RESULTS ON SOME SELECTED TYPES OF STARS

ASTEROSEISMOLOGY OF WHITE DWARF STARS

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Abstract. The theoretical potential of white dwarf asteroseismology is summarized. It is shown how one can derive fundamental parameters on the internal structure and evolution of these stars. The analysis of the non-radial g-modes permits in principle to determine the total mass, the rotation rate, the magnetic field strength. The mass of the outer layers, left on top of the carbon/oxygen core, can be determined as well as the structure of the transition zone between the core and the outer layers, giving an "a posteriori" unique information on the efficiency of the previous mass loss episodes. When measurable, the rate of change of the pulsation periods gives direct access to the evolutionary time scale and to the chemical composition of the core. These theoretical expectations are compared with the observations of variable white dwarfs in the three known instability strips for the planetary nebulae nuclei and PG1159 stars, for the DB and DA white dwarfs. Emphasis is put on results obtained from multi-sites photometric campaigns. Prospects on both theoretical developments and observations conclude the review.

1. Pulsating white dwarfs in the H-R diagram

At the end of their evolution, most of the stars become white dwarfs. Only massive stars, more massive than 6 to 8 $M_\odot$ do explode as supernovae. Less massive stars, after having exhausted their nuclear energy, cool down in radiating the thermal energy stored in their presumed carbon/oxygen core. They form in the H-R diagram a cooling sequence, going from the hot, luminous side, at an effective temperature exceeding 100 kK and a luminosity reaching $\log L/L_\odot = 4$, to the cool, faint end where they will eventually reach the invisibility limit when their internal temperature gets below the Debye temperature. In between these two extremes, they

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cross various instability strips, according to their effective temperature and chemical composition. These instability strips are precious windows which allow us to check many aspects of stellar evolution, dense matter physics, and galactic evolution.

Following the cooling sequence, the first instability strip to be found concerns the very hot stars of PG1159 type. Some of them are central stars of planetary nebulae still embedded in their nebulae while for others the nebulae is no longer visible. There is now a compelling evidence for an evolutionary link between the planetary nebulae nuclei and the white dwarfs. An interesting transition object is the hottest known pulsating PG1159 star, discovered during the ROSAT all sky survey, RXJ2117+3412 (Motch *et al.* 1993) whose very faint nebula was subsequently discovered (Appleton *et al.* 1993). But, another proportion of the hot white dwarfs comes from the evolution of the subdwarf stars, which evolve from the extended horizontal branch directly to the white dwarf sequence. What are the fractions of the white dwarfs coming from the Planetary nebulae or from the subdwarfs, and whether the final white dwarfs show any difference in mass and in structure is still not known. The asteroseismology of these objects may reveal some clues. Today one knows about 7 variable planetary nebulae nuclei (PNNV) and 4 variable PG1159 stars (also called DOV) and one transition object (RXJ2117+3412). The effective temperature range of the instability strip is quite large, from 150 kK for RXJ2117+3412, to 75 kK for PG0122+200 (Dreizler *et al.* 1995). Typical cooling time at this hot end of the cooling sequence is of the order of 10^6 years.

While both the PNNV and variable PG1159 stars show a very similar chemical atmospheric composition, made of a mixture of helium, carbon and oxygen, the chemical composition of the coolest white dwarfs differs significantly, with one helium rich family, the DB type white dwarfs, and one hydrogen rich family, the DA type white dwarfs. The origin of these two types is not entirely understood. But, because one PG1159 star was recently discovered to show hydrogen in its spectrum (HS2324+3944, Dreizler *et al.* 1996), it may be speculated that some of the PG1159 stars have still some hydrogen left in their atmosphere ("PG1159-Hybrid"), while the others have lost all their hydrogen in the previous phases of nuclear burning and mass loss. At the very beginning of the white dwarf sequence, diffusion processes become very efficient with typical time scales much shorter than the cooling time. PG1159 stars with hydrogen left would evolve into DA white dwarfs while those without hydrogen left would evolve into DBs. The following evolution is made more complex because of other intervening physical processes which may play a role in changing the atmospheric chemical composition: convective mixing, accretion, etc.

The DB white dwarfs are the next ones along the cooling sequence to cross an instability strip, at effective temperature around 23-25 kK. One knows 8 variable DB white dwarfs (DBV). At this effective temperature, the luminosity has decreased to about 10^{-1} $L_{\odot}$. The cooling time has increased to about 10^8 years.

The DA white dwarfs cross their instability strip at an effective temperature of about 11-12 kK. Their luminosity is of about 10^{-3} $L_{\odot}$ and their cooling time is of the order of 10^9 years. A total number of 26 variable DAs (DAV) is known (they are also called ZZ Cetis).

The instability mechanism triggering the pulsations in the three instability strips is the $\kappa - \gamma$ mechanism. In the PNNV and PG1159 stars, the instability is produced by the ionization of the heavy elements, mainly oxygen (Bradley and Dziembowski 1996, Saio 1996) following earlier suggestions by Starrfield and collaborators (Starrfield *et al.* 1983, 1984; Stanghellini *et al.* 1991). In the DBV the instability is related to the recombination of the helium (Winget *et al.* 1982b) while in the DAV it is related to the recombination of the hydrogen (Dolez and Vauclair 1981; Winget *et al.* 1982a).

Historically, the first variable white dwarf to be discovered was the DA HL Tau 76 (Landolt 1968), followed by the first PG1159 variable object PG1159-035 which gave its name to this new class of variable stars (McGraw *et al.* 1979). The last class of variable white dwarfs to be discovered, the DBV, is the only one which was theoretically predicted by pulsation theory previously to their observation (Winget *et al.* 1982b).

2. Non radial pulsations in white dwarfs

It is not my purpose to explain what non radial pulsations are to the participants of this symposium entirely devoted to helio- and asteroseismology. I just need to remind the very basic dispersion relation obtained after proceeding to all the simplifications one can think of and which takes the local form:

$$k^2 = \frac{1}{\sigma^2 v_s^2} \left(\sigma^2 - L_\ell^2\right) \left(\sigma^2 - N^2\right)$$

where L_ℓ is the Lamb frequency:

$$L_\ell^2 = \frac{\ell(\ell+1)v_s^2}{r^2}$$

N is the Brunt-Väisälä frequency:

$$N^2 = -g\left[\frac{d\ln\rho}{dr} + \frac{\rho g}{\Gamma_1 P}\right]$$

and k is the wave number of the mode, σ its frequency, and v_s the sound velocity.

The dispersion relation is useful to explain the differences between the non-radial modes in white dwarfs and in other types of stars. The non-radial modes can only propagate in those regions of the stars where the wave number k is real. This condition defines two types of waves and their related propagation regions. The modes whose frequencies are higher than both the Lamb and the Brunt-Väisälä frequencies are the p-modes, while those with lower frequencies are the g-modes. Because white dwarfs are high gravity, degenerate, stratified stars, their non-radial modes have some specific properties. The p-modes would have periods of the order of 1 s and less for higher harmonics: they are not observed in spite of serious and repeated efforts to detect them (Robinson 1984). The g-modes have periods in the range 100s<P<1500s. So, curiously, g-modes in white dwarfs have periods of the same order as p-modes in "normal", non degenerate stars.

In the degenerate core, the Brunt-Väisälä frequency drops to 0 with increasing depth. As a consequence, the g-modes can not propagate deep in the interior. Contrarily to "normal" stars where the g-modes give information on their deep interior, in white dwarfs they inform on their outer layers. The stratified chemical structure of the white dwarfs has interesting signature on the g-modes. It is predicted by stellar evolution that white dwarfs of average mass (about 0.6 $M_\odot$) have a C/O core. What surrounds this core depends on the previous evolutionary phases which are not yet satisfactorily understood. The outer layers are a mixture of He/C/O in PG1159 stars with some admixture of H in the PG1159-Hybrid stars, pure He in the DBs, while in DAs the He layers are surrounded by a pure H layer. Determining the mass of these remainings of previous evolution is one of the challenges of white dwarf asteroseismology. The chemical changes as a function of depth occur on short height scales, determined by the diffusion equilibrium. They produce discontinuities in the run of the Brunt-Väisälä frequency which in turn act as a selection mechanism between modes (trapping mechanism), as will be discussed below.

3. What can we learn from white dwarf asteroseismology?

If a large enough number of g-modes may be identified, one can derive some fundamental parameters for the star:

1 – the total stellar mass: the mass of the star may be derived from the period spacing. In the asymptotic regime, consecutive g-modes are regularly spaced in periods. For g-modes belonging to spherical harmonics of degree ℓ and order k, their periods are approximately given by:

$$P_{\ell,k} = \frac{\Pi_0}{\sqrt{\ell(\ell+1)}}\,(k+\varepsilon)$$

where

$$\Pi_0 = 2\Pi^2 \left(\int \frac{N}{r} dr \right)^{-1}$$

The integral has to be taken on the propagation zone. From this expression one deduces directly that because the Brunt-Väisälä frequency is proportional to the gravity, the period spacing Π_0 is inversely proportional to the gravity. As is well known, the mass and the radius are related for white dwarfs, a consequence of the degenerate equation of state. If a period spacing can be measured, it is a direct measurement of the mass. For the hot white dwarfs, there is a small dependence of Π_0 on the stellar luminosity, which reflects the variation of the propagation cavity as the star evolves along the cooling sequence: the degeneracy boundary moves towards the surface, pushing the propagation zone outwards; the integral of the Brunt-Väisälä frequency on the propagation region decreases as the star evolves; as a consequence Π_0 increases slightly for a given mass. For typical white dwarf masses, Π_0 varies between 38s for $0.5 M_\odot$ and 25s for 0.7 $M_\odot$, with a typical value of 30s for 0.6 $M_\odot$, the white dwarf average mass (Kawaler, Bradley 1994). Of course, what is measured from the observation is the period spacing between modes $P_{\ell,k}$. Deducing Π_0 requires the identification of ℓ.

2 – the mass of the outer layers: the mass of the layer surrounding the degenerate core may be derived from the deviation of the period distribution from a regular distribution. In a chemically homogeneous star, successive g- modes are regularly spaced in periods, as discussed above. In a chemically stratified star, the period distribution is slightly changed because of the discontinuities in chemical composition which reflect into discontinuities of the Brunt-Väisälä frequency. In white dwarfs, such transition zones between regions of different chemical composition exist because of the stratification induced by the gravitational settling. In PG1159 stars, there is such a transition zone between the He rich surface layer and the C/O core. In DB white dwarfs the transition occurs between the pure He outer layer and the core (C/O). In DA white dwarfs there must be two such transition regions, one between the pure H outer layer and the underlying helium layer, and the second one between the He and the C/O core.

Those modes which have one node of their eigenfunction in, or close to, one of these transition zones keep a negligible amplitude below the transition zone; these modes have an appreciable amplitude only above the transition zone. They are trapped in the outer layers. For this reason this mechanism is called mode trapping. Other modes which do not behave like trapped modes and do have an amplitude below the transition zone are called confined modes. The trapped modes and their neighbours in terms of k values are more closely spaced in period than the average period spacing Π_0. The reverse is true for the confined modes. In a plot of the period differences between successive modes as a function of the period for modes of a given ℓ, the trapped modes are easily identified because they correspond to minima.

Such $\Delta\Pi$ versus Π diagrams contain interesting information on the structure of the outer layers. On one hand, they present regularly spaced minima corresponding to successive trapped modes. The differences of period between successive trapped modes, or trapping cycle, is related to the depth of the transition zone; the trapping cycle measures the mass of the outer layers. On the other hand, the amplitude of the variation of $\Delta\Pi$, around the average value Π_0 is related to the gradient of mean molecular weight at the transition zone. It depends on the difference in chemical composition on the two sides of the transition and on its width. Discussion and examples of mode trapping predicted in various models may be found in Brassard *et al.* (1992a, b) and in Bradley (1993, 1996).

3 – the stellar luminosity: once the total mass is known from the period spacing, the mass of the outer layers from mode trapping, with the effective temperature derived from spectrophotometry, the stellar model is entirely determined: one can then derive the luminosity. As an interesting by-product, the distance can be estimated with a much larger accuracy than through other methods. For instance, the distance of the planetary nebulae whose central star is a pulsating objet, may be measured. This may have important implications on the distance scale problem in improving the planetary nebulae distance scale.

4 – the rotation period: this can be deduced from the rotational splitting. It is known that rotation removes the degeneracy of the modes of identical degree ℓ and order k but different azimuthal index m. In the limit of slow rotation, which proves to be a good assumption in the case of the white dwarfs, the frequency of one mode of indices ℓ, k, m in the rotating case is related to the frequency of the mode in the non-rotating case by:

$$\sigma_{k,\ell,m} = \sigma_{k,\ell} + m\,(1 - C_{k,\ell})\,\Omega + 0\left(\Omega^2\right)$$

where $\sigma_{k,\ell}$ is the frequency in the non-rotating case; $C_{k,\ell}$ comes from the Coriolis force term in the momentum equation and Ω is the rotation frequency. This is the classical development to the first order. In the asymptotic limit for g-modes, $C_{k,\ell}$ is simply related to the degree of the mode by:

$$C_{k,\ell} = \frac{1}{\ell(\ell+1)}$$

As m can take all values between $-\ell$ and $+\ell$, each mode of degree ℓ is split into $2\ell + 1$ components in the presence of rotation. The number of components allows the identification of ℓ, which in turn, allows the determination of the mass from the period spacing. The frequency separation between the components is a measure of the rotation period, once ℓ is unambiguously identified.

5 – the magnetic field: from the magnetic splitting. In a way similar to the rotation, the magnetic field removes the degeneracy of the modes. But, in contrast to the rotation, the magnetic field splits one mode into only $\ell + 1$ components. In addition, the frequency shift is proportional to m^2 (Jones *et al.* 1989).

6 – the evolutionary time scale: from the measurement of $\dot{\Pi}$.

As the white dwarfs evolve along the cooling sequence, their structure changes. The whole spectrum of their non-radial eigenvalues changes at the same rate. Roughly, the rate of change of the pulsation period, $\dot{\Pi}$, is related to the rates of change of the core temperature $\dot{T}_m$ and of the radius $\dot{R}$, by the relation (Winget *et al.* 1983):

$$\frac{\dot{\Pi}}{\Pi} \simeq -\frac{1}{2}\frac{\dot{T}_m}{T_m} + \frac{\dot{R}}{R}$$

The first term corresponds to the rate of change in period induced by the cooling of the white dwarf and is a positive contribution, while the second term corresponds to the rate of change induced by the contraction and is a negative contribution. In the PG1159 stars, still in the phase of final contraction towards the cooling sequence, both the cooling and the contraction are acting on $\dot{\Pi}$. The trapped modes, whose eigenfunctions are concentrated in the outer layers, are more sensitive to the contraction of these layers: their $\dot{\Pi}$ are negative. On the contrary, for the confined modes whose eigenfunctions are more sensitive to the region underneath the transition zone, the $\dot{\Pi}$ is dominated by the cooling, due, for a large domain of luminosity, to the neutrino emission: their $\dot{\Pi}$ is positive. In DBV and DAV, the influence of the contraction on $\dot{\Pi}$ has become negligible, $\dot{\Pi}$ is dominated by the cooling and is positive for all modes.

A measurement of $\dot{\Pi}$ is of major interest as it is a direct measurement of the cooling time which, in turn, depends on the chemical composition of the degenerate core. It is a direct test of the validity of the predictions of the stellar evolution theory. Would the $\dot{\Pi}$ for the confined modes in PG1159 stars be accessible to measurement, it would be a test on the neutrino physics. Measuring $\dot{\Pi}$ of the pulsating white dwarfs in the three instability strips allows us to calibrate the cooling sequence age. This should give the age of the galactic disk in the solar neighbourhood (Winget *et al.* 1987).

4. Observations vs. theory

The full use of the asteroseismological tools developed in the framework of the pulsation theory requires that: 1) one can detect as many modes as possible, and 2) one can identify these modes. The first condition implies that one is able to reach a sufficiently high S/N ratio to detect small amplitude modes. To achieve that goal requires having access to large telescopes or going to space. Achieving the second condition requires getting long, uninterrupted, time-series. This is possible from the ground with the organization of multi-site observational campaigns, or again from space.

From the ground, some time of relatively large telescope (i.e. the CFHT 3.60m at Mauna Kea) has been devoted to the asteroseismology of white dwarfs and has demonstrated the impact that the use of such a large telescope may have in this field (Fontaine *et al.* 1991, Fontaine *et al.* 1995). However, it is not yet possible to use network of large telescopes well distributed in longitude to carry on multi- site campaigns. The Whole Earth Telescope (WET; Nather *et al.* 1990) is a compromise to fulfill at the best the two required conditions. WET is an international collaboration which organizes one or two multi-site campaigns a year. Telescopes of various size (from 60cm up to 3.6m) well located around the Earth observe as simultaneously as possible the same target.

Some examples of the results obtained in the field of the white dwarf asteroseismology will be discussed below. Because of the space available, this is of course a selection.

1. PNNV and PG1159 stars

Recent reviews on that part of the H-R diagram may be found in Werner et al (1996) and Vauclair (1996). The observations of the PNNV have been mainly performed through CCD photometry, because of the needed correction from the nebulae brightness (Bond and Meakes 1990; Bond and Ciardullo 1993; Bond 1995) with the exception of K1-16 whose dilute nebulae allowed the discovery of its central star pulsations through photo- multiplier (PM) photometry (Grauer and Bond 1984; Grauer *et al.* 1992). The same is true for the transition object RXJ2117+3412, whose nebulae is in the process of dispersing into the interstellar matter: its pulsations were discovered with classical PM photometry (Watson 1992, Vauclair et al. 1993). For cooler pulsating PG1159 stars, as for RXJ2117+3412, WET campaigns have been organized, which use PM photometers (Winget *et al.* 1991; Kawaler *et al.* 1995). The main results on the prototype star of this group, PG1159-035, are now summarized. The full results of the 1989 WET campaign on PG1159-035 may be found in Winget *et al.* (1991). A total of 264h of high-speed photometry was accumulated on this star with an excellent coverage. The resulting window function, together with a frequency resolution of 1μHz, allowed the detection of about 125 frequencies in the power spectrum. Most of them are members of multiplets: triplets and quintuplets, which are interpreted as modes of degree $\ell = 1$ and 2, respectively, split by rotation. When averaging the triplets and the quintuplets separately, it is found that the ratio of the frequency shifts between the components of the triplets and those of the quintuplets is 0.60, in excellent agreement with the expected theoretical value for the ratio of split modes $\ell = 1$ and $\ell = 2$ which is 0.61 in the asymptotic regime. The deduced rotation period is 1.38 days, a slow enough rotation rate justifying the approximation used to calculate the rotational splitting. A search for period spacing reveals two values: 21.5s and 12.5s. The first value is interpreted as the period spacing for modes $\ell = 1$, while the second value corresponds to the period spacing for modes $\ell = 2$. Their ratio, 1.72, is also in excellent agreement with the theoretical expected value of 1.73, in the asymptotic regime. The total mass derived from these period spacings is $M/M_{\odot}= .586\pm.003$. Note the unusual accuracy on the mass determination of the asteroseismological method. The $\Delta\Pi$ vs. Π diagram shows the typical structure with minima, which is the signature of mode trapping induced by the chemical stratification. The fit with theoretical models leads to a fractional mass in the outer layers (above the C/O core) of $3 - 4 \times 10^{-3}$. From these values, one can derive the luminosity: log $L/L_{\odot}= 2.29 \pm0.05$, and a distance of 440 ± 40 pc.

The largest amplitude mode, with a period of 516s, has a measured time derivative of: $\dot{\Pi}/\Pi = (-4.8 \pm 0.1) \times 10^{-14}$ s^{-1}. This period corresponds to a trapped mode in the best fit model (Kawaler and Bradley 1994) with a period very close to the observed one. This is why $\dot{\Pi}$ is negative: this mode is sensitive to the still ongoing contraction of the outer layers. The characteristic contraction time measured by $\dot{\Pi}$ corresponds to 6.6×10^5 y. Unfortunately, the confined modes are expected to have much smaller amplitudes and the corresponding $\dot{\Pi}$ are not yet available.

2. DBVs

The prototype of this class of pulsating white dwarfs is GD358. It is also the best studied DBV. The results of the WET campaign on this object may be found in Winget at al. (1994). In this star, more than 180 modes were found, most of them identified as modes of degree $\ell = 1$. The analysis of the period spacing suggests a mass of 0.61 ± 0.03 $M_{\odot}$. From the deviation to the average period spacing (the $\Delta\Pi$ vs. Π diagram) the derived helium layer mass is determined as $M_{He}/M_* = 2.0\pm1.0\times10^{-6}$. From these values, a luminosity log $L/L_{\odot} = -1.30$ was derived, which leads to a distance of 42 ± 3 pc, in agreement with the value derived from the parallax. The rotational splitting reveals a non uniformity which was interpreted as a differential rotation within the outer layers, with its outer part rotating about 1.8 times faster than its most interior part. This fact, together with the unexpected low value of the Helium layer mass, were intriguing enough to motivate a second WET campaign, conducted 2 years after the first one, still under analysis.

3- DAVs

In the DAVs, as in the DBVs, the location of the instability strip in the H-R diagram depends on the adopted description of the convection and on its efficiency. A contrario, a precise observational location of the instability strip may be used as a constraint on the convection theory. The ZZ Ceti instability strip is better defined observationally owing to the larger number of known DAVs and to the greater accuracy in the atmospheric parameters for pure hydrogen atmospheres. For these reasons, efforts are continuously made to improve both the statistics by searching for new DAVs, and the accuracy of model atmospheres.

In contrast with the two cases discussed above, most of the DAVs show only a small number of modes compared to their potential rich non radial g-modes spectrum. This prevents any determination of the total stellar mass from the period spacing since there is not enough periods available to derive such a period spacing. For instance, G117-B15A, one of the best studied ZZ Ceti stars, has only three independent modes in its power spectrum. All the other power present in the spectrum are due to linear combinations of these three frequencies. The pulsations of this particular star are followed for almost the last 20 years because the short period (215 s) of its dominant mode and the relatively clean power spectrum around the corresponding frequency, were thought to offer a good opportunity to measure $\dot{\Pi}$. However, the interpretation of the $\dot{\Pi}$ measurement in such stars (Kepler *et al.* 1991) may not be trivial as there may be some other effects than the simple thermal cooling to take into account. The physical process responsible for the efficient mode selection could also affect both the amplitude and the frequency of the observed modes. From high S/N ratio data, Fontaine *et al.* (1995), following Brassard *et al.* 1995, have shown how a linear perturbation in the surface temperature induced by the g-modes propagation lead to non linear perturbations in the emergent flux. They interpret that way the occurrence of small amplitude peaks in the power spectrum corresponding to linear combinations of the fundamental three independent modes. This interpretation leads to a hydrogen mass of $M_H/M_* = 10^{-6}$. But in other cases, the non linear effects could be more fundamentally present in the star, involving for instance interactions between the modes themselves. In such cases, it is not known what would be the frequencies and amplitudes of the finally observed modes.

The mode selection could result either from the earlier discussed mode trapping or from the interaction of the pulsations with the convection, or both. Mode trapping has been thoroughly explored by Brassard *et al.* (1992a, b) and Bradley (1996) in DA models. The "natural" assumption is that the observed largest amplitude modes are trapped modes. However, in some cases, in the best fit models, i.e. in the models where the calculated eigenvalues do correspond the best to the observed frequencies, the modes which are observed with the largest amplitude do not correspond to trapped modes. The other physical process able to act as a selection mechanism is the interaction with convection. Gautschy *et al.* (1996) have recently explored this effect for the ZZ Cetis and found that a large number of modes which would be otherwise unstable, are stable if one takes into account the interaction with convection. Clearly, the question of the mode selection mechanism needs more work.

Closer to the ZZ Ceti instability red edge, GD154 is again a white dwarf which has very efficiently selected its pulsation modes: only three independent modes are identified in the data obtained during a dedicated WET campaign, as well as during subsequent observations (Pfeiffer *et al.* 1996). The much longer period of the dominant mode in GD154 (1186 s) is suggesting a smaller hydrogen envelope mass, $M_H/M_* = 2 \times 10^{-10}$, than in G117-B15A.

Determining the age of the white dwarfs along their cooling sequence is a way of measuring the age of the galactic disk in the solar vicinity. The coolest white dwarfs observed are not old enough to have reached the Debye temperature in their core, a limit at which an abrupt drop in the white dwarf luminosity function is predicted by evolutionary models (Wood 1992). As the lack of faint, cool white dwarfs does occur at a higher luminosity than the one which would correspond to the Debye cooling, it is concluded that these coolest white dwarfs do indicate the age of the galactic disk (Winget *et al.* 1987). However, this age is made rather uncertain as a consequence of our incomplete understanding of the crystallization phase which takes place at the cool end of the white dwarf sequence. The latent heat produced during the crystallization slows down the global cooling in a proportion which depends on the core composition and on how it crystallizes in details. The luminosity at which the crystallization starts does depend on the white dwarf mass: a massive white dwarf starts crystallization at higher luminosity than a smaller mass white dwarf. As discussed above, if one is able to measure through the asteroseismological method, i.e. $\dot{\Pi}$, the age of white dwarfs in the process of crystallization, one could infer how the crystallization proceeds and how much it slows the cooling rate. Unfortunately, the large majority of the white dwarfs have a mass around 0.6 $M_\odot$ Such white dwarfs crystallize at much cooler effective temperature than the red edge of the relevant instability strip. However, one DAV was recently found to be much more massive than the average white dwarf: BPM 37093

(Kanaan *et al.* 1992) has a mass in the range 1.0-1.1 $M_\odot$ (Bergeron *et al.* 1995). Such a massive DA is predicted to start crystallization within the ZZ Ceti instability strip (Winget, private communication). It will be of course a first priority target of a next WET campaign.

5. Conclusions and prospects

The asteroseismology of white dwarfs has been shown to be a powerful tool in the field of stellar physics. It also provides unique constraints in other fields like the physics of the galaxy and the dense matter physics.

In stellar physics, knowing accurately the mass of the white dwarfs and the mass of their remaining outer layers provides constraints on various processes occurring in the previous evolutionary phases, like the mass loss efficiency. It also helps understanding the evolutionary links between various kinds of objects in the final stages of stellar evolution. The rotation rates deduced from the asteroseismology of white dwarfs confirm that the white dwarfs are slow rotators. Together with a similar information coming from the spectroscopy of bright white dwarfs, this result is interesting in itself as it points to the question of the fate of the angular momentum during stellar evolution.

The age of white dwarfs, hopefully deduced from the determination of the cooling time, via the measurement of $\dot{\Pi}$, leads to a direct exploration of the core composition. It is a unique check of the stellar evolution theory. The age-calibration of the white dwarf cooling sequence gives us the age of the galactic disk in the solar vicinity. This is of course of importance for our understanding of the galaxy formation and evolution. The analysis of the crystallization phase should give information on the way crystallization takes place in dense, degenerate matter, a still controversial subject in the field of dense matter physics.

The problem of locating the instability strips in the H-R diagram and of understanding the mode selection mechanism is intimately related to some long standing questions linked to thermal convection: its efficiency and its interaction with pulsation.

Now, that a large amount of data starts to be available on the pulsation properties of white dwarfs, it becomes evident that all kinds of variable white dwarfs do exhibit time dependent variations of their amplitudes. These variations are clearly related to the details of the excitation mechanism and/or to the non linear effects. To extract the information implicitly hidden in these variations will require long term observations and theoretical developments. It will also require the development of new tools for data analysis, better adapted to the new physics involved, like time-frequency analysis, etc (Dolez *et al.* 1996).

The asteroseismology of white dwarfs will continue to be a very active and growing field in astrophysics, taking full advantage of ground-based networks, and of the availability of larger telescopes in a near future. In the same time, the need for a space mission dedicated to the asteroseismology which has been expressed in many occasions during this symposium, is largely demonstrated by the limit of what can be achieved from the ground; it would largely contribute to the progress in our understanding of the pulsating stars.

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