 2n\eta(X)\} = 0.\end{aligned} \tag{17}$$

On the other hand, differentiating (13), using (4) and then applying the Poincare lemma, $g(\nabla_X Df, Y) = g(\nabla_Y Df, X)$, provides

$$\begin{aligned}&g((\nabla_X Q)\xi, Y) - g((\nabla_Y Q)\xi, X) - g((Q\varphi + Q\varphi h)X, Y) - g((\varphi Q + h\varphi Q)X, Y)\\&- (\lambda - Trl)g(\varphi X, Y) = \frac{1}{2}\{(XTrl)\eta(Y) - (YTrl)\eta(X).\end{aligned} \tag{18}$$

Now differentiating the first equation of (7) and applying (4) shows that

$$g((\nabla_X Q)\xi, \xi) = (XTrl) + 2g(Q\varphi X + Q\varphi hX, \xi). \tag{19}$$

Setting $Y = \xi$ in (18) and by virtue of (19) it follows that

$$g((\nabla_\xi Q)X, \xi) - g(Q\varphi X + Q\varphi hX, \xi) = \frac{1}{2}\{(XTrl) + (\xi Trl)\eta(X)\}.$$

Utilising this in (17) and using (12), we find

$$2g(Q\varphi X, \xi) - f(Trh^2)\eta(X) - f\{g(QX, \xi) - 2n\eta(X)\} - ((h\varphi)X)f = 0. \tag{20}$$

Substituting X by φX in (20) and recalling (14) gives

$$f((\varphi X)f) + 2((hX)f) - 2((\varphi^2 X)f) = 0. \tag{21}$$

Taking hX instead of X in (21), making use of $h^2 = (\kappa - 1)\varphi^2$ and then subtracting the resulting equation from (21) yields

$$f((\varphi X)f) + f((h\varphi X)f) - 2\kappa((\varphi^2 X)f) = 0. \tag{22}$$

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Next, replacing X by φX in (21), multiplying the resulting equation by f, we obtain

$$f^2((\varphi^2 X)f) + 2f((h\varphi X)f) + 2f((\varphi X)f) = 0.$$

Finally, subtracting the last equation from twice of (22) yields

$$(f^2 + 4\kappa)((\varphi^2 X)f) = 0. \tag{23}$$

We now prove that f is constant on M. First, we note that if $(\varphi^2 X)f = 0$, then by Lemma 1 it follows that f is constant on M. So we assume that f is not constant (equivalently $((\varphi^2 X)f) \neq 0$) in some open set N of M. Therefore, from (23) we see that $f^2 + 4\kappa = 0$ on N. Covariant differentiation of this equation along ξ and since $\xi\kappa = 0$ (proved earlier) we at once obtain $\xi f = 0$ (as f is non-zero). Consequently (10) shows that $\kappa(= \frac{Trl}{2n})$ is constant on N. This implies that $f^2(= -4\kappa)$ is constant on N, i.e. f is constant on N. Thus, we arrive at a contradiction. Hence, f is constant on M. Therefore, equation (9) reduces to $QX + fh\varphi X + \lambda X = 0$ for all vector fields Y in M. Differentiating this equation along Y, contracting the resulting equation over Y and then recalling equation (8) we find $\frac{1}{2}(Xr) + f\{g(QX, \xi) - 2n\eta(X)\} = 0$. Since f is constant, r is also (follows from (11)) constant and hence the foregoing equation implies $Q\xi = 2n\xi$. This shows that M is K-contact and Einstein (see [**15**]) with $\lambda = -2n$. Making use of these in equation (9) we complete the proof of the first part. Now, if M is complete then using the result of Sharma [**15**] it is easy to see that M is compact, and from Boyer–Galicki's result [**5**], *a compact Einstein K-contact manifold is Sasakian*; we complete the proof.

Proof of Corollary 1*:* Since M is a Jacobi (κ, μ)-space, we see that $Trl(= 2n\kappa)$ is constant and $h^2 = (\kappa - 1)\varphi^2$. Hence the proof follows from Theorem 1.

Proof of Theorem 2*:* Since M admits a generalised Ricci soliton with potential vector field V collinear with ξ, we have from equation (9)

$$S(X, Y) + fg(h\varphi X, Y) + \lambda g(X, Y) = 0,$$

for all X, Y orthogonal to ξ. This is equivalent to (15) for all vector fields Y and for any vector field X. Hence, equation (16) also holds in this case. By hypothesis M is conformally flat. So we have

$$\begin{aligned} R(X, Y)Z = \frac{1}{2n-1}[\{g(QY, Z)X - g(QX, Z)Y \\ + g(Y, Z)QX - g(X, Z)QY\} - \frac{r}{2n}\{g(Y, Z)X - g(X, Z)Y\}]. \end{aligned} \tag{24}$$

Setting $Y = Z = \xi$ in (24) and recalling (7), gives

$$(2n-1)lX = QX + (Trl)X - g(QX, \xi)\xi - \eta(X)Q\xi + \frac{r}{2n}\varphi^2 X. \tag{25}$$

Feeding equation (16) into (25) yields

$$(2n-1)lX = (\lambda - Trl + \frac{r}{2n})\varphi^2 X - fh\varphi X. \tag{26}$$

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Now the contraction of equation (16) shows that $r - Trl + 2n\lambda = 0$. Through this equation, (26) reduces to

$$lX = -\kappa\varphi^2 - \frac{f}{2n-1}h\varphi X, \tag{27}$$

where $\kappa = \frac{Trl}{2n}$. Using (27) in (5) shows $h^2 = (\kappa - 1)\varphi^2$. By virtue of these equations, (27) and (6), we at once obtain $(2n-1)\nabla_\xi h = fh$. Next, we differentiate (27) along an arbitrary vector field Y and contract the resulting equation over Y with respect to an orthonormal frame $\{e_i : i = 1, 2, 3, ...\}$ to get

$$\begin{aligned}(divR)(X,\xi)\xi - g(R(X, \varphi e_i + \varphi h e_i)\xi, e_i) - g(R(X,\xi)(\varphi e_i + \varphi h e_i), e_i)\\ = -((\varphi^2 X)\kappa) - \frac{f}{2n-1}((h\varphi X)f) - \frac{f}{2n-1}\{g(QX,\xi) - 2n\eta(X)\},\end{aligned} \tag{28}$$

where we have used $(div\varphi^2) = 0$ and equation (8). As $C = 0$, we have $divC = 0$ or equivalently

$$g((\nabla_X Q)Y, Z) - g((\nabla_Y Q)X, Z) = \frac{1}{4n}\{(Xr)g(Y,Z) - (Yr)g(X,Z)\}. \tag{29}$$

Also, the contraction of the second Bianchi identity and equation (29) together implies

$$(divR)(X,\xi)\xi = \frac{1}{4n}\{(Xr) - (\xi r)\eta(X)\}. \tag{30}$$

Taking into account (24) we compute the following:

$$(2n-1)g(R(X, \varphi e_i + \varphi h e_i)\xi, e_i) = g(Q\varphi X + Q\varphi h X, \xi) - (TrQ\varphi h)\eta(X). \tag{31}$$

$$(2n-1)g(R(X,\xi)(\varphi e_i + \varphi h e_i), e_i) = 2g(Q\varphi X, \xi). \tag{32}$$

Making use of (30)–(32) in (28) and then replacing X by φX provides

$$\frac{2n-1}{4n}((\varphi X)r) - 3g(Q\varphi^2 X, \xi) - g(QhX,\xi) = ((\varphi X)\kappa) + ((hX)f) - fg(Q\varphi X, \xi). \tag{33}$$

Setting $Y = Z = \xi$ in equation (29) and using (19), we obtain

$$(XTrl) + 2g(Q\varphi X + Q\varphi h X, \xi) - g((\nabla_\xi Q)X, \xi) = \frac{1}{4n}\{(Xr) - (\xi r)\eta(X)\}. \tag{34}$$

Subtracting (34) from equation (16) (in this case equation (16) also holds) and then replacing X by φX it follows that

$$\frac{2n+1}{4n}((\varphi X)r) - ((\varphi X)Trl) - 3g(Q\varphi^2 X, \xi) - g(QhX,\xi) = ((hX)f) - fg(Q\varphi X, \xi). \tag{35}$$

Subtracting (35) from (33), using (12) and noting that $Trl = 2n\kappa$, it is immediate that $(\varphi X)Trl = 0$, as $n > 1$. Taking φX instead of X and remembering that $\xi Trl = 0$

shows $Trl = 2n\kappa$ is constant. Consequently, differentiating $h^2 = (\kappa - 1)\varphi^2$ along ξ gives $\nabla_\xi h^2 = 0$. On the other hand, we note that

$$0 = \nabla_\xi h^2 = h(\nabla_\xi h) + (\nabla_\xi h)h = \frac{2f}{2n-1}h^2.$$

Thus, we have $f(\kappa - 1)\varphi^2 = 0$. Differentiating this along an arbitrary vector field X and then contracting the resulting equation over X, we obtain $(\kappa - 1)((\varphi^2 X)f) = 0$, where we have used div $\varphi^2 = 0$. At this point, suppose that $\kappa \neq 1$. Then the last equation shows that $(\varphi^2 X)f = 0$. This implies that f is constant and since V is non-zero, f is non-zero constant on M and hence $\kappa = 1$, a contradiction. Thus, the only possibility is that $\kappa = 1$. This shows that M is K-contact and being conformally flat, by Tanno's theorem [**18**] it is of constant curvature $+1$, and hence Sasakian. This completes the proof.

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